pgRouting Manual (2.6)

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pgRouting extends the PostGIS/PostgreSQL geospatial database to provide geospatial routing and other network analysis functionality.

This is the manual for pgRouting v2.6.2.



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General

Introduction¶

pgRouting is an extension of PostGIS and PostgreSQL geospatial database and adds routing and other network analysis functionality. A predecessor of pgRouting – pgDijkstra, written by Sylvain Pasche from Camptocamp, was later extended by Orkney and renamed to pgRouting. The project is now supported and maintained by Georepublic, iMaptools and a broad user community.

pgRouting is part of OSGeo Community Projects from the OSGeo Foundation and included on OSGeo Live.

Licensing¶ The following licenses can be found in pgRouting:

License	
GNU General Public License, version 2	Most features of pgRouting are available under GNU General Public License, version 2.
Boost Software License - Version 1.0	Some Boost extensions are available under Boost Software License - Version 1.0.
MIT-X License	Some code contributed by iMaptools.com is available under MIT-X license.
Creative Commons Attribution-Share Alike 3.0 License	The pgRouting Manual is licensed under a Creative Commons Attribution-Share Alike 3.0 License.

In general license information should be included in the header of each source file.

${\bf Contributors} \P$

This Release Contributors¶ Individuals (in alphabetical order)¶

Anthony Tasca, Virginia Vergara

And all the people that give us a little of their time making comments, finding issues, making pull requests etc.

Corporate Sponsors (in alphabetical order) \P

These are corporate entities that have contributed developer time, hosting, or direct monetary funding to the pgRouting project:

- Georepublic
- Google Summer of Code
- iMaptools
- Leopark
- Paragon Corporation
- Versaterm Inc.

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- Camptocamp
- CSIS (University of Tokyo)
- Georepublic
- Google Summer of Code
- iMaptools
- Orkney
- Paragon Corporation
- Versaterm Inc.

More Information¶

- The latest software, documentation and news items are available at the pgRouting web site http://pgrouting.org.
- PostgreSQL database server at the PostgreSQL main site http://www.postgresql.org.
- PostGIS extension at the PostGIS project web site http://postgis.net.
- Boost C++ source libraries at http://www.boost.org.
- Computational Geometry Algorithms Library (CGAL) at http://www.cgal.org.
- \bullet The Migration guide can be found at https://github.com/pgRouting/pgrouting/wiki/Migration-Guide.

Installation¶

Table of Contents

- Short Version
- Get the sources
- Enabling and upgrading in the database
- Dependencies
- Configuring

- Building
- Testing

Instructions for downloading and installing binaries for different Operative systems instructions and additional notes and corrections not included in this documentation can be found in Installation wiki

To use pgRouting postGIS needs to be installed, please read the information about installation in this Install Guide

Short Version¶ Extracting the tar ball

```
tar xvfz pgrouting-2.6.2.tar.gz
cd pgrouting-2.6.2
```

To compile assuming you have all the dependencies in your search path:

```
mkdir build
cd build
cmake ..
make
sudo make install
```

Once pgRouting is installed, it needs to be enabled in each individual database you want to use it in.

```
createdb routing
psql routing -c 'CREATE EXTENSION postGIS'
psql routing -c 'CREATE EXTENSION pgRouting'
```

 $\begin{tabular}{lll} \bf Get & the & sources \P & The & pgRouting & latest & release & can & be & found & in https://github.com/pgRouting/pgrouting/releases/latest & latest &$

wge

To download this release:

```
wget -0 pgrouting-2.6.2.tar.gz https://github.com/pgRouting/pgrouting/archive/v2.6.2.tar.
```

Goto Short Version to the extract and compile instructions.

git

To download the repository

```
git clone git://github.com/pgRouting/pgrouting.git
cd pgrouting
git checkout v2.6.2
```

Goto *Short Version* to the compile instructions (there is no tar ball).

Enabling and upgrading in the database¶ Enabling the database

pgRouting is an extension and depends on postGIS. Enabling postGIS before enabling pgRouting in the database

```
CREATE EXTENSION postgis;
CREATE EXTENSION pgrouting;
```

Upgrading the database

To upgrade pgRouting in the database to version 2.6.2 use the following command:

```
ALTER EXTENSION pgrouting UPDATE TO "2.6.2";
```

More information can be found in https://www.postgresql.org/docs/current/static/sql-createextension.html

Dependencies Compilation Dependencies

To be able to compile pgRouting, make sure that the following dependencies are met:

- C and C++0x compilers * g++ version \geq 4.8
- Postgresql version >= 9.3
- PostGIS version >= 2.2
- The Boost Graph Library (BGL). Version >= 1.53
- CMake >= 3.2
- CGAL >= 4.2

optional dependencies

For user's documentation

- Sphinx >= 1.1
- Latex

For developer's documentation

```
• Doxygen >= 1.7
```

```
For testing
```

```
• pgtap
  • pg_prove
Example: Installing dependencies on linux
Installing the compilation dependencies
Database dependencies
sudo apt-get install
    postgresql-10 \
    postgresql-server-dev-10 \
    postgresql-10-postgis
Build dependencies
sudo apt-get install
    cmake \
    g++ \
    libboost-graph-dev \
    libcgal-dev
Optional dependencies
For documentation and testing
\verb|sudo| apt-get| install -y | python-sphinx | |
    texlive \
    doxygen \
    libtap-parser-sourcehandler-pgtap-perl \
    postgresql-10-pgtap
```

Configuring¶ pgRouting uses the cmake system to do the configuration.

The build directory is different from the source directory

Create the build directory

\$ mkdir build

Configurable variables¶ To see the variables that can be configured

```
$ cd build
$ cmake -L ...
```

Configuring The Documentation

Most of the effort of the documentation has being on the HTML files. Some variables for the documentation:

Variable	Default	Comment
WITH_DOC	BOOL=OFF	Turn on/off building t
BUILD_HTMI	LBOOL=ON	If ON, turn on/off bui documentation
BUILD_DOXY	YBOOL=ON	If ON, turn on/off bui documentation
BUILD_LATE	ZXBOOL=OFF	If ON, turn on/off bui
BUILD_MAN	BOOL=OFF	If ON, turn on/off bui
DOC_USE_B	OBOTSTRAPF	If ON, use sphinx-bootusers documentation

Configuring with documentation

```
$ cmake -DWITH_DOC=ON ..
```

Note

Most of the effort of the documentation has being on the html files.

$\mathbf{Building}\P$ Using make to build the code and the documentation

The following instructions start from path/to/pgrouting/build

We have tested on several platforms, For installing or reinstalling all the steps are needed.

Warning

The sql signatures are configured and build in the cmake command.

MinGW on Windows

```
$ mkdir build
$ cd build
$ cmake -G"MSYS Makefiles" ..
$ make
$ make install
```

Linux

The following instructions start from path/to/pgrouting

```
mkdir build
cd build
cmake ..
make
sudo make install
```

When the configuration changes:

```
rm -rf build
```

and start the build process as mentioned above.

 $\textbf{Testing}\P\quad \text{Currently there is no } \textbf{make test} \text{ and testing is done as follows}$

The following instructions start from path/to/pgrouting/

```
tools/testers/algorithm-tester.pl
createdb -U <user> __pgr__test__
sh ./tools/testers/pg_prove_tests.sh <user>
dropdb -U <user> __pgr__test__
```

See Also¶ Indices and tables

- Index
- Search Page

$\mathbf{Support}\P$

pgRouting community support is available through the pgRouting website, documentation, tutorials, mailing lists and others. If you're looking for *commercial support*, find below a list of companies providing pgRouting development and consulting services.

Reporting Problems¶ Bugs are reported and managed in an issue tracker. Please follow these steps:

- 1. Search the tickets to see if your problem has already been reported. If so, add any extra context you might have found, or at least indicate that you too are having the problem. This will help us prioritize common issues.
- 2. If your problem is unreported, create a new issue for it.
- 3. In your report include explicit instructions to replicate your issue. The best tickets include the exact SQL necessary to replicate a problem.
- 4. If you can test older versions of PostGIS for your problem, please do. On your ticket, note the earliest version the problem appears.
- 5. For the versions where you can replicate the problem, note the operating system and version of pgRouting, PostGIS and PostgreSQL.
- 6. It is recommended to use the following wrapper on the problem to pin point the step that is causing the problem.

```
SET client_min_messages TO debug;
  <your code>
SET client_min_messages TO notice;
```

Mailing List and GIS StackExchange¶ There are two mailing lists for pgRouting hosted on OSGeo mailing list server:

- User mailing list: http://lists.osgeo.org/mailman/listinfo/pgrouting-users
- Developer mailing list: http://lists.osgeo.org/mailman/listinfo/pgrouting-dev

For general questions and topics about how to use pgRouting, please write to the user mailing list.

You can also ask at GIS StackExchange and tag the question with pgrouting. Find all questions tagged with pgrouting under http://gis.stackexchange.com/questions/tagged/pgrouting or subscribe to the pgRouting questions feed.

Commercial Support¶ For users who require professional support, development and consulting services, consider contacting any of the following organizations, which have significantly contributed to the development of pgRouting:

Company	Offices in	Website
Georepublic	Germany, Japan	https://georepublic.info
iMaptools	United States	http://imaptools.com

Paragon	United States	http://www.paragoncorporation.com
Corporation		
Camptocamp	Switzerland, France	http://www.camptocamp.com

• Sample Data that is used in the examples of this manual.

Sample Data¶

The documentation provides very simple example queries based on a small sample network. To be able to execute the sample queries, run the following SQL commands to create a table with a small network data set.

Create table

```
CREATE TABLE edge_table (
    id BIGSERIAL,
   dir character varying,
   source BIGINT,
   target BIGINT,
    cost FLOAT,
   reverse_cost FLOAT,
   capacity BIGINT,
   reverse_capacity BIGINT,
    category_id INTEGER,
   reverse_category_id INTEGER,
   x1 FLOAT,
   y1 FLOAT,
   x2 FLOAT,
   y2 FLOAT,
    the_geom geometry
);
Insert data
INSERT INTO edge_table (
   category_id, reverse_category_id,
    cost, reverse_cost,
    capacity, reverse_capacity,
   x1, y1,
    x2, y2) VALUES
(3, 1,
         1, 1, 80, 130,
                                  0,
                                         2, 1),
                             2,
(3, 2,
         -1, 1, -1, 100,
                             2,
                                         3, 1),
                                  1,
(2, 1,
                                         4, 1),
        -1, 1, -1, 130,
                             3,
                                  1,
(2, 4,
        1, 1, 100, 50,
                             2,
                                  1,
                                         2, 2),
```

```
1,
         1, -1, 130, -1,
                                       3, 2),
(4, 2,
         1, 1, 50, 100,
                            0,
                                 2,
                                       1, 2),
(4, 1,
         1, 1, 50, 130,
                                       2, 2),
                                 2,
                            1,
         1, 1, 100, 130,
                                       3, 2),
(2, 1,
                            2,
                                 2,
                                2,
(1, 3,
         1, 1, 130, 80,
                            3,
                                       4, 2),
(1, 4,
         1, 1, 130, 50,
                            2,
                                2,
                                       2, 3),
         1, -1, 130, -1,
(1, 2,
                            3, 2,
                                       3, 3),
(2, 3,
         1, -1, 100, -1,
                            2, 3,
                                       3, 3),
(2, 4,
         1, -1, 100, -1,
                            3, 3,
                                       4, 3),
(3, 1,
         1, 1,
                 80, 130,
                            2,
                                 3,
                                       2, 4),
(3, 4,
         1, 1,
                 80, 50,
                            4,
                                 2,
                                       4, 3),
(3, 3,
                                       4, 2),
         1, 1,
                 80, 80,
                            4,
                                1,
         1, 1, 130, 100,
(1, 2,
                            0.5, 3.5, 1.99999999999,3.5),
(4, 1,
         1, 1, 50, 130,
                            3.5, 2.3, 3.5,4);
UPDATE edge_table SET the_geom = st_makeline(st_point(x1,y1),st_point(x2,y2)),
dir = CASE WHEN (cost>0 AND reverse_cost>0) THEN 'B' -- both ways
       WHEN (cost>0 AND reverse_cost<0) THEN 'FT' -- direction of the LINESSTRING
       WHEN (cost<0 AND reverse_cost>0) THEN 'TF' -- reverse direction of the LINESTRING
          ELSE '' END;
                                                      -- unknown
```

Topology

• Before you test a routing function use this query to create a topology (fills the source and target columns).

```
SELECT pgr_createTopology('edge_table',0.001);
```

Points of interest

- When points outside of the graph.
- Used with the with Points Family of functions functions.

```
CREATE TABLE pointsOfInterest(
   pid BIGSERIAL,
   x FLOAT,
   y FLOAT,
   edge_id BIGINT,
   side CHAR,
   fraction FLOAT,
   the_geom geometry,
   newPoint geometry
);
```

```
INSERT INTO pointsOfInterest (x, y, edge_id, side, fraction) VALUES
(1.8, 0.4, 1, '1', 0.4),
(4.2, 2.4, 15, 'r', 0.4),
(2.6, 3.2, 12, '1', 0.6),
(0.3, 1.8, 6, 'r', 0.3),
(2.9, 1.8, 5, 'l', 0.8),
(2.2, 1.7, 4, 'b', 0.7);
UPDATE pointsOfInterest SET the_geom = st_makePoint(x,y);
UPDATE pointsOfInterest
    SET newPoint = ST_LineInterpolatePoint(e.the_geom, fraction)
    FROM edge_table AS e WHERE edge_id = id;
Restrictions
  • Used with the pgr_trsp - Turn Restriction Shortest Path (TRSP) func-
     tions.
CREATE TABLE restrictions (
    rid BIGINT NOT NULL,
    to_cost FLOAT,
    target_id BIGINT,
    from_edge BIGINT,
    via_path TEXT
);
INSERT INTO restrictions (rid, to_cost, target_id, from_edge, via_path) VALUES
(1, 100, 7, 4, NULL),
(1, 100, 11, 8, NULL),
(1, 100, 10, 7, NULL),
(2, 4, 8, 3, 5),
(3, 100, 9, 16, NULL);
Categories
  • Used with the Flow - Family of functions functions.
CREATE TABLE categories (
    category_id INTEGER,
    category text,
    capacity BIGINT
);
```

```
INSERT INTO categories VALUES
(1, 'Category 1', 130),
(2, 'Category 2', 100),
(3, 'Category 3', 80),
(4, 'Category 4', 50);
*/
```

Vertex table

• Used in some deprecated signatures or deprecated functions.

```
-- TODO check if this table is still used

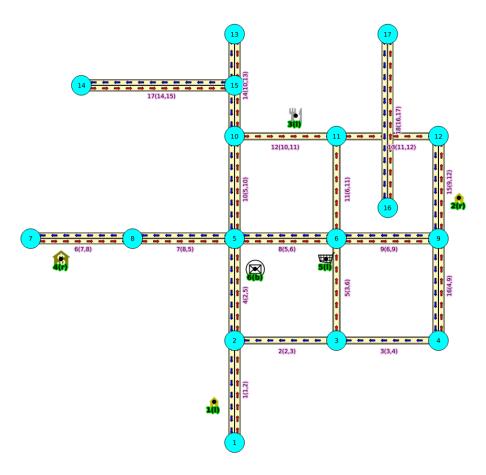
CREATE TABLE vertex_table (
    id SERIAL,
    x FLOAT,
    y FLOAT
);

INSERT INTO vertex_table VALUES
(1,2,0), (2,2,1), (3,3,1), (4,4,1), (5,0,2), (6,1,2), (7,2,2), (8,3,2), (9,4,2), (10,2,3), (11,3,3), (12,4,3), (13,2,4);
```

$Images\P$

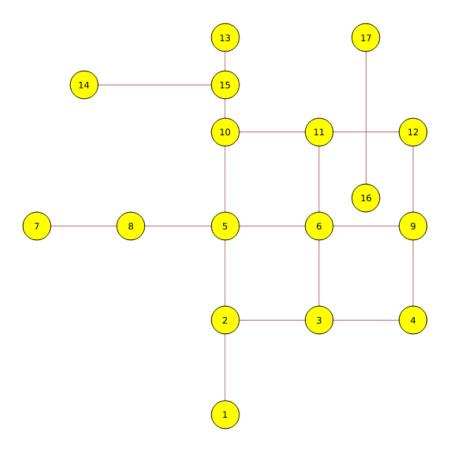
- Red arrows correspond when cost > 0 in the edge table.
- Blue arrows correspond when $reverse_cost > 0$ in the edge table.
- Points are outside the graph.
- Click on the graph to enlarge.

Network for queries marked as directed and cost and reverse_cost columns are used¶ When working with city networks, this is recommended for point of view of vehicles.



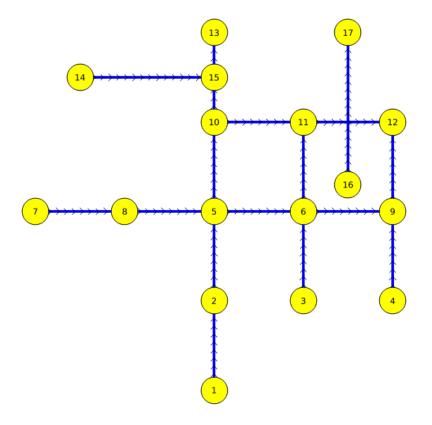
Graph 1: Directed, with cost and reverse cost

Network for queries marked as undirected and cost and reverse_cost columns are used \P When working with city networks, this is recommended for point of view of pedestrians.



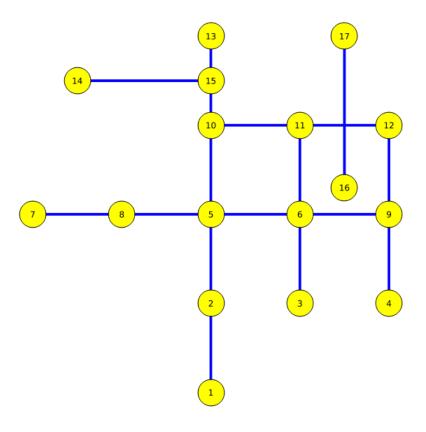
Graph 2: Undirected, with cost and reverse cost

Network for queries marked as directed and only cost column is used \P



Graph 3: Directed, with cost

Network for queries marked as undirected and only cost column is



$\mathbf{used}\P$

Graph 4: Undirected, with cost

Pick & Deliver Data¶

```
DROP TABLE IF EXISTS customer CASCADE;
CREATE table customer (
   id BIGINT not null primary key,
   x DOUBLE PRECISION,
   y DOUBLE PRECISION,
   demand INTEGER,
   opentime INTEGER,
   closetime INTEGER,
   servicetime INTEGER,
   pindex BIGINT,
   dindex BIGINT
);
```

```
INSERT INTO customer(
               y, demand, opentime, closetime, servicetime, pindex, dindex) VALUES
 id,
         х,
   0,
           40,
                  50,
                            Ο,
                                     Ο,
                                          1236,
                                                     0,
                                                            Ο,
                                                                    0),
(
   1,
           45,
                  68,
                         -10,
                                  912,
                                           967,
                                                    90,
                                                           11,
                                                                    0),
(
   2,
           45,
                  70,
                         -20,
                                  825,
                                           870,
                                                    90,
                                                            6,
                                                                    0),
                                                    90,
           42,
                  66,
                           10,
                                   65,
                                           146,
                                                            Ο,
                                                                   75),
(
   3,
   4,
           42,
                  68,
                         -10,
                                  727,
                                           782,
                                                    90,
                                                            9,
                                                                    0),
(
   5,
           42,
                  65,
                           10,
                                            67,
                                                    90,
                                                                    7),
                                   15,
                                                            0,
   6,
           40,
                  69,
                           20,
                                  621,
                                           702,
                                                    90,
                                                            Ο,
                                                                    2),
   7,
           40,
                  66,
                                                    90,
(
                         -10,
                                  170,
                                           225,
                                                            5,
                                                                    0),
(
   8,
           38,
                  68,
                           20,
                                  255,
                                           324,
                                                    90,
                                                            0,
                                                                   10),
                           10,
(
   9,
           38,
                  70,
                                  534,
                                           605,
                                                    90,
                                                                    4),
                                                            0,
(10,
           35,
                  66,
                         -20,
                                  357,
                                           410,
                                                    90,
                                                                    0),
                                                            8,
(11,
           35,
                  69,
                           10,
                                  448,
                                           505,
                                                    90,
                                                            0,
                                                                    1),
(12,
           25,
                  85,
                         -20,
                                  652,
                                           721,
                                                    90,
                                                           18,
                                                                    0),
                                                                  17),
(13,
           22,
                  75,
                           30,
                                   30,
                                            92,
                                                    90,
                                                            0,
           22,
                                                    90,
(14,
                  85,
                         -40,
                                  567,
                                           620,
                                                           16,
                                                                    0),
(15,
           20,
                  80,
                                  384,
                                           429,
                                                    90,
                                                           19,
                         -10,
                                                                    0),
(16,
           20,
                  85,
                           40,
                                  475,
                                           528,
                                                    90,
                                                            0,
                                                                   14),
(17,
           18,
                  75,
                          -30,
                                   99,
                                           148,
                                                    90,
                                                           13,
                                                                    0),
                           20,
                                  179,
                                           254,
                                                    90,
                                                            Ο,
(18,
           15,
                  75,
                                                                   12),
                                                    90,
(19,
           15,
                  80,
                           10,
                                  278,
                                           345,
                                                            0,
                                                                   15),
(20,
           30,
                  50,
                           10,
                                   10,
                                            73,
                                                    90,
                                                            0,
                                                                  24),
                                                                    0),
(21,
           30,
                  52,
                         -10,
                                  914,
                                           965,
                                                    90,
                                                           30,
(
  22,
           28,
                  52,
                         -20,
                                  812,
                                           883,
                                                    90,
                                                           28,
                                                                    0),
  23,
                                                            Ο,
(
           28,
                  55,
                           10,
                                  732,
                                           777,
                                                     0,
                                                                 103),
(24,
           25,
                  50,
                         -10,
                                   65,
                                           144,
                                                    90,
                                                           20,
                                                                    0),
                                                                  27),
(25,
           25,
                  52,
                           40,
                                  169,
                                           224,
                                                    90,
                                                            0,
           25,
(26,
                         -10,
                                           701,
                                                    90,
                  55,
                                  622,
                                                           29,
                                                                    0),
(27,
           23,
                  52,
                         -40,
                                  261,
                                           316,
                                                    90,
                                                           25,
                                                                    0),
(28,
           23,
                  55,
                           20,
                                  546,
                                           593,
                                                    90,
                                                            0,
                                                                  22),
(
  29,
           20,
                  50,
                           10,
                                  358,
                                           405,
                                                    90,
                                                            0,
                                                                   26),
           20,
                                                    90,
(
  30,
                  55,
                           10,
                                  449,
                                           504,
                                                            0,
                                                                  21),
(
  31,
           10,
                  35,
                          -30,
                                  200,
                                           237,
                                                    90,
                                                           32,
                                                                    0),
           10,
(32,
                  40,
                           30,
                                   31,
                                           100,
                                                    90,
                                                            0,
                                                                   31),
                                   87,
                                                                  37),
(33,
            8,
                  40,
                           40,
                                           158,
                                                    90,
                                                            0,
(34,
                                                    90,
            8,
                  45,
                         -30,
                                  751,
                                           816,
                                                           38,
                                                                    0),
(
  35,
            5,
                  35,
                           10,
                                  283,
                                           344,
                                                    90,
                                                            0,
                                                                   39),
            5,
(36,
                  45,
                           10,
                                           716,
                                                     0,
                                                            0,
                                  665,
                                                                 105),
            2,
(37,
                  40,
                         -40,
                                  383,
                                           434,
                                                    90,
                                                           33,
                                                                    0),
(38,
            0,
                  40,
                           30,
                                  479,
                                           522,
                                                    90,
                                                            0,
                                                                  34),
(39,
            0,
                  45,
                         -10,
                                  567,
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                                                    90,
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                         -20,
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149,

90,

0,

40),

68,

(42,

33,

32,

20,

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                                  448,
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                                                           44,
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                                                    90,
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                                                    90,
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                           30,
                                  385,
                                           436,
                                                    90,
                                                            0,
                                                                   59),
                   5,
(57,
           40,
                  15,
                           40,
                                   35,
                                            87,
                                                    90,
                                                            0,
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                         -20,
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                                                    90,
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                         -30,
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                         -40,
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(79,
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                           10,
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                                           731,
                                                    90,
                                                            Ο,
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(80,
                                  769,
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                                                    90,
           85,
                  25,
                         -10,
                                                           79,
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(81,
                                                    90,
           85,
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(85,
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                         -20,
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                                           612,
                                                    90,
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(87,
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                  55,
                           20,
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                         -10,
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                                                                    0),
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                                  285,
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(95,
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                                          726,
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                  30,
                         -10,
                                  725,
                                          786,
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                                                          51,
                                                                   0),
                                                                   0),
(102,
          48,
                  30,
                         -10,
                                  632,
                                          693,
                                                   90,
                                                          64,
(103,
          28,
                  55,
                         -10,
                                  732,
                                          777,
                                                   90,
                                                          23,
                                                                   0),
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          88,
                  35,
                         -20,
                                  109,
                                          170,
                                                   90,
                                                          78,
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                  45,
                         -10,
                                  665,
                                          716,
                                                   90,
                                                          36,
                                                                   0),
(106,
          60,
                  85,
                         -30,
                                  561,
                                          622,
                                                   90,
                                                          97,
                                                                   0);
```

Pgrouting Concepts¶

pgRouting Concepts¶

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Getting Started¶ This is a simple guide to walk you through the steps of getting started with pgRouting. In this guide we will cover:

- Create a routing Database
- Load Data
- Build a Routing Topology
- Check the Routing Topology
- Compute a Path

Create a routing Database¶ The first thing we need to do is create a database and load perouting in the database. Typically you will create a database for each project. Once you have a database to work in, your can load your data and build your application in that database. This makes it easy to move your project later if you want to to say a production server.

For Postgresql 9.2 and later versions

```
createdb mydatabase
psql mydatabase -c "create extension postgis"
psql mydatabase -c "create extension pgrouting"
```

 ${\bf Load~Data\P~~How~you~load~your~data~will~depend~in~what~form~it~comes~it.}$ There are various OpenSource tools that can help you, like:

osm2pgrouting:

this is a tool for loading OSM data into postgresql with pgRouting requirements

shp2pgsql:

• this is the postgresql shapefile loader

ogr2ogr:

• this is a vector data conversion utility

osm2pgsql:

• this is a tool for loading OSM data into postgresql

So these tools and probably others will allow you to read vector data so that you may then load that data into your database as a table of some kind. At this point you need to know a little about your data structure and content. One easy way to browse your new data table is with pgAdmin3 or phpPgAdmin.

Build a Routing Topology¶ Next we need to build a topology for our street data. What this means is that for any given edge in your street data the ends of that edge will be connected to a unique node and to other edges that are also connected to that same unique node. Once all the edges are connected to nodes we have a graph that can be used for routing with pgrouting. We provide a tool that will help with this:

Note

this step is not needed if data is loaded with osm2pgrouting

```
select pgr_createTopology('myroads', 0.000001);
```

• pgr_createTopology

Check the Routing Topology¶ There are lots of possible sources for errors in a graph. The data that you started with may not have been designed with routing in mind. A graph has some very specific requirements. One is that it is *NODED*, this means that except for some very specific use cases, each road segment starts and ends at a node and that in general is does not cross another road segment that it should be connected to.

There can be other errors like the direction of a one-way street being entered in the wrong direction. We do not have tools to search for all possible errors but we have some basic tools that might help.

- pgr_analyzeGraph
- pgr_analyzeOneway
- pgr_nodeNetwork

Compute a Path¶ Once you have all the preparation work done above, computing a route is fairly easy. We have a lot of different algorithms that can work with your prepared road network. The general form of a route query is:

```
select pgr_dijkstra(`SELECT * FROM myroads', 1, 2)
```

As you can see this is fairly straight forward and you can look and the specific algorithms for the details of the signatures and how to use them. These results have information like edge id and/or the node id along with the cost or geometry for the step in the path from *start* to *end*. Using the ids you can join these result back to your edge table to get more information about each step in the path.

• pgr_dijkstra

Inner Queries¶

- Description of the edges_sql query for dijkstra like functions
- Description of the edges_sql query (id is not necessary)
- Description of the parameters of the signatures
- Description of the edges_sql query for a tar like functions
- Description of the edges_sql query for Max-flow like functions
- Description of the Points SQL query

There are several kinds of valid inner queries and also the columns returned are depending of the function. Which kind of inner query will depend on the function(s) requirements. To simplify variety of types, ANY-INTEGER and ANY-NUMERICAL is used.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the edges_sql query for dijkstra like functions¶ edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGI	ER	Identifier of the edge.
source	ANY-INTEGI	ER	Identifier of the first end point vertex edge.
target	ANY-INTEGI	ER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMER:	ICAL	Weight of the edge (source, target)
			• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	cant-numer:	ICAL	Weight of the edge (target, source),
			• When negative: edge (target, so not exist, therefore it's not part graph.

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

$\label{lem:constraint} \textbf{Description of the edges_sql query (id is not necessary)} \P \quad \text{edges_sql:}$

an SQL query, which should return a set of rows with the following columns: $\,$

Column	Type Default	Description
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point ver- edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)

• When negative: edge (source, to not exist, therefore it's not part graph.

Column	Type	Default	Description
reverse_c	cant-numer	ICIAL	Weight of the edge (target, source),
			• When negative: edge (target, so not exist, therefore it's not part graph.

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Parameter	Туре
edges_sql	TEXT
via_vertices	ARRAY[ANY-INTEGER]
directed	BOOLEAN
strict	BOOLEAN
$U_turn_on_edge$	BOOLEAN

Description of the parameters of the signatures \P

Description of the edges_sql query for a star like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vertedge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
mayanga	CANA NUMEDICAL	• When negative: edge (source, to not exist, therefore it's not part graph.
reverse_	_c ant -numerical	 Weight of the edge (target, source), When negative: edge (target, so not exist, therefore it's not part graph.
x 1	ANY-NUMERICAL	X coordinate of source vertex.
y1	ANY-NUMERICAL	Y coordinate of source vertex.
x2	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the edges_sql query for Max-flow like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathrm{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.

Column	Type	Default	Description
target	ANY-INTEG	ER	Identifier of the second end point ver
capacity	ANY-INTEG	ER	Weight of the edge (source, target)
			• When negative: edge (source, t does not exist, therefore it's no the graph.
reverse_c	ca4pNevcit.NyTEG	ERI	Weight of the edge (target, source),
			• When negative: edge (target, so does not exist, therefore it's no the graph.

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

$\begin{tabular}{ll} \textbf{Description of the Points SQL query} \P & points_sql: \\ \end{tabular}$

an SQL query, which should return a set of rows with the following columns: $\,$

Column	Type	Description				
pid	ANY-INTEGER (optional) Identifier of the point.					
		 If column present, it can not be NULL. If column not present, a sequential identifier will be given automatically. 				
${\bf edge_id}$ ANY-INTEGERIdentifier of the "closest" edge to the point.						
fraction ANY-NUMERICWalue in $<0,1>$ that indicates the relative postition from the first end point of the edge.						
side	CHAR	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is:				
		 In the right, left of the edge or If it doesn't matter with 'b' or NULL. If column not present 'b' is considered. 				

Where:

ANY-INTEGER:

smallint, int, bigint

ANY-NUMERICAL:

smallint, int, bigint, real, float

Return columns & values \P

- Description of the return values for a path
- Description of the return values for a Cost function
- Description of the Return Values

There are several kinds of columns returned are depending of the function.

Description of the return values for a path¶ Returns set of (seq, path_seq[, start_vid][, end_vid], node, edge, cost, agg_cost)

Column	Type
\overline{seq}	INT
path _id	INT
path_seq	INT
start_vid	BIGINT
564 0_71d	DIGINI
${ m end_vid}$	BIGINT
node	BIGINT
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

Description of the return values for a Cost function¶ Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vidBIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vert ending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

Column	Type	Description
$\overline{\text{seq}}$	INT	Sequential value starting from 1.
$edge_id$	BIGINT	Identifier of the edge in the original query(edges_sql).
source	BIGINT	Identifier of the first end point vertex of the edge.
target	BIGINT	Identifier of the second end point vertex of the edge.
flow	BIGINT	Flow through the edge in the direction (source, target).
residual_ca	pad i tgint	Residual capacity of the edge in the direction (source, target).

Description of the Return Values¶

Advanced Topics \P

- Routing Topology
- Graph Analytics
- Analyze a Graph
- Analyze One Way Streets
 - Example

Routing Topology¶ Overview

Typically when GIS files are loaded into the data database for use with pgRouting they do not have topology information associated with them. To create a useful topology the data needs to be "noded". This means that where two or more roads form an intersection there it needs to be a node at the intersection and all the road segments need to be broken at the intersection, assuming that

you can navigate from any of these segments to any other segment via that intersection.

You can use the *graph analysis functions* to help you see where you might have topology problems in your data. If you need to node your data, we also have a function $pgr_nodeNetwork()$ that might work for you. This function splits ALL crossing segments and nodes them. There are some cases where this might NOT be the right thing to do.

For example, when you have an overpass and underpass intersection, you do not want these noded, but pgr_nodeNetwork does not know that is the case and will node them which is not good because then the router will be able to turn off the overpass onto the underpass like it was a flat 2D intersection. To deal with this problem some data sets use z-levels at these types of intersections and other data might not node these intersection which would be ok.

For those cases where topology needs to be added the following functions may be useful. One way to prep the data for pgRouting is to add the following columns to your table and then populate them as appropriate. This example makes a lot of assumption like that you original data tables already has certain columns in it like <code>one_way</code>, <code>fcc</code>, and possibly others and that they contain specific data values. This is only to give you an idea of what you can do with your data.

```
ALTER TABLE edge_table

ADD COLUMN source integer,

ADD COLUMN target integer,

ADD COLUMN cost_len double precision,

ADD COLUMN cost_time double precision,

ADD COLUMN rcost_len double precision,

ADD COLUMN rcost_time double precision,

ADD COLUMN x1 double precision,

ADD COLUMN x1 double precision,

ADD COLUMN y1 double precision,

ADD COLUMN x2 double precision,

ADD COLUMN y2 double precision,

ADD COLUMN to_cost double precision,

ADD COLUMN to_cost double precision,

ADD COLUMN isolated integer;

SELECT pgr_createTopology('edge_table', 0.000001, 'the_geom', 'id');
```

The function $pgr_createTopology()$ will create the vertices_tmp table and populate the source and target columns. The following example populated the remaining columns. In this example, the fcc column contains feature class code and the CASE statements converts it to an average speed.

```
x2 = st_x(st_endpoint(the_geom)),
                       y2 = st_y(st_endpoint(the_geom)),
  cost_len = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]'),
 rcost_len = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]'),
 len_km = st_length_spheroid(the_geom, 'SPHEROID["WGS84", 6378137, 298.25728]')/1000.0,
  len_miles = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]')
              / 1000.0 * 0.6213712,
  speed_mph = CASE WHEN fcc='A10' THEN 65
                   WHEN fcc='A15' THEN 65
                   WHEN fcc='A20' THEN 55
                   WHEN fcc='A25' THEN 55
                   WHEN fcc='A30' THEN 45
                   WHEN fcc='A35' THEN 45
                   WHEN fcc='A40' THEN 35
                   WHEN fcc='A45' THEN 35
                   WHEN fcc='A50' THEN 25
                   WHEN fcc='A60' THEN 25
                   WHEN fcc='A61' THEN 25
                   WHEN fcc='A62' THEN 25
                   WHEN fcc='A64' THEN 25
                   WHEN fcc='A70' THEN 15
                   WHEN fcc='A69' THEN 10
                   ELSE null END,
  speed_kmh = CASE WHEN fcc='A10' THEN 104
                   WHEN fcc='A15' THEN 104
                   WHEN fcc='A20' THEN 88
                   WHEN fcc='A25' THEN 88
                   WHEN fcc='A30' THEN 72
                   WHEN fcc='A35' THEN 72
                   WHEN fcc='A40' THEN 56
                   WHEN fcc='A45' THEN 56
                   WHEN fcc='A50' THEN 40
                   WHEN fcc='A60' THEN 50
                   WHEN fcc='A61' THEN 40
                   WHEN fcc='A62' THEN 40
                   WHEN fcc='A64' THEN 40
                   WHEN fcc='A70' THEN 25
                   WHEN fcc='A69' THEN 15
                   ELSE null END;
-- UPDATE the cost information based on oneway streets
```

```
UPDATE edge_table SET
    cost_time = CASE
        WHEN one_way='TF' THEN 10000.0
        ELSE cost_len/1000.0/speed_kmh::numeric*3600.0
```

```
END,
rcost_time = CASE
WHEN one_way='FT' THEN 10000.0
ELSE cost_len/1000.0/speed_kmh::numeric*3600.0
END;
```

-- clean up the database because we have updated a lot of records

VACUUM ANALYZE VERBOSE edge_table;

Now your database should be ready to use any (most?) of the pgRouting algorithms.

Graph Analytics¶ Overview

It is common to find problems with graphs that have not been constructed fully noded or in graphs with z-levels at intersection that have been entered incorrectly. An other problem is one way streets that have been entered in the wrong direction. We can not detect errors with respect to "ground" truth, but we can look for inconsistencies and some anomalies in a graph and report them for additional inspections.

We do not current have any visualization tools for these problems, but I have used mapserver to render the graph and highlight potential problem areas. Someone familiar with graphviz might contribute tools for generating images with that.

Analyze a Graph¶ With *pgr_analyzeGraph* the graph can be checked for errors. For example for table "mytab" that has "mytab_vertices_pgr" as the vertices table:

```
SELECT pgr_analyzeGraph('mytab', 0.000002);
NOTICE: Performing checks, pelase wait...
NOTICE: Analyzing for dead ends. Please wait...
NOTICE: Analyzing for gaps. Please wait...
NOTICE: Analyzing for isolated edges. Please wait...
NOTICE: Analyzing for ring geometries. Please wait...
NOTICE: Analyzing for intersections. Please wait...
NOTICE:
                     ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:
                           Isolated segments: 158
NOTICE:
                                   Dead ends: 20028
NOTICE: Potential gaps found near dead ends: 527
NOTICE:
                      Intersections detected: 2560
NOTICE:
                             Ring geometries: 0
pgr_analyzeGraph
```

```
OK
(1 row)
```

(1 row)

In the vertices table "mytab_vertices_pgr":

- Deadends are identified by cnt=1
- Potencial gap problems are identified with chk=1.

```
SELECT count(*) as deadends FROM mytab_vertices_pgr WHERE cnt = 1;
deadends
-----
20028
(1 row)

SELECT count(*) as gaps FROM mytab_vertices_pgr WHERE chk = 1;
gaps
-----
527
```

For isolated road segments, for example, a segment where both ends are deadends. you can find these with the following query:

```
SELECT *
   FROM mytab a, mytab_vertices_pgr b, mytab_vertices_pgr c
   WHERE a.source=b.id AND b.cnt=1 AND a.target=c.id AND c.cnt=1;
```

If you want to visualize these on a graphic image, then you can use something like mapserver to render the edges and the vertices and style based on cnt or if they are isolated, etc. You can also do this with a tool like graphviz, or geoserver or other similar tools.

Analyze One Way Streets¶ *pgr_analyzeOneway* analyzes one way streets in a graph and identifies any flipped segments. Basically if you count the edges coming into a node and the edges exiting a node the number has to be greater than one.

This query will add two columns to the vertices_tmp table ein int and eout int and populate it with the appropriate counts. After running this on a graph you can identify nodes with potential problems with the following query.

The rules are defined as an array of text strings that if match the col value would be counted as true for the source or target in or out condition.

Example¶

Lets assume we have a table "st" of edges and a column "one_way" that might have values like:

- 'FT' oneway from the source to the target node.
- 'TF' oneway from the target to the source node.
- 'B' two way street.
- "- empty field, assume twoway.
- <NULL> NULL field, use two_way_if_null flag.

Then we could form the following query to analyze the oneway streets for errors.

Typically these problems are generated by a break in the network, the one way direction set wrong, maybe an error related to z-levels or a network that is not properly noded.

The above tools do not detect all network issues, but they will identify some common problems. There are other problems that are hard to detect because they are more global in nature like multiple disconnected networks. Think of an island with a road network that is not connected to the mainland network because the bridge or ferry routes are missing.

Performance Tips¶

- For the Routing functions
- For the topology functions:

For the Routing functions¶ To get faster results bound your queries to the area of interest of routing to have, for example, no more than one million rows.

Use an inner query SQL that does not include some edges in the routing function

```
SELECT id, source, target from edge_table WHERE
    id < 17 and
    the_geom && (select st_buffer(the_geom,1) as myarea FROM edge_table where id = 5)</pre>
```

Integrating the inner query to the pgRouting function:

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target from edge_table WHERE
        id < 17 and
        the_geom && (select st_buffer(the_geom,1) as myarea FROM edge_table where id = 5)';
1, 2)</pre>
```

For the topology functions:¶ When "you know" that you are going to remove a set of edges from the edges table, and without those edges you are going to use a routing function you can do the following:

Analize the new topology based on the actual topology:

```
pgr_analyzegraph('edge_table',rows_where:='id < 17');</pre>
```

Or create a new topology if the change is permanent:

```
pgr_createTopology('edge_table',rows_where:='id < 17');
pgr_analyzegraph('edge_table',rows_where:='id < 17');</pre>
```

How to contribute¶ Wiki

- Edit an existing pgRouting Wiki page.
- Or create a new Wiki page
 - Create a page on the pgRouting Wiki
 - Give the title an appropriate name
- Example

Adding Functionaity to pgRouting

Consult the developer's documentation

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Reference

 $pgr_version$ - to get pgRouting's version information.

$pgr_version\P$

 $\mathbf{Name} \P \quad \mathsf{pgr_version} \longrightarrow \mathsf{Query} \text{ for pgRouting version information}.$

 ${\bf Synopsis}\P \quad {\bf Returns~a~table~with~pg} \\ {\bf Routing~version~information}.$

table() pgr_version();

Description ¶ Returns a table with:

Column	Type	Description
version	varchar	pgRouting version
tag	varchar	Git tag of pgRouting build
hash	varchar	Git hash of pgRouting build
branch	varchar	Git branch of pgRouting build
\mathbf{boost}	varchar	Boost version

History

• New in version 2.0.0

$Examples \P$

• Query for the version string

SELECT version FROM pgr_version();
version

2.6.2 (1 row)

See Also¶ Indices and tables

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Data Types¶

- pgr_costResult[] A set of records to describe a path result with cost attribute.
- pgr_costResult3[] A set of records to describe a path result with cost attribute.
- pgr_geomResult A set of records to describe a path result with geometry attribute.

pgRouting Data Types¶

The following are commonly used data types for some of the pgRouting functions.

- $pgr_costResult[]$ A set of records to describe a path result with cost attribute.
- pgr_costResult3[] A set of records to describe a path result with cost attribute.
- *pgr_geomResult* A set of records to describe a path result with geometry attribute.

$pgr_costResult[]$ ¶

Name¶ pgr_costResult[] — A set of records to describe a path result with cost attribute.

$\mathbf{Description}\P$

```
CREATE TYPE pgr_costResult AS
(
    seq integer,
    id1 integer,
    id2 integer,
    cost float8
);
```

```
seq:
sequential ID indicating the path order
id1:
generic name, to be specified by the function, typically the node id
id2:
generic name, to be specified by the function, typically the edge id
cost:
cost attribute
```

pgr_costResult3[] - Multiple Path Results with Cost¶

Name¶ pgr_costResult3[] — A set of records to describe a path result with cost attribute.

$\mathbf{Description}\P$

```
CREATE TYPE pgr_costResult3 AS

(
    seq integer,
    id1 integer,
    id2 integer,
    id3 integer,
    cost float8
);

seq:
sequential ID indicating the path order
id1:
generic name, to be specified by the function, typically the path id
id2:
generic name, to be specified by the function, typically the node id
id3:
generic name, to be specified by the function, typically the edge id
cost:
cost attribute
History
```

- New in version 2.0.0
- Replaces path_result

See Also \P

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$pgr_geomResult[]\P$

 $\label{lem:name} \textbf{Name} \P \quad \texttt{pgr_geomResult[]} \ -- \ A \ set \ of \ records \ to \ describe \ a \ path \ result \ with \ geometry \ attribute.$

${\bf Description}\P$

```
CREATE TYPE pgr_geomResult AS
(
    seq integer,
    id1 integer,
    id2 integer,
    geom geometry
);
seq:
sequential ID indicating the path order
id1:
generic name, to be specified by the function
id2:
generic name, to be specified by the function
geom:
geometry attribute
History
```

- New in version 2.0.0
- Replaces geoms

See Also¶

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See Also¶ Indices and tables

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$Topology Functions \P$

- $pgr_createTopology$ to create a topology based on the geometry.
- pgr_createVerticesTable to reconstruct the vertices table based on the source and target information.
- $pgr_analyzeGraph$ to analyze the edges and vertices of the edge table.
- pgr_analyzeOneway to analyze directionality of the edges.
- pgr_nodeNetwork -to create nodes to a not noded edge table.

Topology - Family of Functions¶

The pgRouting's topology of a network, represented with an edge table with source and target attributes and a vertices table associated with it. Depending on the algorithm, you can create a topology or just reconstruct the vertices table, You can analyze the topology, We also provide a function to node an unoded network.

- pgr_createTopology to create a topology based on the geometry.
- $\bullet \ pgr_createVerticesTable$ to reconstruct the vertices table based on the source and target information.
- pgr_analyzeGraph to analyze the edges and vertices of the edge table.
- pgr_analyzeOneway to analyze directionality of the edges.
- pgr_nodeNetwork -to create nodes to a not noded edge table.

$pgr_createTopology\P$

Name¶ pgr_createTopology — Builds a network topology based on the geometry information.

Synopsis¶ The function returns:

- OK after the network topology has been built and the vertices table created.
- FAIL when the network topology was not built due to an error.

Description¶ Parameters

The topology creation function accepts the following parameters:

edge_table:

text Network table name. (may contain the schema name AS well)

tolerance:

float8 Snapping tolerance of disconnected edges. (in projection unit)

the geom:

text Geometry column name of the network table. Default value is the geom.

id:

text Primary key column name of the network table. Default value is id.

source:

text Source column name of the network table. Default value is source.

target:

text Target column name of the network table. Default value is target.

rows_where:

text Condition to SELECT a subset or rows. Default value is true to indicate all rows that where source or target have a null value, otherwise the condition is used.

clean:

boolean Clean any previous topology. Default value is false.

Warning

The edge_table will be affected

- The source column values will change.
- The target column values will change.

- An index will be created, if it doesn't exists, to speed up the process to the following columns:
 - * id
 - * the_geom
 - * source
 - * target

The function returns:

- OK after the network topology has been built.
 - Creates a vertices table: <edge_table>_vertices_pgr.
 - $-% \frac{1}{2}$ Fills id and the $\mbox{\tt geom}$ columns of the vertices table.
 - Fills the source and target columns of the edge table referencing the id of the vertices table.
- FAIL when the network topology was not built due to an error:
 - A required column of the Network table is not found or is not of the appropriate type.
 - The condition is not well formed.
 - The names of source , target or id are the same.
 - The SRID of the geometry could not be determined.

The Vertices Table

The vertices table is a requirement of the $pgr_analyzeGraph$ and the $pgr_analyzeOneway$ functions.

The structure of the vertices table is:

id.

bigint Identifier of the vertex.

cnt:

<code>integer</code> Number of vertices in the edge_table that reference this vertex. See $pgr_analyzeGraph$.

chk:

integer Indicator that the vertex might have a problem. See *pgr_analyzeGraph*.

ein:

<code>integer</code> Number of vertices in the edge_table that reference this vertex AS incoming. See $pgr_analyzeOneway$.

eout:

integer Number of vertices in the edge_table that reference this vertex AS outgoing. See *pgr_analyzeOneway*.

the geom:

geometry Point geometry of the vertex.

History

• Renamed in version 2.0.0

Usage when the edge table's columns MATCH the default values:¶

The simplest way to use pgr createTopology is:

When the arguments are given in the order described in the parameters:

We get the same result AS the simplest way to use the function.

Warning

An error would occur when the arguments are not given in the appropriate order:

In this example, the column id of the table ege_table is passed to the function as the geometry column,

and the geometry column the geom is passed to the function as the id column.

When using the named notation

Parameters defined with a default value can be omitted, as long as the value matches the default And The order of the parameters would not matter.

```
SELECT pgr_createTopology('edge_table', 0.001,
    the_geom:='the_geom', id:='id', source:='source', target:='target');
    pgr_createtopology

OK
(1 row)

SELECT pgr_createTopology('edge_table', 0.001,
    source:='source', id:='id', target:='target', the_geom:='the_geom');
    pgr_createtopology

OK
(1 row)

SELECT pgr_createTopology('edge_table', 0.001, source:='source');
    pgr_createtopology

OK
(1 row)

OK
(1 row)
```

```
SELECT pgr_createTopology('edge_table', 0.001, rows_where:='id < 10');</pre>
pgr_createtopology
._____
OK
(1 row)
Selecting the rows where the geometry is near the geometry of row with id =
SELECT pgr_createTopology('edge_table', 0.001,
  rows_where:='the_geom && (SELECT st_buffer(the_geom, 0.05) FROM edge_table WHERE id=5)')
pgr_createtopology
OK
(1 row)
Selecting the rows where the geometry is near the geometry of the row with gid
=100 of the table othertable.
CREATE TABLE otherTable AS (SELECT 100 AS gid, st_point(2.5, 2.5) AS other_geom);
SELECT 1
SELECT pgr_createTopology('edge_table', 0.001,
  rows_where:='the_geom && (SELECT st_buffer(other_geom, 1) FROM otherTable WHERE gid=100)
pgr_createtopology
ΠK
(1 row)
Usage when the edge table's columns DO NOT MATCH the default
values:¶ For the following table
```

CREATE TABLE mytable AS (SELECT id AS gid, the geom AS mygeom, source AS src , target AS tgt H

Using positional notation:

SELECT 18

Selecting rows using rows_where parameter

Selecting rows based on the id.

The arguments need to be given in the order described in the parameters.

Note that this example uses clean flag. So it recreates the whole vertices table.

```
SELECT pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt', clean := TRUE);
pgr_createtopology
 OK
(1 row)
Warning
An error would occur when the arguments are not given in the appropriate
order:
In this example, the column gid of the table mytable is passed to the function
AS the geometry column,
and the geometry column mygeom is passed to the function AS the id column.
SELECT pgr_createTopology('mytable', 0.001, 'gid', 'mygeom', 'src', 'tgt');
NOTICE: PROCESSING:
NOTICE: pgr_createTopology('mytable', 0.001, 'gid', 'mygeom', 'src', 'tgt', rows_where := '
NOTICE: Performing checks, please wait .....
NOTICE: ----> PGR ERROR in pgr_createTopology: Wrong type of Column id:mygeom
NOTICE: Unexpected error raise_exception
pgr_createtopology
FATI.
(1 row)
When using the named notation
In this scenario omitting a parameter would create an error because the default
values for the column names do not match the column names of the table. The
order of the parameters do not matter:
SELECT pgr_createTopology('mytable', 0.001, the_geom:='mygeom', id:='gid', source:='src',
pgr_createtopology
 OK
(1 row)
SELECT pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt', the_{2}
pgr_createtopology
OK
(1 row)
```

Selecting rows using rows_where parameter

Based on id:

```
SELECT pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt', rows_where:='gi
 pgr_createtopology
 OK
(1 row)
SELECT pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt', the_g
 pgr_createtopology
 OK
(1 row)
SELECT pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt',
  rows_where:='mygeom && (SELECT st_buffer(mygeom, 1) FROM mytable WHERE gid=5)');
 pgr_createtopology
_____
 OK
(1 row)
SELECT pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt', the_g
  rows_where:='mygeom && (SELECT st_buffer(mygeom, 1) FROM mytable WHERE gid=5)');
 pgr_createtopology
 OK
(1 row)
Selecting the rows where the geometry is near the geometry of the row with gid
=100 of the table othertable.
SELECT pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt',
  rows_where:='mygeom && (SELECT st_buffer(other_geom, 1) FROM otherTable WHERE gid=100)')
 pgr_createtopology
_____
 OK
(1 row)
SELECT pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt', the_g
  rows_where:='mygeom && (SELECT st_buffer(other_geom, 1) FROM otherTable WHERE gid=100)')
 pgr_createtopology
 OK
(1 row)
```

Examples with full output¶ This example start a clean topology, with 5 edges, and then its incremented to the rest of the edges.

```
SELECT pgr_createTopology('edge_table', 0.001, rows_where:='id < 6', clean := true);
NOTICE: PROCESSING:
NOTICE: pgr_createTopology('edge_table', 0.001, 'the_geom', 'id', 'source', 'target', rows
NOTICE: Performing checks, please wait .....
NOTICE: Creating Topology, Please wait...
NOTICE: ----> TOPOLOGY CREATED FOR 5 edges
NOTICE: Rows with NULL geometry or NULL id: 0
NOTICE: Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
pgr_createtopology
OK
(1 row)
SELECT pgr_createTopology('edge_table', 0.001);
NOTICE: PROCESSING:
NOTICE: pgr_createTopology('edge_table', 0.001, 'the_geom', 'id', 'source', 'target', rows
NOTICE: Performing checks, please wait .....
NOTICE: Creating Topology, Please wait...
NOTICE: ----> TOPOLOGY CREATED FOR 13 edges
NOTICE: Rows with NULL geometry or NULL id: 0
NOTICE: Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
NOTICE: ----
pgr_createtopology
ΠK
(1 row)
```

The example uses the Sample Data network.

See Also \P

- Routing Topology for an overview of a topology for routing algorithms.
- $\bullet \ pgr_createVerticesTable$ to reconstruct the vertices table based on the source and target information.
- pgr_analyzeGraph to analyze the edges and vertices of the edge table.

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pgr createVerticesTable¶

Name¶ pgr_createVerticesTable — Reconstructs the vertices table based on the source and target information.

Synopsis¶ The function returns:

- OK after the vertices table has been reconstructed.
- FAIL when the vertices table was not reconstructed due to an error.

pgr_createVerticesTable(edge_table, the_geom, source, target, rows_where)
RETURNS VARCHAR

Description¶ Parameters

The reconstruction of the vertices table function accepts the following parameters:

edge_table:

text Network table name. (may contain the schema name as well)

the_geom:

text Geometry column name of the network table. Default value is the geom.

source:

text Source column name of the network table. Default value is source.

target:

text Target column name of the network table. Default value is target.

rows where:

 ${\tt text}$ Condition to SELECT a subset or rows. Default value is ${\tt true}$ to indicate all rows.

Warning

The edge_table will be affected

- An index will be created, if it doesn't exists, to speed up the process to the following columns:
 - the_geom
 - source
 - target

The function returns:

- OK after the vertices table has been reconstructed.
 - Creates a vertices table: <edge_table>_vertices_pgr.
 - Fills id and the_geom columns of the vertices table based on the source and target columns of the edge table.
- FAIL when the vertices table was not reconstructed due to an error.
 - A required column of the Network table is not found or is not of the appropriate type.
 - The condition is not well formed.
 - The names of source, target are the same.
 - The SRID of the geometry could not be determined.

The Vertices Table

The vertices table is a requierment of the $pgr_analyzeGraph$ and the $pgr_analyzeOneway$ functions.

The structure of the vertices table is:

id:

bigint Identifier of the vertex.

cnt:

integer Number of vertices in the edge_table that reference this vertex. See $pgr_analyzeGraph$.

chk:

integer Indicator that the vertex might have a problem. See *pgr_analyzeGraph*. ein:

integer Number of vertices in the edge_table that reference this vertex as incoming. See pgr_analyzeOneway.

eout:

integer Number of vertices in the edge_table that reference this vertex as outgoing. See *pgr_analyzeOneway*.

the geom:

geometry Point geometry of the vertex.

History

• Renamed in version 2.0.0

Usage when the edge table's columns MATCH the default values:¶

The simplest way to use pgr_createVerticesTable is:

```
SELECT pgr_createVerticesTable('edge_table');
```

When the arguments are given in the order described in the parameters:

```
SELECT pgr_createVerticesTable('edge_table','the_geom','source','target');
```

We get the same result as the simplest way to use the function.

Warning

An error would occur when the arguments are not given in the appropriate order: In this example, the column source column source of the table mytable is passed to the function as the geometry column, and the geometry column the geom is passed to the function as the source column.

When using the named notation

The order of the parameters do not matter:

```
SELECT pgr_createVerticesTable('edge_table',the_geom:='the_geom',source:='source',target

SELECT pgr_createVerticesTable('edge_table',source:='source',target:='target',the_geom:=
```

Parameters defined with a default value can be omitted, as long as the value matches the default:

```
SELECT pgr_createVerticesTable('edge_table',source:='source');
```

Selecting rows using rows_where parameter

Selecting rows based on the id.

```
SELECT pgr_createVerticesTable('edge_table',rows_where:='id < 10');</pre>
Selecting the rows where the geometry is near the geometry of row with id = 5
SELECT pgr_createVerticesTable('edge_table',rows_where:='the_geom && (select st_buffer(th
Selecting the rows where the geometry is near the geometry of the row with gid
=100 of the table othertable.
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS (SELECT 100 AS gid, st_point(2.5,2.5) AS other_geom);
SELECT pgr_createVerticesTable('edge_table',rows_where:='the_geom && (select st_buffer(ot
Usage when the edge table's columns DO NOT MATCH the default
values:¶ For the following table
DROP TABLE IF EXISTS mytable;
CREATE TABLE mytable AS (SELECT id AS gid, the geom AS mygeom, source AS src , target AS tgt FRO
Using positional notation:
The arguments need to be given in the order described in the parameters:
SELECT pgr_createVerticesTable('mytable','mygeom','src','tgt');
Warning
An error would occur when the arguments are not given in the appropriate
order: In this example, the column src of the table mytable is passed to the
function as the geometry column, and the geometry column mygeom is passed
to the function as the source column.
SELECT pgr_createVerticesTable('mytable','src','mygeom','tgt');
NOTICE: PROCESSING:
NOTICE: pgr_createVerticesTable('mytable','src','mygeom','tgt','true')
NOTICE: Performing checks, please wait .....
NOTICE: ---> PGR ERROR in pgr_createVerticesTable: Table mytable not found
         ----> Check your table name
HINT:
NOTICE: Unexpected error raise_exception
pgr_createverticestable
 FAIL
```

(1 row)

```
When using the named notation

The order of the parameters do not matter:

SELECT pgr_createVerticesTable('mytable',the_geom:='mygeom',source:='src',target:='tgt')

SELECT pgr_createVerticesTable('mytable',source:='src',target:='tgt',the_geom:='mygeom')

In this scenario omitting a parameter would create an error because the default
```

Selecting rows using rows_where parameter

Selecting rows based on the gid.

Selecting the rows where the geometry is near the geometry of the row with \mathtt{gid} =100 of the table othertable.

SELECT pgr_createVerticesTable('mytable','mygeom','src','tgt',

values for the column names do not match the column names of the table.

```
DROP TABLE IF EXISTS otherTable;

CREATE TABLE otherTable AS (SELECT 100 AS gid, st_point(2.5,2.5) AS other_geom);

SELECT pgr_createVerticesTable('mytable', 'mygeom', 'src', 'tgt',

rows_where:='the_geom && (SELECT st_buffer(othergeom,0.5) FROM otherTab

SELECT pgr_createVerticesTable('mytable', source:='src', target:='tgt', the_geom:='mygeom',
```

rows_where:='the_geom && (SELECT st_buffer(othergeom, 0.5) FROM otherTab

Examples¶

```
SELECT pgr_createVerticesTable('edge_table');
    NOTICE: PROCESSING:
NOTICE: pgr_createVerticesTable('edge_table','the_geom','source','target','true')
NOTICE: Performing checks, pelase wait .....
NOTICE: Populating public.edge_table_vertices_pgr, please wait...
          ---->
                  VERTICES TABLE CREATED WITH 17 VERTICES
NOTICE:
NOTICE:
                                               FOR
                                                    18 EDGES
NOTICE:
          Edges with NULL geometry, source or target: 0
NOTICE:
                                    Edges processed: 18
NOTICE: Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
    pgr_createVerticesTable
    -----
    OK
    (1 row)
```

The example uses the Sample Data network.

See Also¶

- Routing Topology for an overview of a topology for routing algorithms.
- pgr_createTopology to create a topology based on the geometry.
- $pgr_analyzeGraph$ to analyze the edges and vertices of the edge table.
- pgr_analyzeOneway to analyze directionality of the edges.

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$pgr_analyzeGraph\P$

Name¶ pgr_analyzeGraph — Analyzes the network topology.

Synopsis¶ The function returns:

- OK after the analysis has finished.
- FAIL when the analysis was not completed due to an error.

Description ¶ Prerequisites

The edge table to be analyzed must contain a source column and a target column filled with id's of the vertices of the segments and the corresponding vertices table <edge_table>_vertices_pgr that stores the vertices information.

- Use pgr_createVerticesTable to create the vertices table.
- Use pgr_createTopology to create the topology and the vertices table.

Parameters

The analyze graph function accepts the following parameters:

edge_table:

text Network table name. (may contain the schema name as well)

tolerance:

float8 Snapping tolerance of disconnected edges. (in projection unit)

the geom:

text Geometry column name of the network table. Default value is the geom.

id:

text Primary key column name of the network table. Default value is id.

source:

text Source column name of the network table. Default value is source.

target:

text Target column name of the network table. Default value is target.

rows_where:

text Condition to select a subset or rows. Default value is true to indicate all rows.

The function returns:

- OK after the analysis has finished.
 - Uses the vertices table: <edge_table>_vertices_pgr.
 - Fills completely the cnt and chk columns of the vertices table.
 - Returns the analysis of the section of the network defined by rows_where
- FAIL when the analysis was not completed due to an error.
 - The vertices table is not found.

- A required column of the Network table is not found or is not of the appropriate type.
- The condition is not well formed.
- The names of source , target or id are the same.
- The SRID of the geometry could not be determined.

The Vertices Table

The vertices table can be created with pgr_createVerticesTable or pgr_createTopology

The structure of the vertices table is:

id:

bigint Identifier of the vertex.

cnt:

integer Number of vertices in the edge_table that reference this vertex.

chk:

integer Indicator that the vertex might have a problem.

ein

integer Number of vertices in the edge_table that reference this vertex as incoming. See pgr_analyzeOneway.

eout:

<code>integer</code> Number of vertices in the edge_table that reference this vertex as outgoing. See $pgr_analyzeOneway$.

the_geom:

geometry Point geometry of the vertex.

History

• New in version 2.0.0

Usage when the edge table's columns MATCH the default values:¶

The simplest way to use pgr_analyzeGraph is:

```
SELECT pgr_createTopology('edge_table',0.001);
SELECT pgr_analyzeGraph('edge_table',0.001);
```

When the arguments are given in the order described in the parameters:

```
SELECT pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target');
```

We get the same result as the simplest way to use the function.

Warning

An error would occur when the arguments are not given in the appropriate order: In this example, the column id of the table mytable is passed to the function as the geometry column, and the geometry column the geom is passed to the function as the id column.

```
SELECT pgr_analyzeGraph('edge_table',0.001,'id','the_geom','source','target');
NOTICE: PROCESSING:
NOTICE: pgr_analyzeGraph('edge_table',0.001,'id','the_geom','source','target','true')
NOTICE: Performing checks, please wait ...
NOTICE: Got function st_srid(bigint) does not exist
NOTICE: ERROR: something went wrong when checking for SRID of id in table public.edge_table
pgr_analyzegraph
 FAIL
(1 row)
When using the named notation
The order of the parameters do not matter:
SELECT pgr_analyzeGraph('edge_table',0.001,the_geom:='the_geom',id:='id',source:='source
SELECT pgr_analyzeGraph('edge_table',0.001,source:='source',id:='id',target:='target',th
Parameters defined with a default value can be omitted, as long as the value
matches the default:
SELECT pgr_analyzeGraph('edge_table',0.001,source:='source');
Selecting rows using rows where parameter
Selecting rows based on the id. Displays the analysis a the section of the network.
SELECT pgr_analyzeGraph('edge_table',0.001,rows_where:='id < 10');</pre>
Selecting the rows where the geometry is near the geometry of row with id =5
SELECT pgr_analyzeGraph('edge_table',0.001,rows_where:='the_geom && (SELECT st_buffer(the
Selecting the rows where the geometry is near the geometry of the row with gid
=100 of the table othertable.
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS (SELECT 100 AS gid, st_point(2.5,2.5) AS other_geom);
SELECT pgr_analyzeGraph('edge_table',0.001,rows_where:='the_geom && (SELECT st_buffer(oth
```

Usage when the edge table's columns DO NOT MATCH the default values:¶ For the following table

```
DROP TABLE IF EXISTS mytable;
```

CREATE TABLE mytable AS (SELECT id AS gid, source AS src ,target AS tgt , the geom AS mygeom F. SELECT pgr_createTopology('mytable',0.001,'mygeom','gid','src','tgt');

Using positional notation:

The arguments need to be given in the order described in the parameters:

```
SELECT pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt');
```

Warning

An error would occur when the arguments are not given in the appropriate order: In this example, the column gid of the table mytable is passed to the function as the geometry column, and the geometry column mygeom is passed to the function as the id column.

FAIL (1 row)

When using the named notation

The order of the parameters do not matter:

```
SELECT pgr_analyzeGraph('mytable',0.001,the_geom:='mygeom',id:='gid',source:='src',targe
SELECT pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_geom:=
```

In this scenario omitting a parameter would create an error because the default values for the column names do not match the column names of the table.

Selecting rows using rows_where parameter

Selecting rows based on the id.

```
SELECT pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt',rows_where:='gid < 10
SELECT pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_geom:=
Selecting the rows WHERE the geometry is near the geometry of row with id
=5 .
SELECT pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt',
                  rows_where:='mygeom && (SELECT st_buffer(mygeom,1) FROM mytable WHERE gio
SELECT pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_geom:=
                  rows_where:='mygeom && (SELECT st_buffer(mygeom,1) FROM mytable WHERE gio
Selecting the rows WHERE the geometry is near the place='myhouse' of the
table othertable. (note the use of quote_literal)
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS (SELECT 'myhouse'::text AS place, st_point(2.5,2.5) AS other_geo
SELECT pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt',
      rows_where:='mygeom && (SELECT st_buffer(other_geom,1) FROM otherTable WHERE place='
SELECT pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_geom:=
     rows_where:='mygeom && (SELECT st_buffer(other_geom,1) FROM otherTable WHERE place='|
Examples \P
    SELECT pgr_createTopology('edge_table',0.001);
    SELECT pgr_analyzeGraph('edge_table', 0.001);
   NOTICE: PROCESSING:
  NOTICE: pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','true')
   NOTICE: Performing checks, pelase wait...
   NOTICE: Analyzing for dead ends. Please wait...
   NOTICE: Analyzing for gaps. Please wait...
   NOTICE: Analyzing for isolated edges. Please wait...
   NOTICE: Analyzing for ring geometries. Please wait...
   NOTICE: Analyzing for intersections. Please wait...
                         ANALYSIS RESULTS FOR SELECTED EDGES:
   NOTICE:
   NOTICE:
                               Isolated segments: 2
   NOTICE:
                                        Dead ends: 7
   NOTICE: Potential gaps found near dead ends: 1
    NOTICE:
                          Intersections detected: 1
   NOTICE:
                                 Ring geometries: 0
```

```
pgr_analyzeGraph
  OK
 (1 row)
SELECT pgr_analyzeGraph('edge_table',0.001,rows_where:='id < 10');</pre>
 NOTICE: PROCESSING:
NOTICE: pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','id < 10
 NOTICE: Performing checks, pelase wait...
 NOTICE: Analyzing for dead ends. Please wait...
 NOTICE: Analyzing for gaps. Please wait...
 NOTICE: Analyzing for isolated edges. Please wait...
 NOTICE: Analyzing for ring geometries. Please wait...
 NOTICE: Analyzing for intersections. Please wait...
 NOTICE:
                      ANALYSIS RESULTS FOR SELECTED EDGES:
 NOTICE:
                            Isolated segments: 0
 NOTICE:
                                    Dead ends: 4
 NOTICE: Potential gaps found near dead ends: 0
 NOTICE:
                       Intersections detected: 0
 NOTICE:
                              Ring geometries: 0
  pgr_analyzeGraph
  OK
 (1 row)
SELECT pgr_analyzeGraph('edge_table',0.001,rows_where:='id >= 10');
 NOTICE: PROCESSING:
NOTICE: pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','id >= 1
 NOTICE: Performing checks, pelase wait...
 NOTICE: Analyzing for dead ends. Please wait...
 NOTICE: Analyzing for gaps. Please wait...
 NOTICE: Analyzing for isolated edges. Please wait...
 NOTICE: Analyzing for ring geometries. Please wait...
 NOTICE: Analyzing for intersections. Please wait...
 NOTICE:
                      ANALYSIS RESULTS FOR SELECTED EDGES:
 NOTICE:
                            Isolated segments: 2
 NOTICE:
                                    Dead ends: 8
 NOTICE: Potential gaps found near dead ends: 1
 NOTICE:
                       Intersections detected: 1
 NOTICE:
                              Ring geometries: 0
  pgr_analyzeGraph
  OK
 (1 row)
```

```
-- Simulate removal of edges
  SELECT pgr_createTopology('edge_table', 0.001,rows_where:='id <17');</pre>
    SELECT pgr_analyzeGraph('edge_table', 0.001);
    NOTICE: PROCESSING:
  NOTICE: pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','true')
    NOTICE: Performing checks, pelase wait...
    NOTICE: Analyzing for dead ends. Please wait...
   NOTICE: Analyzing for gaps. Please wait...
   NOTICE: Analyzing for isolated edges. Please wait...
   NOTICE: Analyzing for ring geometries. Please wait...
   NOTICE: Analyzing for intersections. Please wait...
   NOTICE:
                         ANALYSIS RESULTS FOR SELECTED EDGES:
   NOTICE:
                               Isolated segments: 0
   NOTICE:
                                      Dead ends: 3
   NOTICE: Potential gaps found near dead ends: 0
   NOTICE:
                         Intersections detected: 0
   NOTICE:
                                Ring geometries: 0
    pgr_analyzeGraph
    ΠK
    (1 row)
SELECT pgr_createTopology('edge_table', 0.001,rows_where:='id <17');</pre>
NOTICE: PROCESSING:
NOTICE: pgr_createTopology('edge_table',0.001,'the_geom','id','source','target','id <17'
NOTICE: Performing checks, pelase wait .....
NOTICE: Creating Topology, Please wait...
NOTICE: ----> TOPOLOGY CREATED FOR 16 edges
NOTICE: Rows with NULL geometry or NULL id: 0
NOTICE: Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
    pgr_analyzeGraph
    ______
    OK
    (1 row)
SELECT pgr_analyzeGraph('edge_table', 0.001);
NOTICE: PROCESSING:
NOTICE: pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','true')
NOTICE: Performing checks, pelase wait...
NOTICE: Analyzing for dead ends. Please wait...
NOTICE: Analyzing for gaps. Please wait...
NOTICE: Analyzing for isolated edges. Please wait...
NOTICE: Analyzing for ring geometries. Please wait...
```

```
NOTICE:
        Analyzing for intersections. Please wait...
NOTICE:
                     ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:
                           Isolated segments: 0
NOTICE:
                                   Dead ends: 3
NOTICE: Potential gaps found near dead ends: 0
NOTICE:
                      Intersections detected: 0
NOTICE:
                             Ring geometries: 0
    pgr_analyzeGraph
     OK
    (1 row)
```

The examples use the Sample Data network.

See Also¶

- Routing Topology for an overview of a topology for routing algorithms.
- pgr_analyzeOneway to analyze directionality of the edges.
- pgr_createVerticesTable to reconstruct the vertices table based on the source and target information.
- pgr_nodeNetwork to create nodes to a not noded edge table.

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pgr_analyzeOneway¶

 ${\bf Name}\P$ pgr_analyze Oneway — Analyzes oneway S
streets and identifies flipped segments.

Synopsis¶ This function analyzes oneway streets in a graph and identifies any flipped segments.

Description¶ The analyses of one way segments is pretty simple but can be a powerful tools to identifying some the potential problems created by setting the direction of a segment the wrong way. A node is a source if it has edges the exit from that node and no edges enter that node. Conversely, a node is a sink if all edges enter the node but none exit that node. For a source type node it is logically impossible to exist because no vehicle can exit the node if no vehicle and enter the node. Likewise, if you had a sink node you would have an infinite number of vehicle piling up on this node because you can enter it but not leave it.

So why do we care if the are not feasible? Well if the direction of an edge was reversed by mistake we could generate exactly these conditions. Think about a divided highway and on the north bound lane one segment got entered wrong or maybe a sequence of multiple segments got entered wrong or maybe this happened on a round-about. The result would be potentially a source and/or a sink node.

So by counting the number of edges entering and exiting each node we can identify both source and sink nodes so that you can look at those areas of your network to make repairs and/or report the problem back to your data vendor.

Prerequisites

The edge table to be analyzed must contain a source column and a target column filled with id's of the vertices of the segments and the corresponding vertices table <edge_table>_vertices_pgr that stores the vertices information.

- Use $pgr_createVerticesTable$ to create the vertices table.
- Use $pgr_createTopology$ to create the topology and the vertices table.

```
Parameters
edge_table:

text Network table name. (may contain the schema name as well)
s_in_rules:

text[] source node in rules
s_out_rules:
text[] source node out rules
t_in_rules:
text[] target node in rules
t_out_rules:
text[] target node out rules
oneway:
```

text oneway column name name of the network table. Default value is oneway. source:

text Source column name of the network table. Default value is source.

target:

text Target column name of the network table. Default value is target.

two way if null:

boolean flag to treat oneway NULL values as bi-directional. Default value is true.

Note

It is strongly recommended to use the named notation. See *pgr_createVerticesTable* or *pgr_createTopology* for examples.

The function returns:

- OK after the analysis has finished.
 - Uses the vertices table: <edge_table>_vertices_pgr.
 - Fills completely the ein and eout columns of the vertices table.
- FAIL when the analysis was not completed due to an error.
 - The vertices table is not found.
 - A required column of the Network table is not found or is not of the appropriate type.
 - The names of source , target or oneway are the same.

The rules are defined as an array of text strings that if match the **oneway** value would be counted as **true** for the source or target **in** or **out** condition.

The Vertices Table

The vertices table can be created with pgr createVerticesTable or pgr createTopology

The structure of the vertices table is:

id:

bigint Identifier of the vertex.

cnt

integer Number of vertices in the edge_table that reference this vertex. See $pgr_analyzeGgraph$.

chk:

integer Indicator that the vertex might have a problem. See pgr_analyzeGraph.

ein:

integer Number of vertices in the edge_table that reference this vertex as incoming.

eout:

integer Number of vertices in the edge_table that reference this vertex as outgoing.

the_geom:

geometry Point geometry of the vertex.

History

• New in version 2.0.0

$Examples \P$

```
SELECT pgr_analyzeOneway('edge_table',
ARRAY['', 'B', 'TF'],
ARRAY['', 'B', 'FT'],
ARRAY['', 'B', 'FT'],
ARRAY['', 'B', 'TF'],
oneway:='dir');
NOTICE: PROCESSING:
NOTICE: pgr_analyzeGraph('edge_table','{"",B,TF}','{"",B,FT}','{"",B,FT}','{"",B,TF}','d
NOTICE: Analyzing graph for one way street errors.
NOTICE: Analysis 25% complete ...
NOTICE: Analysis 50% complete ...
NOTICE: Analysis 75% complete ...
NOTICE: Analysis 100% complete ...
NOTICE: Found 0 potential problems in directionality
pgr_analyzeoneway
OK
(1 row)
```

The queries use the Sample Data network.

See Also¶

 \bullet $\ \textit{Routing Topology}$ for an overview of a topology for routing algorithms.

- *Graph Analytics* for an overview of the analysis of a graph.
- pgr_analyzeGraph to analyze the edges and vertices of the edge table.
- pgr_createVerticesTable to reconstruct the vertices table based on the source and target information.

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pgr_nodeNetwork¶

Name¶ pgr_nodeNetwork - Nodes an network edge table.

Author:

Nicolas Ribot

Copyright:

Nicolas Ribot, The source code is released under the MIT-X license.

Synopsis¶ The function reads edges from a not "noded" network table and writes the "noded" edges into a new table.

pgr_nodenetwork(edge_table, tolerance, id, text the_geom, table_ending, rows_where, outall)
RETURNS TEXT

Description¶ A common problem associated with bringing GIS data into pgRouting is the fact that the data is often not "noded" correctly. This will create invalid topologies, which will result in routes that are incorrect.

What we mean by "noded" is that at every intersection in the road network all the edges will be broken into separate road segments. There are cases like an over-pass and under-pass intersection where you can not traverse from the over-pass to the under-pass, but this function does not have the ability to detect and accommodate those situations.

This function reads the edge_table table, that has a primary key column id and geometry column named the_geom and intersect all the segments in it against all the other segments and then creates a table edge_table_noded. It uses the tolerance for deciding that multiple nodes within the tolerance are considered the same node.

Parameters

```
edge table:
text Network table name. (may contain the schema name as well)
tolerance:
float8 tolerance for coincident points (in projection unit)dd
id:
text Primary key column name of the network table. Default value is id.
the geom:
text Geometry column name of the network table. Default value is the geom.
table_ending:
text Suffix for the new table's. Default value is noded.
The output table will have for edge_table_noded
bigint Unique identifier for the table
old id:
bigint Identifier of the edge in original table
sub id:
integer Segment number of the original edge
source:
integer Empty source column to be used with pgr_createTopology function
target:
integer Empty target column to be used with pgr_createTopology function
geometry Geometry column of the noded network
History
```

• New in version 2.0.0

Example¶ Let's create the topology for the data in *Sample Data*

```
SELECT pgr_createTopology('edge_table', 0.001);
NOTICE: PROCESSING:
NOTICE: pgr_createTopology('edge_table',0.001,'the_geom','id','source','target','true')
NOTICE: Performing checks, pelase wait .....
```

```
NOTICE: Creating Topology, Please wait...
NOTICE: ----> TOPOLOGY CREATED FOR 18 edges
NOTICE: Rows with NULL geometry or NULL id: 0
NOTICE: Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
        _____
NOTICE:
pgr_createtopology
OK
(1 row)
Now we can analyze the network.
SELECT pgr_analyzegraph('edge_table', 0.001);
NOTICE: PROCESSING:
NOTICE: pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','true')
NOTICE: Performing checks, pelase wait...
NOTICE: Analyzing for dead ends. Please wait...
NOTICE: Analyzing for gaps. Please wait...
NOTICE: Analyzing for isolated edges. Please wait...
NOTICE: Analyzing for ring geometries. Please wait...
NOTICE: Analyzing for intersections. Please wait...
NOTICE:
                    ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:
                          Isolated segments: 2
NOTICE:
                                  Dead ends: 7
NOTICE: Potential gaps found near dead ends: 1
NOTICE:
                     Intersections detected: 1
NOTICE:
                            Ring geometries: 0
pgr_analyzegraph
ΠK
(1 row)
The analysis tell us that the network has a gap and an intersection. We try to
fix the problem using:
SELECT pgr_nodeNetwork('edge_table', 0.001);
NOTICE: PROCESSING:
NOTICE: pgr_nodeNetwork('edge_table',0.001,'the_geom','id','noded')
NOTICE: Performing checks, pelase wait .....
NOTICE: Processing, pelase wait .....
NOTICE:
          Split Edges: 3
NOTICE:
         Untouched Edges: 15
NOTICE:
             Total original Edges: 18
         Edges generated: 6
NOTICE:
```

Untouched Edges: 15

NOTICE:

```
NOTICE: Total New segments: 21
NOTICE: New Table: public.edge_table_noded
NOTICE: ------
pgr_nodenetwork
OK
(1 row)
```

Inspecting the generated table, we can see that edges 13,14 and 18 has been segmented

SELECT old_id,sub_id FROM edge_table_noded ORDER BY old_id,sub_id;

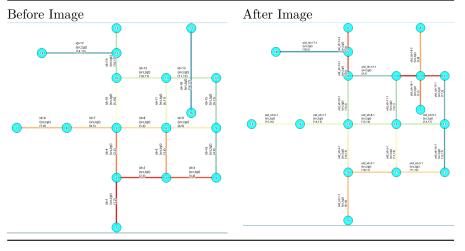
old_id	l sub_id
1	-+
1 2	1 1
3	
4	1
5	1
6	1
7	1
8	1
9	1
10	1 1
11	1
12	1
13	1
13	1 2
14	1
14	2
15	1
16	1
17	1
18	1
18	2
(21 row	
(21 10	

We can create the topology of the new network

```
SELECT pgr_createTopology('edge_table_noded', 0.001);
NOTICE: PROCESSING:
NOTICE: pgr_createTopology('edge_table_noded',0.001,'the_geom','id','source','target','t
NOTICE: Performing checks, pelase wait .....
NOTICE: Creating Topology, Please wait...
NOTICE: ------> TOPOLOGY CREATED FOR 21 edges
```

```
NOTICE: Vertices table for table public.edge_table_noded is: public.edge_table_noded_verti
NOTICE:
pgr_createtopology
OK
(1 row)
Now let's analyze the new topology
SELECT pgr_analyzegraph('edge_table_noded', 0.001);
NOTICE: PROCESSING:
NOTICE: pgr_analyzeGraph('edge_table_noded',0.001,'the_geom','id','source','target','tru
NOTICE: Performing checks, pelase wait...
NOTICE: Analyzing for dead ends. Please wait...
NOTICE: Analyzing for gaps. Please wait...
NOTICE:
        Analyzing for isolated edges. Please wait...
NOTICE: Analyzing for ring geometries. Please wait...
NOTICE:
        Analyzing for intersections. Please wait...
NOTICE:
                     ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:
                           Isolated segments: 0
NOTICE:
                                   Dead ends: 6
NOTICE:
        Potential gaps found near dead ends: 0
NOTICE:
                      Intersections detected: 0
NOTICE:
                             Ring geometries: 0
pgr_createtopology
 OK
(1 row)
```

NOTICE: Rows with NULL geometry or NULL id: 0



 ${\bf Images}\P$

Comparing the results \P Comparing with the Analysis in the original edge_table, we see that.

	Before	After
Table name	edge_table	edge_table_noded
Fields	All original fields	Has only basic fields to do a topology analysis
Dead ends	 Edges with 1 dead end: 1,6,24 Edges with 2 dead ends 17,18 	Edges with 1 dead end: 1-1 ,6-1,14-2, 18-1 17-1 18-2
	Edge 17's right node is a dead end because there is no other edge sharing that same node. (cnt=1)	
Isolated segments	two isolated segments: 17 and 18 both they have 2 dead ends	No Isolated segments • Edge 17 now shares a node with edges 14-1 and 14-2 • Edges 18-1 and 18-2 share a node with edges 13-1 and 13-2
Gaps	There is a gap between edge 17 and 14 because edge 14 is near to the right node of edge 17	Edge 14 was segmented Now edges: 14-1 14-2 17 share the same node The tolerance value was taken in account
Intersections	Edges 13 and 18 were intersecting	Edges were segmented, So, now in the interection's point there is a node and the following edges share it: 13-1 13-2 18-1 18-2

Now, we are going to include the segments 13-1, 13-2 14-1, 14-2 ,18-1 and 18-2 into our edge-table, copying the data for dir,cost,and reverse cost with the following steps:

- Add a column old_id into edge_table, this column is going to keep track the id of the original edge
- Insert only the segmented edges, that is, the ones whose $\max(\text{sub_id}) > 1$

```
alter table edge_table drop column if exists old_id;
alter table edge_table add column old_id integer;
insert into edge_table (old_id,dir,cost,reverse_cost,the_geom)
        (with
     segmented as (select old_id,count(*) as i from edge_table_noded group by old_id)
     select segments.old_id,dir,cost,reverse_cost,segments.the_geom
          from edge_table as edges join edge_table_noded as segments on (edges.id = segment
          where edges.id in (select old_id from segmented where i>1));
We recreate the topology:
SELECT pgr createTopology('edge table', 0.001);
NOTICE: PROCESSING:
NOTICE: pgr_createTopology('edge_table',0.001,'the_geom','id','source','target','true')
NOTICE: Performing checks, pelase wait .....
NOTICE: Creating Topology, Please wait...
NOTICE: ----> TOPOLOGY CREATED FOR 24 edges
NOTICE: Rows with NULL geometry or NULL id: 0 \,
NOTICE: Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
NOTICE: -----
pgr_createtopology
_____
OK
(1 row)
To get the same analysis results as the topology of edge_table_noded, we do
the following query:
SELECT pgr_analyzegraph('edge_table', 0.001,rows_where:='id not in (select old_id from edge
NOTICE: PROCESSING:
NOTICE: pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
                  'id not in (select old_id from edge_table where old_id is not null)')
NOTICE: Performing checks, pelase wait...
NOTICE: Analyzing for dead ends. Please wait...
NOTICE: Analyzing for gaps. Please wait...
NOTICE: Analyzing for isolated edges. Please wait...
NOTICE: Analyzing for ring geometries. Please wait...
NOTICE: Analyzing for intersections. Please wait...
NOTICE:
                    ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:
                          Isolated segments: 0
NOTICE:
                                  Dead ends: 6
NOTICE: Potential gaps found near dead ends: 0
NOTICE:
                     Intersections detected: 0
```

```
NOTICE:
                             Ring geometries: 0
pgr_createtopology
OK
(1 row)
To get the same analysis results as the original edge table, we do the following
query:
SELECT pgr_analyzegraph('edge_table', 0.001,rows_where:='old_id is null')
NOTICE: PROCESSING:
NOTICE: pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','old_id is
NOTICE: Performing checks, pelase wait...
NOTICE: Analyzing for dead ends. Please wait...
NOTICE: Analyzing for gaps. Please wait...
NOTICE: Analyzing for isolated edges. Please wait...
NOTICE: Analyzing for ring geometries. Please wait...
NOTICE: Analyzing for intersections. Please wait...
NOTICE:
                     ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:
                            Isolated segments: 2
                                    Dead ends: 7
NOTICE:
NOTICE: Potential gaps found near dead ends: 1
                      Intersections detected: 1
NOTICE:
NOTICE:
                              Ring geometries: 0
pgr_createtopology
OK
(1 row)
Or we can analyze everything because, maybe edge 18 is an overpass, edge 14
is an under pass and there is also a street level juction, and the same happens
with edges 17 and 13.
SELECT pgr_analyzegraph('edge_table', 0.001);
NOTICE: PROCESSING:
NOTICE: pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','true')
NOTICE: Performing checks, pelase wait...
NOTICE: Analyzing for dead ends. Please wait...
NOTICE: Analyzing for gaps. Please wait...
NOTICE: Analyzing for isolated edges. Please wait...
NOTICE: Analyzing for ring geometries. Please wait...
NOTICE: Analyzing for intersections. Please wait...
```

ANALYSIS RESULTS FOR SELECTED EDGES:

NOTICE:

```
NOTICE: Isolated segments: 0
NOTICE: Dead ends: 3
NOTICE: Potential gaps found near dead ends: 0
NOTICE: Intersections detected: 5
NOTICE: Ring geometries: 0
pgr_createtopology
-----
OK
(1 row)
```

See Also¶ Routing Topology for an overview of a topology for routing algorithms. $pgr_analyzeOneway$ to analyze directionality of the edges. $pgr_createTopology$ to create a topology based on the geometry. $pgr_analyzeGraph$ to analyze the edges and vertices of the edge table.

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See Also¶ Indices and tables

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$Routing\ functions \P$

Routing Functions¶

All Pairs - Family of Functions

- $pgr_floydWarshall$ Floyd-Warshall's Algorithm
- pgr_johnson- Johnson's Algorithm

```
pgr\_aStar - Shortest Path A* pgr\_bdAstar - Bi-directional A* Shortest Path pgr\_bdDijkstra - Bi-directional Dijkstra Shortest Path Dijkstra - Family of functions
```

- pgr_dijkstra Dijkstra's algorithm for the shortest paths.
- $pgr_dijkstraCost$ Get the aggregate cost of the shortest paths.

- pgr_dijkstraCostMatrix proposed Use pgr_dijkstra to create a costs matrix.
- pgr_drivingDistance Use pgr_dijkstra to calculate catchament information.
- pgr_KSP Use Yen algorithm with pgr_dijkstra to get the K shortest paths.
- $pgr_dijkstraVia$ Proposed Get a route of a seuence of vertices.

 pgr_KSP - K-Shortest Path

pgr_trsp - Turn Restriction Shortest Path (TRSP)

Traveling Sales Person - Family of functions

- pgr_TSP When input is given as matrix cell information.
- $pgr_eucledianTSP$ When input are coordinates.

Driving Distance - Category

- pgr_drivingDistance Driving Distance based on pgr_dijkstra
- pgr_withPointsDD Proposed Driving Distance based on pgr_withPoints
- · Post pocessing
 - pgr_alphaShape Alpha shape computation
 - pgr_pointsAsPolygon Polygon around a set of points

All Pairs - Family of Functions \P The following functions work on all vertices pair combinations

pgr_floydWarshall¶ Synopsis¶

pgr_floydWarshall - Returns the sum of the costs of the shortest path for each pair of nodes in the graph using Floyd-Warshall algorithm.



Boost Graph Inside

Availability: 2.0.0

• Renamed on 2.2.0, previous name pgr_apspWarshall

The Floyd-Warshall algorithm, also known as Floyd's algorithm, is a good choice to calculate the sum of the costs of the shortest path for each pair of nodes in the graph, for *dense graphs*. We use Boost's implementation which runs in (\hat{V}^3) time,

Characteristics

The main Characteristics are: • It does not return a path.

- Returns the sum of the costs of the shortest path for each pair of nodes in the graph.
- Process is done only on edges with positive costs.
- Boost returns a \(V \times V\) matrix, where the infinity values. Represent the distance between vertices for which there is no path.
 - We return only the non infinity values in form of a set of (start_vid, end_vid, agg_cost).
- Let be the case the values returned are stored in a table, so the unique index would be the pair: (start_vid, end_vid).
- For the undirected graph, the results are symmetric.
 - The agg_cost of (u, v) is the same as for (v, u).
- When start_vid = end_vid, the $agg_cost = 0$.
- Recommended, use a bounding box of no more than 3500 edges.

```
Signature Summary¶

pgr_floydWarshall(edges_sql)
pgr floydWarshall(edges_sql, directed)

RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET

Signatures¶

Minimal Signature¶

pgr_floydWarshall(edges_sql)
RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET

Example 1:

On a directed graph.

SELECT * FROM pgr_floydWarshall(
    'SELECT id, source, target, cost FROM edge_table where id < 5');
    start_vid | end_vid | agg_cost
```

```
1 | 2 | 1
1 | 5 | 2
2 | 5 | 1
(3 rows)
```

 $Complete\ Signature \P$

```
pgr_floydWarshall(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

Example 2:

On an undirected graph.

```
SELECT * FROM pgr_floydWarshall(
    'SELECT id, source, target, cost FROM edge_table where id < 5',
    false
);</pre>
```

${\tt start_vid}$		${\tt end_vid}$		agg_cost
	+-		+-	
1		2	1	1
1		5		2
2		1	1	1
2		5	1	1
5		1		2
5		2	1	1
(6 rows)				

Description of the Signatures \P

Description of the edges_sql query (id is not necessary)¶

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)

• When negative: edge (source, to not exist, therefore it's not part graph.

Column Type Default	Description
reverse_cant-numerical	Weight of the edge (target, source),
	• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Receives (edges_sql, directed)

ParameterType	Description
edges_sq T EXT	SQL query as described above.
directed BOOLEAN	(optional) Default is true (is directed). When set to false the graph is considered as ${\it Undirected}$

Description of the return values¶

Returns set of (start_vid, end_vid, agg_cost)

Column	Type	Description	
start_vid	BIGINT	Identifier of the starting vertex.	
$\mathbf{end}\mathbf{_vid}$	BIGINT	Identifier of the ending vertex.	
agg_cost	FLOAT	Total cost from start_vid to end_vid.	

History

- Re-design of pgr_apspWarshall in Version 2.2.0

See Also \P

- pgr_johnson
- Boost floyd-Warshall algorithm
- Queries uses the Sample Data network.

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pgr_johnson¶ Synopsis¶

pgr_johnson - Returns the sum of the costs of the shortest path for each pair of nodes in the graph using Floyd-Warshall algorithm.



Boost Graph Inside

Availability: 2.0.0

• Renamed on 2.2.0, previous name pgr_apspJohnson

The Johnson algorithm, is a good choice to calculate the sum of the costs of the shortest path for each pair of nodes in the graph, for *sparse graphs*. It usees the Boost's implementation which runs in $\langle O(V \to \log V) \rangle$ time,

Characteristics

The main Characteristics are: • It does not return a path.

- Returns the sum of the costs of the shortest path for each pair of nodes in the graph.
- Process is done only on edges with positive costs.
- Boost returns a $\setminus (V \setminus V)$ matrix, where the infinity values. Represent the distance between vertices for which there is no path.
 - We return only the non infinity values in form of a set of (start_vid, end_vid, agg_cost).
- Let be the case the values returned are stored in a table, so the unique index would be the pair: (start_vid, end_vid).
- For the undirected graph, the results are symmetric.
 - The agg_cost of (u, v) is the same as for (v, u).
- When start_vid = end_vid, the $agg_cost = 0$.

```
Signature Summary¶
pgr_johnson(edges_sql)
pgr johnson(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
Signatures¶
Minimal Signature¶
pgr_johnson(edges_sql)
RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
Example 1:
On a directed graph.
SELECT * FROM pgr_johnson(
    'SELECT source, target, cost FROM edge_table WHERE id < 5
        ORDER BY id'
);
 start_vid | end_vid | agg_cost
-----
        1 | 2 | 1
1 | 5 | 2
2 | 5 | 1
(3 rows)
Complete Signature¶
pgr_johnson(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
Example 2:
On an undirected graph.
SELECT * FROM pgr_johnson(
    'SELECT source, target, cost FROM edge_table WHERE id < 5
         ORDER BY id',
    false
);
start_vid | end_vid | agg_cost
        1 | 2 | 1
```

(6 rows)

Description of the Signatures \P

Description of the edges_sql query (id is not necessary) \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	_c any -numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Receives (edges_sql, directed)

ParameterType	Description
edges_sq t ext	SQL query as described above.

Parameter Type	Description
directed BOOLEAN	(optional) Default is true (is directed). When set to false the graph is considered as Undirected

Description of the return values \P

Returns set of (start_vid, end_vid, agg_cost)

Column	Type	Description
start_vid	BIGINT	Identifier of the starting vertex.
${\bf end_vid}$	BIGINT	Identifier of the ending vertex.
agg_cost	FLOAT	Total cost from start_vid to end_vid.

History

 - Re-design of pgr_apsp Johnson in Version 2.2.0

See Also \P

- $\bullet \quad pgr_floydWarshall$
- \bullet Boost Johnson algorithm implementation.
- Queries uses the Sample Data network.

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$\mathbf{Performance}\P$

The following tests: \bullet non server computer

- with AMD 64 CPU
- 4G memory
- trusty
- \bullet posgreSQL version 9.3

Data¶

The following data was used

```
BBOX="-122.8,45.4,-122.5,45.6"
wget --progress=dot:mega -0 "sampledata.osm" "http://www.overpass-api.de/api/xapi?*[bbox=]
Data processing was done with osm2pgrouting-alpha
createdb portland
psql -c "create extension postgis" portland
psql -c "create extension pgrouting" portland
osm2pgrouting -f sampledata.osm -d portland -s 0
Results¶
Test:
One
This test is not with a bounding box The density of the passed graph is extremely
low. For each <SIZE> 30 tests were executed to get the average The tested
query is:
SELECT count(*) FROM pgr_floydWarshall(
  'SELECT gid as id, source, target, cost, reverse_cost FROM ways where id <= <SIZE>');
SELECT count(*) FROM pgr_johnson(
  'SELECT gid as id, source, target, cost, reverse_cost FROM ways where id <= <SIZE>');
The results of this tests are presented as:
SIZE:
is the number of edges given as input.
EDGES:
is the total number of records in the query.
is the density of the data \( dfrac{E}{V \times (V-1)} \).
OUT ROWS:
is the number of records returned by the queries.
Floyd-Warshall:
is the average execution time in seconds of pgr floydWarshall.
Johnson:
is the average execution time in seconds of pgr_johnson.
```

			OUT		
SIZE	EDGES	DENSITY	ROWS	Floyd-Warshall	Johnson
500	500	0.18E-7	1346	0.14	0.13
1000	1000	0.36E-7	2655	0.23	0.18
1500	1500	0.55E-7	4110	0.37	0.34
2000	2000	0.73E-7	5676	0.56	0.37
2500	2500	0.89E-7	7177	0.84	0.51
3000	3000	1.07E-7	8778	1.28	0.68
3500	3500	1.24E-7	10526	2.08	0.95
4000	4000	1.41E-7	12484	3.16	1.24
4500	4500	1.58E-7	14354	4.49	1.47
5000	5000	1.76E-7	16503	6.05	1.78
5500	5500	1.93E-7	18623	7.53	2.03
6000	6000	2.11E-7	20710	8.47	2.37
6500	6500	2.28E-7	22752	9.99	2.68
7000	7000	2.46E-7	24687	11.82	3.12
7500	7500	2.64E-7	26861	13.94	3.60
8000	8000	2.83E-7	29050	15.61	4.09
8500	8500	3.01E-7	31693	17.43	4.63
9000	9000	3.17E-7	33879	19.19	5.34
9500	9500	3.35E-7	36287	20.77	6.24
10000	10000	3.52E-7	38491	23.26	6.51

Test:

Two

This test is with a bounding box The density of the passed graph higher than of the Test One. For each $\langle \text{SIZE} \rangle$ 30 tests were executed to get the average The tested edge query is:

WITH

buffer AS (SELECT ST_Buffer(ST_Centroid(ST_Extent(the_geom)), SIZE) AS geom FROM ways), bbox AS (SELECT ST_Envelope(ST_Extent(geom)) as box from buffer)
SELECT gid as id, source, target, cost, reverse_cost FROM ways where the_geom && (SELECT box)

The tested queries

```
SELECT count(*) FROM pgr_floydWarshall(<edge query>)
SELECT count(*) FROM pgr_johnson(<edge query>)
```

The results of this tests are presented as:

SIZE:

is the size of the bounding box.

EDGES:

is the total number of records in the query.

DENSITY:

is the density of the data $\(\dfrac{E}{V \times (V-1)}\)$.

OUT ROWS:

is the number of records returned by the queries.

Floyd-Warshall:

is the average execution time in seconds of pgr_floydWarshall.

Johnson:

is the average execution time in seconds of pgr_johnson.

			OUT	
SIZE	EDGES	DENSITY	ROWS	Floyd-W
0.001	44	0.0608	1197	0.10
0.002	99	0.0251	4330	0.10
0.003	223	0.0122	18849	0.12
0.004	358	0.0085	71834	0.16
0.005	470	0.0070	116290	0.22
0.006	639	0.0055	207030	0.37
0.007	843	0.0043	346930	0.64
0.008	996	0.0037	469936	0.90
0.009	1146	0.0032	613135	1.26
0.010	1360	0.0027	849304	1.87
0.011	1573	0.0024	1147101	2.65
0.012	1789	0.0021	1483629	3.72
0.013	1975	0.0019	1846897	4.86
0.014	2281	0.0017	2438298	7.08

SIZE	EDGES	DENSITY	OUT ROWS	Floyd-Wa
0.015	2588	0.0015	3156007	10.28
0.016	2958	0.0013	4090618	14.67
0.017	3247	0.0012	4868919	18.12

See Also¶

- pgr_johnson
- pgr_floydWarshall
- ullet Boost floyd-Warshall algorithm

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$pgr_bdAstar\P$

Name¶ pgr_bdAstar — Returns the shortest path using A* algorithm.



Boost Graph Inside

Availability:

- pgr_bdAstar(one to one) 2.0.0, Signature change on 2.5.0
- pgr_bdAstar(other signatures) 2.5.0

Signature Summary \P

```
pgr_bdAstar(edges_sql, start_vid, end_vid)
pgr_bdAstar(edges_sql, start_vid, end_vid, directed [, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq , node, edge, cost, agg_cost)
    OR EMPTY SET
```

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

```
pgr_bdAstar(edges_sql, start_vid, end_vids [, directed, heuristic, factor, epsilon])
pgr_bdAstar(edges_sql, start_vids, end_vid [, directed, heuristic, factor, epsilon])
pgr_bdAstar(edges_sql, start_vids, end_vids [, directed, heuristic, factor, epsilon])
```

RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost) OR EMPTY SET

Using these signatures, will load once the graph and perform several one to one pgr $\,$ bdAstar

- The result is the union of the results of the one to one pgr bdAStar.
- The extra start_vid and/or end_vid in the result is used to distinguish to which path it belongs.

Avaliability

- pgr_bdAstar(one to one) 2.0, signature change on 2.5
- $pgr_bdAstar(other signatures) 2.5$

Signatures Minimal Signature

```
pgr_bdAstar(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

This usage finds the shortest path from the start_vid to the end_vid on a directed graph

- with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example:

(6 rows)

Using the defaults

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
                             4 |
   1 |
              1 |
                      2 |
                                    1 |
   2 |
              2 |
                      5 I
                             8 |
                                    1 |
                                                3
              3 |
   3 |
                      6 |
                             9 |
                                    1 |
                                                5
   4 |
              4 |
                      9 |
                            16 |
                                    1 |
                                                8
                      4 |
   5 I
              5 I
                            3 |
                                    1 |
                                                9
   6 |
              6 I
                      3 |
                            -1 |
                                    0 |
                                               10
```

pgr_bdAstar One to One \P

```
pgr_bdAstar(edges_sql, start_vid, end_vid, directed [, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

This usage finds the shortest path from the start_vid to the end_vid allowing the user to cheuristic,

- and/or factor
- and/or epsilon.

Note

In the One to One signature, because of the deprecated signature existence, it is compulsory to indicate if the graph is **directed** or **undirected**.

Example:

Directed using Heuristic 2

```
SELECT * FROM pgr_bdAstar(
```

```
'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
   FROM edge_table',
   2, 3,
   true, heuristic := 2
);
 seq | path_seq | node | edge | cost | agg_cost
----+----
          1 | 2 | 4 | 1 |
          2 |
              5 | 8 | 1 |
  2 |
  3 I
          3 l
               6 l
                    9 |
                          1 |
                                   3
  4 |
          4 | 9 | 16 |
                         1 |
          5 |
                         1 |
                                   5
  5 I
                4 |
                    3 |
  6 |
           6 I
                3 |
                     -1 |
                           0 |
(6 rows)
```

pgr_bdAstar One to many¶

pgr_bdAstar(edges_sql, start_vid, end_vids [, directed, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET

This usage finds the shortest path from the start_vid to each end_vid in end_vids allowing t if the graph is directed or undirected

- and/or heuristic,
- and/or factor
- and/or epsilon.

Example:

Directed using Heuristic 3 and a factor of 3.5

```
SELECT * FROM pgr_bdAstar(
   'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
   FROM edge_table',
   2, ARRAY[3, 11],
   heuristic := 3, factor := 3.5
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
____+
           1 | 3 | 2 | 4 | 1 |
  2 |
           2 |
                  3 l
                       5 l
                             8 I
                                  1 |
                                         25.5
                             9 |
  3 |
           3 |
                  3 |
                        6 I
                                  1 |
                                         38.75
          4 |
                  3 l
                                   1 |
  4 |
                        9 |
                             16 |
                                         64.25
                            3 |
                                 1 |
  5 I
          5 I
                  3 l
                        4 |
                                         65.25
  6 |
          6 l
                  3 |
                        3 |
                             -1 | 0 |
                                         66.25
         1 |
                 11 | 2 | 4 | 1 |
  7 |
                                         0
```

```
2 |
                     11 |
                            5 |
                                 8 |
                                       1 |
  9 |
            3 I
                     11 |
                            6 l
                                 11 |
                                                   2
 10 |
            4 |
                     11 |
                                         0 |
                                                   3
                           11 |
                                 -1 |
(10 rows)
```

pgr_bdAstar Many to One¶

pgr_bdAstar(edges_sql, start_vids, end_vid [, directed, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET

This usage finds the shortest path from each start_vid in start_vids to the end_vid allowing if the graph is directed or undirected

- and/or heuristic,
- and/or factor
- and/or epsilon.

Example:

Undirected graph with Heuristic 4

```
SELECT * FROM pgr_bdAstar(
   'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
   FROM edge_table',
   ARRAY[2, 7], 3,
   false, heuristic := 4
);
seq | path_seq | start_vid | node | edge | cost |
____+___
  1 |
           1 |
                     2 |
                           2 |
                                2 |
                                      1 |
                                                      0
           2 |
                     2 |
                           3 |
                                -1 |
                                     0 |
  2 |
                                                      1
                    7 |
           1 |
                          7 |
                                6 I
  3 l
                                      1 |
                     7 |
                                7 |
  4 |
           2 |
                          8 I
                                      1 | 3.23606797749979
                     7 |
                                    1 | 5.65028153987288
  5 I
           3 l
                          5 I
                                8 I
  6 |
           4 |
                     7 | 6 |
                                5 | 1 | 6.65028153987288
                     7 |
  7 |
           5 |
                          3 |
                               -1 |
                                    0 | 7.65028153987288
(7 rows)
```

pgr_bdAstar Many to Many¶

pgr_bdAstar(edges_sql, start_vids, end_vids [, directed, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET

This usage finds the shortest path from each start_vid in start_vids to each end_vid in end_ if the graph is directed or undirected

- and/or heuristic,
- and/or factor
- and/or **epsilon**.

Example:

Directed graph with a factor of 0.5

```
SELECT * FROM pgr_bdAstar(
     'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
     ARRAY[2, 7], ARRAY[3, 11],
    factor := 0.5
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
   1 |
               1 |
                             2 |
                                        3 |
                                               2 |
                                                       4 |
                                                               1 |
                                                                           0
   2 |
               2 |
                             2 |
                                               5
                                                                           2
                                        3 |
                                                       8 |
                                                               1 |
   3 |
               3 |
                             2 |
                                        3 |
                                               6
                                                       9 |
                                                               1
                                                                         3.5
   4 |
               4
                             2 |
                                        3 I
                                               9
                                                      16 I
                                                               1 |
                                                                         4.5
               5 I
                             2 |
                                                                         5.5
   5 I
                                        3 |
                                               4
                                                       3 |
                                                               1 |
   6 |
               6 |
                             2 |
                                        3 |
                                               3 |
                                                      -1 |
                                                               0 |
                                                                         6.5
   7 |
               1 |
                             2 |
                                       11 |
                                                2 |
                                                        4 |
                                                               1 |
                                                                           0
               2 |
                             2 |
                                               5 I
                                                                           1
   8 |
                                       11 l
                                                        8 I
   9 I
               3 |
                             2 |
                                                                           2
                                       11 |
                                               6
                                                      11 |
                                                                           3
               4 |
                             2 |
  10 |
                                       11 |
                                               11
                                                      -1 |
                                                               0 |
  11 |
               1
                             7 |
                                        3
                                               7
                                                       6
                                                                           0
  12 I
               2 |
                             7 |
                                        3 I
                                               8
                                                       7 |
                                                               1 I
                                                                         2.5
               3 |
                                        3 |
                                               5
                                                                         4.5
  13 |
                             7 |
                                                       8 |
                                                               1 |
                                        3 |
  14 |
               4 |
                             7 |
                                                6 |
                                                       9 |
                                                               1 |
                                                                           6
  15 |
               5 |
                             7 |
                                        3 |
                                                9 |
                                                      16 |
                                                                           7
                                                               1 |
  16 |
               6 |
                             7 |
                                        3 |
                                                4 |
                                                       3 |
                                                                           8
                                                               1 I
                             7 |
                                                                           9
  17 |
               7 |
                                        3 |
                                               3
                                                      -1 l
                                                               0 |
                             7 |
                                                7
                                                                           0
  18 I
               1 |
                                       11 |
                                                        6
                                                               1 |
  19 I
                             7 |
                                       11 |
                                                8
                                                        7
                                                                           1
                                                                           2
  20 |
               3 I
                             7 |
                                       11 |
                                               5 I
                                                               1 I
                                                       8 I
                                                                           3
  21 |
               4 |
                             7 |
                                       11 |
                                               6 |
                                                      11 |
                                                               1 |
               5 I
  22 |
                             7 |
                                       11 |
                                              11 |
                                                      -1 |
                                                                           4
(22 rows)
```

Description of the Signatures ¶ Description of the edges_sql query for a star like functions ¶

edges sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
reverse_	_c ant -numerical	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source), When negative: edge (target, so
		not exist, therefore it's not part graph.
x1	ANY-NUMERICAL	X coordinate of source vertex.
y1	ANY-NUMERICAL	Y coordinate of source vertex.
x2	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\, INTEGER,\, BIGINT}$

ANY-NUMERICAL:

 ${\bf SMALLINT,\, INTEGER,\, BIGINT,\, REAL,\, FLOAT}$

Description of the parameters of the signatures \P

Parameter Type	Description				
edges_sqlTEXT	Edges SQL query as described above.				
${f start_vid}$ ANY-INTEGER	Starting vertex identifier.				
start_vidsARRAY[ANY-INTSCER]ng vertices identifierers.					
end_vid ANY-INTEGER	Ending vertex identifier.				
end_vids ARRAY[ANY-INTEGERING vertices identifiers.					

Paramete	r Type	Description
directed	BOOLEAN	• Optional.
		 When false the graph is considered as Undirected. Default is true which considers the graph as Directed.
heuristic	INTEGER	(optional). Heuristic number. Current valid values 0~5. Default 5
		 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) 1: h(v) abs(max(dx, dy)) 2: h(v) abs(min(dx, dy)) 3: h(v) = dx * dx + dy * dy 4: h(v) = sqrt(dx * dx + dy * dy) 5: h(v) = abs(dx) + abs(dy)
factor	FLOAT	(optional). For units manipulation. \setminus (factor $> 0 \setminus$). Default 1, see <i>Factor</i>
epsilon	FLOAT	(optional). For less restricted results. \(epsilon >= 1\). Default 1.

Description of the return values for a path \P

Returns set of (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost) $\label{eq:cost} % \begin{subarray}{ll} \end{subarray} % \b$

Column	Type
seq	INT
$\operatorname{path_id}$	INT
path_seq	INT
start_vid	BIGINT
end_vid	BIGINT
node	BIGINT

Column	Туре
edge	BIGINT
cost	FLOAT
${ m agg_cost}$	FLOAT

See Also \P

- Bidirectional A* Family of functions
- Sample Data network.
- $\bullet \ \ http://www.boost.org/libs/graph/doc/astar_search.html$
- http://en.wikipedia.org/wiki/A*_search_algorithm

Indices and tables

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pgr_bdDijkstra¶ pgr_bdDijkstra — Returns the shortest path(s) using Bidirectional Dijkstra algorithm.



Boost Graph Inside

Availability:

- pgr_bdDijkstra(one to one) 2.0.0, Signature changed 2.4.0
- pgr_bdDijkstra(other signatures) 2.5.0

Signature Summary \P

```
pgr_bdDijkstra(edges_sql, start_vid, end_vid)
pgr_bdDijkstra(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
OR EMPTY SET
```

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

```
pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)

RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)
OR EMPTY SET
```

Signatures¶ Minimal signature¶

```
pgr_bdDijkstra(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph from one **start_vid** to one end_vid:

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3
);
seq | path_seq | node | edge | cost | agg_cost
```

```
1 |
                            4 |
  2 |
              2 |
                     5 I
                            8 I
                                   1 |
                                              1
  3 I
              3 I
                     6 |
                           9 |
                                              2
                                   1 |
              4 |
  4 |
                     9 |
                         16 |
                                   1 |
                                              3
  5 I
              5 I
                     4 |
                           3 |
                                   1 |
                                              4
  6 |
              6 I
                     3 |
                           -1 |
(6 rows)
```

pgr_bd Dijkstra One to One \P

```
pgr_bdDijkstra(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one start_vid to one end_vid:

- on a directed graph when directed flag is missing or is set to true.
- on an undirected graph when directed flag is set to false.

Example:

pgr_bd Dijkstra One to many
¶

```
pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one start_vid to each end_vid in end_vids:

- on a directed graph when directed flag is missing or is set to true.
- on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform a one to one pgr_dijkstra where the starting vertex is fixed, and stop when all end_vids are reached.

- The result is equivalent to the union of the results of the one to one pgr dijkstra.
- The extra end_vid in the result is used to distinguish to which path it belongs.

Example:

SELECT * FROM pgr_bdDijkstra(

'SELECT id, source, target, cost, reverse_cost FROM edge_table',

2, ARRAY[3, 11]);

	path_seq		end_vid				edge	١	cost	١	agg_cost
+ 1	 1	+- 1	+ 3		2	⊦- ı	4	+- ı	1	⊦- I	0
2	2	i	3	ı I	5	i	8	ï	1	ı	1
3	3	i	3		6	i	9	i	1	i	2
4	4	I	3	١	9	ĺ	16	ĺ	1	ĺ	3
5	5		3	l	4	I	3	I	1	١	4
6 l	6		3	l	3	I	-1		0		5
7	1		11		2		4		1		0
8	2		11		5		8		1		1
9	3		11		6	l	11		1		2
10	4		11		11	I	-1	I	0		3
(10 row	s)										

pgr_bdDijkstra Many to One¶

```
pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each start_vid in start_vids to one end_vid:

- on a directed graph when directed flag is missing or is set to true.
- on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to one pgr_dijkstra where the ending vertex is fixed.

• The result is the union of the results of the one to one pgr_dijkstra.

 The extra start_vid in the result is used to distinguish to which path it belongs.

Example:

SELECT * FROM pgr_bdDijkstra(
 'SELECT id, source, target, cost, reverse_cost FROM edge_table',
 ARRAY[2, 7], 3);

seq	path_seq	start_vid	١	node	1	edge	1	cost	1	agg_cost
+	-+				+-		+-		+-	
1	1	2	ı	2	ı	4	ı	1	١	0
2	2	2		5		8		1	١	1
3	3	2		6	1	9	-	1	-	2
4	4	2		9		16		1	-	3
5	5	2		4		3		1	-	4
6	6 l	2	١	3		-1	1	0	1	5
7	1	7	١	7		6	1	1	1	0
8	2	7		8	1	7	1	1	-	1
9	3	7		5	1	8	1	1	-	2
10	4	7		6	1	9	1	1	-	3
11	5	7		9	1	16	1	1	1	4
12	6 I	7	١	4	1	3	1	1	1	5
13	7	7		3	1	-1	1	0	-	6
(13 row	s)									

pgr_bdDijkstra Many to Many¶

```
pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each start_vid in start_vids to each end_vid in end_vids:

- on a directed graph when directed flag is missing or is set to true.
- on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to Many pgr_dijkstra for all start_vids.

- The result is the union of the results of the one to one pgr_dijkstra.
- The extra start_vid in the result is used to distinguish to which path it belongs.

The extra start_vid and end_vid in the result is used to distinguish to which path it belongs.

Example:

SELECT * FROM pgr_bdDijkstra(
 'SELECT id, source, target, cost, reverse_cost FROM edge_table',
 ARRAY[2, 7], ARRAY[3, 11]);

	path_seq	start_vid		node	edge	e cost	agg_cost	;
1	1	2	3	2	4	. 1	.)
2	2	2] 3	J 5	8	3 1	. 1	L
3 l	3	2] 3	6	9) 1	. 2	2
4	4	2	J 3	9	16	5 1	. 3	3
5 I	5	2] 3	4	3	3 1	. 4	1
6 I	6	2] 3	3	-1	. 0)	5
7	1	2	11	2	4	1	. I)
8	2	2	11	5	8	3 1	. 1	L
9	3	2	11	6	11	. 1	. 2	2
10 l	4	2	11	11	-1	. 0) 3	3
11	1	7] 3	7	1 6	5 1	. I C)
12	2	7] 3	8	7	' 1	. 1	L
13 l	3	7] 3	5	8	3 1	. 2	2
14	4	7] 3	6	9) 1	. 3	3
15 l	5	7] 3	9	16	5 1	. 4	1
16 l	6	7] 3	4	3	3 1	. 5	5
17 l	7	7] 3	3	-1	. 0	1 6	3
18	1	7	11	7	1 6	6 1	. 0)
19	2	7	11	8	1 7	' 1	. 1	L
20 l	3	7	11	5	10) 1	. 2	2
21	4	7	11	10	12	2 1	. 3	3
22	5	7	11	11	-1	. 1 0) 4	1
(22 rd	ows)							

Description of the Signatures \P Description of the edges_sql query for dijkstra like functions \P

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
$\overline{\mathbf{id}}$	ANY-INTEG	ER	Identifier of the edge.
source	ANY-INTEG	ER	Identifier of the first end point vertex edge.

Column	Type Default	Description
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
reverse	c any -numeri€al	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source),
reverse_	CHAU NOMENTARE	 When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

 $SMALLINT,\,INTEGER,\,BIGINT,\,REAL,\,FLOAT$

Description of the parameters of the signatures \P

Column	Type	Default	Description
$\overline{\mathrm{sql}}$	TEXT		SQL query as descr
start_vid	BIGINT		Identifier of the stapath.
start_vid	sARRAY[BIGINT]		Array of identifiers
end_vid	BIGINT		Identifier of the enpath.
end_vids	ARRAY[BIGINT]		Array of identifiers
directed	BOOLEAN	true	 When true (Directed) When false considered as

Description of the return values for a path \P

Returns set of (seq, path_seq [, start_vid] [, end_vid], node, edge,

cost, agg_cost)

Column	Туре
seq	INT
path_id	INT
path _seq	INT
start_vid	BIGINT
${ m end_vid}$	BIGINT
node	BIGINT
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

See Also \P

- The queries use the Sample Data network.
- Bidirectional Dijkstra Family of functions
- $\bullet \ \ http://www.cs.princeton.edu/courses/archive/spr06/cos423/Handouts/EPP\%20 shortest\%20 path\%21/20 path\%2$
- $\bullet \ \ https://en.wikipedia.org/wiki/Bidirectional_search$

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Dijkstra - Family of functions \P

- • $pgr_dijkstra$ - Dijkstra's algorithm for the shortest paths.
- $\bullet \;\; pgr_dijkstraCost$ Get the aggregate cost of the shortest paths.
- • $pgr_dijkstraCostMatrix$ - proposed - Use $pgr_dijkstra$ to create a costs matrix.

- pgr_drivingDistance Use pgr_dijkstra to calculate catchament information.
- pgr_KSP Use Yen algorithm with pgr_dijkstra to get the K shortest paths.
- pgr_dijkstraVia Proposed Get a route of a seuence of vertices.

pgr_dijkstra¶ pgr_dijkstra — Returns the shortest path(s) using Dijkstra algorithm. In particular, the Dijkstra algorithm implemented by Boost.Graph.



Boost Graph Inside

Availability

- pgr_dijkstra(one to one) 2.0.0, signature change 2.1.0
- pgr_dijkstra(other signatures) 2.1.0

Synopsis

Dijkstra's algorithm, conceived by Dutch computer scientist Edsger Dijkstra in 1956. It is a graph search algorithm that solves the shortest path problem for a graph with non-negative edge path costs, producing a shortest path from a starting vertex (start_vid) to an ending vertex (end_vid). This implementation can be used with a directed graph and an undirected graph.

Characteristics

The main Characteristics are: • Process is done only on edges with positive costs.

- Values are returned when there is a path.
 - When the starting vertex and ending vertex are the same, there is no path.
 - * The agg_cost the non included values (v, v) is 0
 - When the starting vertex and ending vertex are the different and there is no path:
 - * The agg_cost the non included values (u, v) is $\setminus (\setminus infty \setminus)$
- For optimization purposes, any duplicated value in the start_vids or end_vids are ignored.
- The returned values are ordered:
 - start vid ascending

```
- end vid ascending
```

• Running time: $\langle O(| \text{start} \vee \text{vids} | * (V \setminus \text{log } V + E)) \rangle$

Signature Summary¶

$Signatures \P$

Minimal signature¶

```
pgr_dijkstra(TEXT edges_sql, BIGINT start_vid, BIGINT end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph from one **start_vid** to one end_vid.

 $\quad \ Example:$

6 I

(6 rows)

```
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, 3
);
seq | path_seq | node | edge | cost | agg_cost
  1 |
             1 |
                    2 |
                           4 |
                                  1 |
                                             0
  2 |
             2 |
                    5 I
                           8 I
                                  1 |
                                             1
  3 |
             3 I
                    6 I
                          9 |
                                  1 |
                          16 l
  4 |
             4 |
                    9 |
                                 1 |
                                             3
             5 |
  5 |
                    4 | 3 |
                                 1 |
                                            4
```

3 | -1 |

pgr_dijkstra One to One¶

6 |

0 |

This signature finds the shortest path from one start_vid to one end_vid:

on a directed graph when directed flag is missing or is set to
true.

• on an undirected graph when directed flag is set to false.

Example:

```
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
   FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
              1 |
                     2 |
                          2 |
                                   1 |
                                               0
   2 |
              2 |
                     3 |
                           -1 |
                                   0 |
                                               1
(2 rows)
```

pgr_dijkstra One to many¶

This signature finds the shortest path from one start_vid to each end_vid in end_vids:

on a directed graph when directed flag is missing or is set to
true.

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform a one to one pgr_dijkstra where the starting vertex is fixed, and stop when all end_vids are reached.

- The result is equivalent to the union of the results of the one to one pgr_dijkstra.
- The extra end_vid in the result is used to distinguish to which path it belongs.

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, ARRAY[3,5],
```

FALSE); seq | path_seq | end_vid | node | edge | cost | agg_cost ---+----1 | 1 | 3 | 2 | 2 | 2 | 3 | 5 I 8 I 1 | 1 3 | 5 I 3 | 3 | 6 | 1 | 2 4 | 3 | 3 l 0 | 3 4 | -1 | 5 I 1 | 5 I 2 | 4 | 1 | 6 | 2 | 5 I 5 I -1 l 0 | 1 (6 rows)

pgr_dijkstra Many to One¶

This signature finds the shortest path from each start_vid in start_vids to one end_vid: on a directed graph when directed flag is missing or is set to true.

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to one pgr_dijkstra where the ending vertex is fixed.

- The result is the union of the results of the one to one pgr dijkstra.
- The extra start_vid in the result is used to distinguish to which path it belongs.

```
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   ARRAY[2,11], 5
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
                          2 |
   1 |
             1 |
                                 2 |
                                        4 |
                                               1 |
                                                          0
   2 |
              2 |
                          2 |
                                       -1 |
                                               0 |
                                 5 I
                                                          1
   3 |
             1 |
                         11 |
                                11 |
                                       13 |
                                               1 |
                                                          0
              2 |
                         11 |
                                12 |
                                       15 |
   4 |
                                               1 |
                                                          1
                                                          2
   5 |
              3 |
                         11 |
                                9 |
                                      9 |
                                               1 |
```

```
6 | 4 | 11 | 6 | 8 | 1 | 3
7 | 5 | 11 | 5 | -1 | 0 | 4
(7 rows)
```

pgr_dijkstra Many to Many¶

RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET

This signature finds the shortest path from each start_vid in start_vids to each end_vid in on a directed graph when directed flag is missing or is set to true.

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to Many pgr_dijkstra for all start_vids.

- The result is the union of the results of the one to one pgr_dijkstra.
- The extra start_vid in the result is used to distinguish to which path it belongs.

The extra start_vid and end_vid in the result is used to distinguish to which path it belongs.

```
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2,11], ARRAY[3,5],
   FALSE
);
seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
1 |
            1 |
                       2 |
                                3 |
                                       2 |
                                             2 |
                                                   1 |
                                                             0
  2 |
            2 |
                       2 |
                                3 |
                                       3 |
                                            -1 l
                                                   0 |
                                                             1
                                5 I
                                       2 |
  3 |
            1 |
                       2 |
                                             4 |
                                                   1 |
                                                             0
            2 |
   4 |
                       2 |
                                5 |
                                       5 |
                                            -1 |
                                                    0 |
                                                             1
   5 I
            1 |
                      11 |
                                3 |
                                      11 |
                                            11 |
                                                    1 |
   6 I
            2 |
                      11 |
                                3 I
                                       6 |
                                             5 I
                                                    1 l
                                                             1
  7 |
            3 |
                      11 |
                                3 |
                                       3 |
                                            -1 |
                                                    0 |
                                                             2
  8 I
            1 |
                      11 |
                                5 |
                                      11 |
                                            11 |
                                                    1 |
                                                             0
            2 |
                      11 |
                                5 I
                                       6 |
  9 |
                                             8 |
                                                    1 |
                                                              1
  10 |
            3 I
                      11 |
                                5 I
                                            -1 |
                                                             2
                                       5 |
                                                    0 |
(10 rows)
```

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description	
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.	
source	ANY-INTEGER	Identifier of the first end point vertex edge.	
target	ANY-INTEGER	Identifier of the second end point vert edge.	
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)	
		• When negative: edge (source, ta not exist, therefore it's not part graph.	
$reverse_$	c ant -numericial	Weight of the edge (target, source),	
		• When negative: edge (target, so not exist, therefore it's not part graph.	

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Column	Type	Default	Description
$\overline{\mathrm{sql}}$	TEXT		SQL query as desc
start_vio	d BIGINT		Identifier of the stapath.
start_vidsARRAY[BIGINT]		·]	Array of identifiers
${ m end_vid}$	BIGINT		Identifier of the enepath.
end _vids	s ARRAY[BIGINT	·]	Array of identifiers

Column	Type	Default	Description
directed	BOOLEAN	true	When true (DirectedWhen false considered as

Description of the return values for a path \P

Returns set of (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)

Column	Туре
\overline{seq}	INT
path _id	INT
path _seq	INT
$start_vid$	BIGINT
${f end_vid}$	BIGINT
node	BIGINT
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

Additional Examples¶

The examples of this section are based on the Sample Data network.

The examples include combinations from starting vertices 2 and 11 to ending vertices 3 and 5 in a directed and undirected graph with and with out reverse_cost.

Examples for queries marked as ${\tt directed}$ with ${\tt cost}$ and ${\tt reverse_cost}$ columns \P

The examples in this section use the following $Network\ for\ queries\ marked\ as\ directed\ and\ cost\ and\ reverse_cost\ columns\ are\ used$

```
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
           1 | 2 | 4 | 1 |
          2 | 5 | 8 | 1 |
  2 |
  3 l
          3 | 6 | 9 | 1 |
          4 | 9 | 16 | 1 |
  4 |
          5 | 4 | 3 | 1 |
  5 |
          6 | 3 | -1 | 0 |
  6 |
(6 rows)
SELECT * FROM pgr_dijkstra(
  'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, 5
);
seq | path_seq | node | edge | cost | agg_cost
----+-----
           1 | 2 | 4 | 1 | 0
2 | 5 | -1 | 0 | 1
  2 |
(2 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, ARRAY[3,5]
);
seq | path_seq | end_vid | node | edge | cost | agg_cost
____+
          2 |
3 |
4 |
5 |
6 |
1 |
         1 | 3 | 2 | 4 | 1 |
  2 |
          2 |
                 3 | 5 | 8 | 1 |
          3 |
                 3 | 6 | 9 |
  3 I
                                  1 |
                                   1 |
          4 |
                  3 | 9 | 16 |
  4 |
                  3 |
          5 |
                       4 |
                             3 |
                                   1 |
  5 |
                       3 | -1 |
  6 |
                  3 |
                                   0 |
                                            5
  7 |
                  5 | 2 | 4 |
                                   1 |
                                            0
                  5 | 5 | -1 |
                                   0 |
  8 |
(8 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   11, 3
);
 seq | path_seq | node | edge | cost | agg_cost
```

```
1 |
                11 |
                     13 |
  2 |
           2 |
               12 |
                    15 l
                           1 |
                                   2
  3 |
           3 |
               9 |
                    16 |
                           1 |
                         1 |
                                   3
               4 |
  4 |
           4 |
                     3 |
  5 I
           5 |
                3 |
                    -1 |
                           0 |
(5 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
);
 seq | path_seq | node | edge | cost | agg_cost
----+-----
           1 | 11 | 13 | 1 |
  2 |
          2 | 12 | 15 | 1 |
                                   1
  3 |
          3 | 9 | 9 | 1 |
          4 | 6 | 8 | 1 |
  4 |
           5 | 5 | -1 | 0 |
  5 |
(5 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   ARRAY[2,11], 5
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
_____
           1 | 2 | 2 | 4 | 1 | 0
                         5 | -1 |
                   2 |
  2 |
           2 |
                                     0 |
                                             1
                   11 | 11 | 13 |
                                   1 |
  3 I
           1 |
                                             1
  4 |
          2 |
                   11 | 12 | 15 | 1 |
                                             2
  5 |
          3 |
                   11 | 9 | 9 | 1 |
  6 |
          4 |
                   11 | 6 | 8 | 1 |
  7 I
                   11 | 5 | -1 |
          5 I
                                     0 I
(7 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   ARRAY[2, 11], ARRAY[3,5]
);
seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
     1 | 2 | 3 | 2 |
                                     4 | 1 | 0
  2 |
          2 |
                   2 |
                          3 |
                                5 |
                                     8 |
                                           1 |
                  2 |
                       3 | 6 | 9 | 1 | 3 | 9 | 16 | 1 |
          3 |
  3 I
                                                   2
          4 |
                  2 |
  4 |
                                                   3
         5 |
                  2 |
  5 |
                         3 | 4 | 3 | 1 |
```

```
6 I
            6 |
                      2 |
                                3 |
                                      3 |
                                           -1 |
                                                  0 I
  7 |
            1 |
                      2 |
                                5 I
                                     2 |
                                           4 |
                                                   1 |
  8 I
            2 |
                      2 |
                               5 |
                                     5 |
                                           -1 |
                                                   0 1
                                                            1
            1 |
                     11 |
  9 |
                                3 |
                                     11 |
                                           13 |
                                                   1 |
                                                            0
 10 |
            2 |
                     11 |
                                3 |
                                   12 |
                                           15 |
                                                   1 |
                                                            1
           3 |
                                   9 l
 11 |
                     11 |
                                3 |
                                           16 |
                                                  1 |
                                                            2
           4 |
                                3 | 4 |
                                                            3
 12 |
                     11 |
                                            3 |
                                                  1 |
           5 l
                                3 | 3 |
                                                            4
 13 |
                     11 |
                                           -1 |
                                                   0 |
           1 |
                     11 |
                               5 | 11 |
                                           13 |
                                                  1 |
 14 |
                                                  1 |
 15 l
           2 |
                     11 |
                               5 | 12 |
                                           15 |
                                                            1
 16 l
           3 |
                     11 |
                               5 I
                                    9 |
                                           9 |
                                                   1 |
                                                            2
 17 |
           4 |
                      11 |
                               5 |
                                     6 |
                                                   1 |
                                                            3
                                            8 |
 18 |
            5 |
                      11 |
                                5 |
                                      5 I
                                           -1 |
                                                            4
(18 rows)
```

Examples for queries marked as $undirected\ with\ cost\ and\ reverse_cost\ columns\P$

The examples in this section use the following Network for queries marked as undirected and cost and reverse_cost columns are used

```
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, 3,
   FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
----+-----
           1 |
                 2 | 2 | 1 |
  1 |
                                     0
  2 |
           2 |
               3 | -1 | 0 |
                                     1
(2 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, 5,
   FALSE
);
seq | path_seq | node | edge | cost | agg_cost
_____
          1 | 2 | 4 | 1 | 0
                 5 | -1 |
                             0 |
  2 |
           2 |
                                     1
(2 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   11, 3,
```

```
FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
----+----
           1 | 11 | 11 | 1 |
2 | 6 | 5 | 1 |
3 | 3 | -1 | 0 |
                               0
1
2
  2 |
  3 |
(3 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   11, 5,
   FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
----+-----
          1 | 11 | 11 | 0
           2 | 6 | 8 | 1 |
                                   1
  2 |
           3 | 5 | -1 | 0 |
                                   2
  3 |
(3 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   ARRAY[2,11], 5,
   FALSE
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
1 |
                   2 | 2 | 4 | 1 |
  1 |
                   2 | 5 | -1 | 0 |
  2 |
          2 |
                                             1
                   11 | 11 | 12 | 1 |
          1 |
  3 |
  4 |
          2 |
                   11 | 10 | 10 | 1 |
  5 I
          3 l
                   11 | 5 | -1 | 0 |
(5 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, ARRAY[3,5],
   FALSE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
                                   1 |
           1 |
                        2 | 2 |
                   3 |
                        3 |
                                 0 |
  2 |
           2 |
                   3 I
                             -1 |
                                            1
  3 |
           1 |
                  5 | 2 | 4 | 1 |
                                            0
                  5 | 5 | -1 | 0 |
          2 |
  4 |
```

```
(4 rows)
SELECT * FROM pgr_dijkstra(
  'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   ARRAY[2, 11], ARRAY[3,5],
   FALSE
);
seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
______
        1 | 2 | 3 | 2 | 2 | 1 |
  2 |
         2 |
                 2 |
                        3 | 3 | -1 | 0 |
                                               1
         1 |
                 2 |
                        5 I
                            2 |
                                 4 |
                                       1 |
  3 |
                            5 l
  4 |
         2 |
                 2 |
                        5 |
                                  -1 |
                                        0 |
                       3 | 11 | 11 |
  5 I
        1 |
                11 |
                                       1 |
  6 |
        2 |
                11 |
                        3 | 6 | 5 |
                                      1 |
                                               1
  7 |
        3 |
                11 |
                       3 | 3 | -1 |
                                      0 |
               11 |
11 |
  8 |
        1 |
                       5 | 11 | 11 |
                                      1 |
  9 I
        2 |
                        5 | 6 |
                                  8 |
                                       1 |
                                               1
         3 |
 10 |
                 11 |
                       5 |
                                      0 |
                                               2
                             5 |
                                  -1 |
(10 rows)
```

Examples for queries marked as directed with cost column¶

The examples in this section use the following Network for queries marked as directed and only cost column is used

```
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost FROM edge_table',
   2, 3
);
seq | path_seq | node | edge | cost | agg_cost
----+-----
(0 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost FROM edge_table',
   2, 5
);
 seq | path_seq | node | edge | cost | agg_cost
            1 | 2 | 4 | 1 |
            2 | 5 | -1 | 0 |
                                       1
  2 |
(2 rows)
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
```

```
11, 3
);
seq | path_seq | node | edge | cost | agg_cost
----+-----
(0 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost FROM edge_table',
   11, 5
);
 seq | path_seq | node | edge | cost | agg_cost
----+----+----+
(0 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost FROM edge_table',
   ARRAY[2,11], 5
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
2 | 2 | 4 | 1 |
2 | 5 | -1 | 0 |
         1 |
  2 |
           2 |
(2 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost FROM edge_table',
   2, ARRAY[3,5]
);
seq | path_seq | end_vid | node | edge | cost | agg_cost
____+
              5 | 2 | 4 | 1 |
           1 |
  2 |
           2 |
                  5 | 5 | -1 |
                                  0 |
(2 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost FROM edge_table',
   ARRAY[2, 11], ARRAY[3,5]
);
seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
          1 |
                   2 | 5 | 2 | 4 |
                                           1 |
                         5 | 5 | -1 |
          2 |
                  2 |
                                         0 |
  2 |
(2 rows)
```

Examples for queries marked as undirected with cost column¶

The examples in this section use the following Network for queries marked as undirected and only cost column is used

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, 3,
   FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
   1 l
           1 | 2 | 4 | 1 | 0
           2 | 5 | 8 | 1 |
3 | 6 | 5 | 1 |
4 | 3 | -1 | 0 |
   2 |
   3 |
   4 |
(4 rows)
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, 5,
   FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
            1 | 2 | 4 | 1 |
             2 | 5 | -1 | 0 |
   2 |
                                         1
(2 rows)
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
   11, 3,
   FALSE
);
seq | path_seq | node | edge | cost | agg_cost
----+-----
            1 | 11 | 11 | 1 |
2 | 6 | 5 | 1 |
3 | 3 | -1 | 0 |
   2 |
   3 |
(3 rows)
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
   11, 5,
   FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
_____
```

```
1 |
               11 |
                    11 | 1 |
  2 |
              6 | 8 | 1 |
           2 |
                                   1
                5 | -1 |
  3 |
           3 |
                            0 |
(3 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost FROM edge_table',
   ARRAY[2,11], 5,
   FALSE
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
1 | 2 |
2 | 2 |
                              4 |
                          2 |
                                     1 |
                                             1
          2 |
                         5 | -1 | 0 |
  2 |
  3 |
          1 |
                   11 | 11 | 12 | 1 |
  4 |
          2 |
                   11 |
                         10 | 10 | 1 |
  5 |
           3 |
                   11 | 5 | -1 |
                                     0 |
(5 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost FROM edge_table',
   2, ARRAY[3,5],
   FALSE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
           1 | 3 | 2 | 4 | 1 |
  2 |
           2 |
                  3 |
                        5 |
                              8 |
                                   1 |
                            5 l
                                  1 l
          3 l
                  3 | 6 |
  3 I
                                  0 |
                                           3
  4 |
          4 |
                  3 | 3 | -1 |
          1 |
                  5 | 2 | 4 |
                                           0
  5 I
                                  1 |
  6 |
           2 |
                  5 | 5 | -1 |
                                   0 |
                                           1
(6 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost FROM edge_table',
   ARRAY[2, 11], ARRAY[3,5],
   FALSE
);
seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
        1 | 2 | 3 | 2 | 4 | 1 | 0
  2 |
          2 |
                   2 |
                           3 |
                                5 I
                                      8 |
                                           1 |
                                                   1
                  2 |
          3 |
                                    5 |
                                         1 |
  3 |
                         3 |
                                6 I
                                                   2
  4 |
          4 |
                  2 |
                         3 | 3 | -1 | 0 |
                                                   3
  5 |
                         5 | 2 | 4 | 1 |
         1 |
                  2 |
```

```
6 |
              2 |
                           2 |
                                      5 |
                                             5 |
                                                    -1 |
  7 |
              1 |
                          11 |
                                      3 |
                                                    11 |
                                            11 |
                                                             1 |
                          11 |
  8 |
              2 |
                                      3 |
                                             6 I
                                                     5 I
                                                             1 |
                                                                        1
              3 |
                                                                        2
  9 |
                          11 |
                                      3 |
                                             3 |
                                                    -1 |
                                                            0 |
 10 |
              1 |
                          11 |
                                      5 I
                                            11 |
                                                    11 |
                                                             1 |
                                                                        0
 11 |
              2 |
                          11 |
                                      5 I
                                             6 I
                                                     8 |
                                                             1 |
                                                                        1
                                      5 I
                                             5 I
                                                                        2
 12 |
              3 |
                          11 |
                                                    -1 |
(12 rows)
```

Equivalences between signatures¶

Examples:

For queries marked as directed with cost and reverse_cost columns

The examples in this section use the following:

6 |

3 |

6 |

• Network for queries marked as directed and cost and reverse_cost columns are used

```
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    TRUE
);
 seq | path_seq | node | edge | cost | agg_cost
   1 |
              1 |
                      2 |
                             4 |
                                    1 |
                                                0
   2 |
              2 |
                      5 I
                             8 |
                                    1 |
                                                1
   3 |
              3 |
                      6 I
                            9 |
                                    1 |
                                                2
   4 |
              4 |
                     9 I
                            16 |
                                                3
                                    1 l
                                                4
   5 |
              5 |
                      4 |
                           3 |
                                    1 |
   6 |
              6 |
                      3 |
                            -1 |
                                    0 |
(6 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2,3
);
 seq | path_seq | node | edge | cost | agg_cost
   1 l
              1 |
                      2 |
                             4 |
                                    1 |
                                                0
   2 |
              2 |
                      5 I
                             8 |
                                    1 |
                                                1
   3 |
              3 |
                      6 |
                                                2
                             9 |
                                     1 |
   4 |
              4 |
                      9 |
                            16 I
                                    1 |
                                                3
   5 I
              5 I
                      4 |
                                                4
                             3 |
                                    1 |
```

0 |

-1 |

```
(6 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, ARRAY[3],
   TRUE
);
seq | path_seq | end_vid | node | edge | cost | agg_cost
           1 l
                   3 | 2 | 4 | 1 |
                                      1 |
  2 |
           2 I
                    3 I
                        5 |
                               8 |
           3 |
                   3 |
                        6 l
  3 |
                              9 |
                                      1 |
                        9 |
  4 |
           4 |
                    3 |
                               16 |
                                      1 |
                        4 l
                              3 l
                    3 I
  5 I
           5 |
                                      1 l
                                               4
                    3 |
                                               5
  6 |
           6 |
                        3 | -1 |
                                      0 |
(6 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, ARRAY[3]
);
seq | path_seq | end_vid | node | edge | cost | agg_cost
                    3 |
                                4 l
           1 |
                          2 |
           2 |
                    3 l
                        5 l
                               8 I
  2 |
                                      1 |
  3 |
           3 |
                   3 |
                        6 |
                              9 |
                                     1 |
  4 |
           4 |
                    3 |
                        9 |
                               16 |
                                    1 |
                                               3
  5 I
           5 |
                    3 |
                         4 |
                               3 |
                                      1 |
                                               4
           6 |
                    3 |
                        3 |
  6 |
                               -1 l
(6 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   ARRAY[2], ARRAY[3],
   TRUE
);
seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
1 |
                  2 |
  1 |
                             3 |
                                  2 |
                                        4 |
                                            1 |
  2 |
           2 |
                   2 |
                           3 |
                                  5 I
                                        8 I
                                            1 |
          3 |
                   2 |
                                  6 |
  3 I
                            3 |
                                       9 |
                                             1 l
           4 |
                   2 |
                                            1 |
  4 |
                           3 |
                                  9 | 16 |
                                                       3
                   2 |
2 |
                        3 |
3 |
           5 I
  5 I
                                  4 |
                                       3 |
                                             1 |
                                                       4
           6 |
                                  3 | -1 |
                                            0 |
(6 rows)
```

```
SELECT * FROM pgr_dijkstra(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     ARRAY[2], ARRAY[3]
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost

    1 |
    2 |
    3 |
    2 |

    2 |
    2 |
    3 |
    5 |

    3 |
    2 |
    3 |
    6 |

    4 |
    2 |
    3 |
    9 |

    5 |
    2 |
    3 |
    4 |

    6 |
    2 |
    3 |
    3 |

                                                                    4 |
    1 |
                                                                      8 I
    2 |
                                                                             1 |
    3 l
                                                                    9 |
                                                                             1 |
                                               3 | 9 | 16 | 1 |
    4 I
                                                                                                3
    5 I
                                                                    3 |
                                                                              1 |
                                                                                                4
                                                            3 | -1 |
    6 |
                                                                               0 |
                                                                                         5
(6 rows)
```

Examples:

For queries marked as undirected with cost and reverse_cost columns. The examples in this section use the following:

 Network for queries marked as undirected and cost and reverse_cost columns are used

```
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, 3,
   FALSE
);
seq | path_seq | node | edge | cost | agg_cost
----+-----
           1 | 2 | 2 | 1 |
                                      0
  1 |
  2 |
            2 | 3 | -1 | 0 |
                                      1
(2 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, ARRAY[3],
   FALSE
);
seq | path_seq | end_vid | node | edge | cost | agg_cost
          1 | 3 | 2 | 2 | 1 | 0
  1 |
  2 |
            2 |
                    3 I
                          3 |
                               -1 |
                                       0 |
(2 rows)
SELECT * FROM pgr_dijkstra(
```

```
'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   ARRAY[2], 3,
   FALSE
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
                     2 | 2 | 2 | 1 |
  1 |
           1 |
                     2 | 3 | -1 |
           2 |
                                       0 |
  2 |
                                               1
(2 rows)
SELECT * FROM pgr_dijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   ARRAY[2], ARRAY[3],
   FALSE
);
seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
2 |
  1 |
           1 |
                   2 |
                            3 |
                                       2 |
                                             1 |
                                                     0
  2 |
           2 |
                   2 |
                          3 |
                                 3 | -1 |
                                           0 |
                                                     1
(2 rows)
```

See Also¶

- http://en.wikipedia.org/wiki/Dijkstra%27s_algorithm
- The queries use the Sample Data network.

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pgr_dijkstraCost¶ Synopsis¶

pgr_dijkstraCost

Using Dijkstra algorithm implemented by Boost.Graph, and extract only the aggregate cost of the shortest path(s) found, for the combination of vertices given.



Boost Graph Inside

Availability

• pgr_dijkstraCost(all signatures) 2.2.0

The pgr_dijkstraCost algorithm, is a good choice to calculate the sum of the costs of the shortest path for a subset of pairs of nodes of the graph. We make use of the Boost's implementation of dijkstra which runs in $(O(V \setminus V + E))$ time.

Characteristics

The main Characteristics are: • It does not return a path.

- Returns the sum of the costs of the shortest path for pair combination of nodes in the graph.
- Process is done only on edges with positive costs.
- Values are returned when there is a path.
 - The returned values are in the form of a set of (start_vid, end_vid, agg_cost).
 - When the starting vertex and ending vertex are the same, there is no path.
 - * The agg cost int the non included values (v, v) is 0
 - When the starting vertex and ending vertex are the different and there is no path.
 - * The agg_cost in the non included values (u, v) is $\setminus (\setminus infty \setminus)$
- Let be the case the values returned are stored in a table, so the unique index would be the pair: (start vid, end vid).
- For undirected graphs, the results are symmetric.
 - The agg cost of (u, v) is the same as for (v, u).
- Any duplicated value in the start vids or end vids is ignored.
- The returned values are ordered:
 - start vid ascending
 - end vid ascending
- Running time: $\langle O(| \text{start} \vee \text{vids} | * (V \setminus \text{log } V + E)) \rangle$

Signature Summary \P

```
pgr_dijkstraCost(edges_sql, start_vid, end_vid);
pgr_dijkstraCost(edges_sql, start_vid, end_vid, directed);
pgr_dijkstraCost(edges_sql, start_vids, end_vid, directed);
pgr_dijkstraCost(edges_sql, start_vid, end_vids, directed);
pgr_dijkstraCost(edges_sql, start_vids, end_vids, directed);
pgr_dijkstraCost(edges_sql, start_vids, end_vids, directed);
RETURNS SET OF (start vid, end vid, agg cost) or EMPTY SET
```

```
Signatures¶
```

Minimal signature

The minimal signature is for a **directed** graph from one **start_vid** to one end_vid:

pgr_dijkstraCost(TEXT edges_sql, BIGINT start_vid, BIGINT end_vid)
 RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET

Example

pgr_dijkstraCost One to One¶

This signature performs a Dijkstra from one start_vid to one end_vid:

on a directed graph when directed flag is missing or is set to
true.

• on an undirected graph when directed flag is set to false.

Example:

pgr_dijkstra Cost One to Many
¶

This signature performs a Dijkstra from one start_vid to each end_vid in end_vids:

on a directed graph when directed flag is missing or is set to true.

• on an undirected graph when directed flag is set to false.

Example:

pgr_dijkstraCost Many to One¶

This signature performs a Dijkstra from each start_vid in start_vids to one end_vid:

on a directed graph when directed flag is missing or is set to

• on an undirected graph when directed flag is set to false.

Example:

pgr_dijkstraCost Many to Many¶

This signature performs a Dijkstra from each start_vid in start_vids to each end_vid in end on a directed graph when directed flag is missing or is set to true.

• on an undirected graph when directed flag is set to false.

Example:

 Column

Description of the Signatures¶

Type

Description of the edges_sql query for dijkstra like functions¶ edges_sql:

Default

an SQL query, which should return a set of rows with the following columns:

id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	_cany-numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Description

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Column	Type	Default	Description
$\overline{\mathrm{sql}}$	TEXT		SQL query as desc
start_vid	BIGINT		Identifier of the stapath.
$start_vid$	sARRAY[BIGINT]]	Array of identifiers
end_vid	BIGINT		Identifier of the enepath.
end_vids	ARRAY[BIGINT]]	Array of identifiers
directed	BOOLEAN	true	When true (DirectedWhen false considered as

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vidBIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vert ending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

Additional Examples \P

Example 1:

Demonstration of repeated values are ignored, and result is sorted.

SELECT * FROM pgr_dijkstraCost(

```
'select id, source, target, cost, reverse_cost from edge_table',

ARRAY[5, 3, 4, 3, 3, 4], ARRAY[3, 5, 3, 4]);

rt wid | and w
```

start_vid	ı	end_vid	ı	agg_cost
	+-		+-	
3		4	-	3
3		5	-	2
4	1	3	-	1
4		5	-	3
5		3	-	4
5		4		3
(6 rows)				

Example 2:

Making start_vids the same as end_vids

'select id, source, target, cost, reverse_cost from edge_table',

ARRAY[5, 3, 4], ARRAY[5, 3, 4]);

agg_cost	 	end_vid	 	start_vid	_
3	т- 	4		3	_
2	1	5	1	3	
1	-	3	1	4	
3	1	5	1	4	
4	-	3	-	5	
3	-	4	1	5	
				/ - \ \	

(6 rows)

See Also \P

- $\bullet \ \ http://en.wikipedia.org/wiki/Dijkstra\%27s_algorithm$
- Sample Data network.

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$\mathbf{pgr_dijkstraCostMatrix} - \mathbf{proposed}\P \quad \mathrm{Name}\P$

 ${\tt pgr_dijkstraCostMatrix}$ - Calculates the a cost matrix using ${\tt pgr_dijktras}$.

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.



Boost Graph Inside

Availability: 2.3.0

Synopsis¶

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary¶

```
pgr_dijkstraCostMatrix(edges_sql, start_vids)
pgr_dijkstraCostMatrix(edges_sql, start_vids, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

 $Signatures \P$

Minimal Signature¶

The minimal signature: • Is for a directed graph.

```
pgr_dijkstraCostMatrix(edges_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example:

Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_dijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
start_vid | end_vid | agg_cost</pre>
```

```
1 |
                  2 |
                            1
        1 |
                  3 |
                            6
                  4 |
                            5
        1 |
        2 |
                  1 |
        2 |
                  3 |
        2 |
                  4 |
                            2
        3 |
                  1 |
        3 I
                  2 |
        3 |
                  4 |
        4 |
                  1 |
                  2 |
                            2
        4 |
                 3 |
        4 |
(12 rows)
```

Complete Signature¶

```
pgr_dijkstraCostMatrix(edges_sql, start_vids, directed:=true)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example:

Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_dijkstraCostMatrix(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
   false
```

);				
${\tt start_vid}$	I	$\verb"end_vid"$		agg_cost
			+-	
1	I	2	1	1
1	١	3		2
1	١	4		3
2		1	1	1
2		3	1	1
2		4	1	2
3		1	1	2
3		2	1	1
3		4	1	1
4		1	1	3
4		2	1	2
4	١	3		1
(40				

(12 rows)

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	_cant-numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

ParametefType	Description
edges_sqEXT	Edges SQL query as described above.
start_vidRRAY[ANY-	INTEGER identifiers of the vertices.
directed BOOLEAN	(optional). When false the graph is considered as Undirected. Default is true which considers the graph as Directed.

Description of the return values for a Cost function¶ Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vid BIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vert ending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

```
\operatorname{Examples} \P
Example:
Use with tsp
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_dijkstraCostMatrix(
      'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    )
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
          1 |
   1 |
                  1 |
                              0
   2 |
          2 |
                  1 |
                              1
          3 |
   3 |
                  1 |
                              2
   4 |
          4 |
                  3 |
                              3
   5 I
          1 |
                  0 |
                              6
(5 rows)
```

See Also \P

- Dijkstra Family of functions
- Cost Matrix Category
- Traveling Sales Person Family of functions
- The queries use the Sample Data network.

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pgr_drivingDistance¶ Name¶

pgr_drivingDistance - Returns the driving distance from a start node.



Boost Graph Inside

Availability

- pgr_drivingDistance(single vertex) 2.0.0, signature change 2.1.0
- pgr_drivingDistance(multiple vertices) 2.1.0

Synopsis¶

Using the Dijkstra algorithm, extracts all the nodes that have costs less than or equal to the value distance. The edges extracted will conform to the corresponding spanning tree.

Signature Summary¶

```
pgr_drivingDistance(edges_sql, start_vid, distance)
pgr_drivingDistance(edges_sql, start_vid, distance, directed)
pgr_drivingDistance(edges_sql, start_vids, distance, directed, equicost)
```

RETURNS SET OF (seq, [start_vid,] node, edge, cost, agg_cost)

Signatures

Minimal Use¶

```
pgr_drivingDistance(edges_sql, start_vid, distance)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Driving Distance From A Single Starting Vertex¶

```
pgr_drivingDistance(edges_sql, start_vid, distance, directed)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Driving Distance From Multiple Starting Vertices¶

pgr_drivingDistance(edges_sql, start_vids, distance, directed, equicost)
RETURNS SET OF (seq, start_vid, node, edge, cost, agg_cost)

Description of the Signatures¶

Description of the edges_sql query for dijkstra like functions \P

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
reverse_	c onv -numerical	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source), When negative: edge (target, so not exist, therefore it's not part.
		not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

 $SMALLINT,\,INTEGER,\,BIGINT,\,REAL,\,FLOAT$

Description of the parameters of the signatures \P

Column	Туре
edges_sql	TEXT
$\operatorname{start_vid}$	BIGINT
start_vids	ARRAY[ANY-INTEGER]

Column	Туре
distance directed	FLOAT BOOLEAN
equicost	BOOLEAN

Description of the return values¶

Returns set of (seq [, start_v], node, edge, cost, agg_cost)

Column	Туре	Description
$\overline{\text{seq}}$	INTEGER	Sequential value starting fr
start_vi	d integer	Identifier of the starting ve
node	BIGINT	Identifier of the node in the start_vid.
edge	BIGINT	Identifier of the edge used to node is the start_vid.
\mathbf{cost}	FLOAT	Cost to traverse edge.
agg_cost	FLOAT	Aggregate cost from start

Additional Examples¶

Examples for queries marked as ${\tt directed}$ with ${\tt cost}$ and ${\tt reverse_cost}$ columns \P

The examples in this section use the following $Network\ for\ queries\ marked\ as\ directed\ and\ cost\ and\ reverse_cost\ columns\ are\ used$

```
3 |
         5 |
                4 |
   4 |
                 8 |
                       1 |
                                  2
          6 |
   5 I
         8 |
                7 |
                       1 |
                                  2
                                  2
   6 |
         10 |
                10 |
                       1 |
   7 |
         7 |
                6 |
                       1 |
                                  3
   8 |
         9 |
                9 |
                       1 |
                                  3
        11 |
                12 |
                                  3
   9 |
                       1 |
  10 |
         13 |
                                  3
                14 |
                       1 |
(10 rows)
SELECT * FROM pgr_drivingDistance(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        13, 3
      );
 seq | node | edge | cost | agg_cost
----+-----
   1 |
       13 |
               -1 |
                       0 |
                       1 |
   2 |
               14 |
        10 |
                                  1
   3 |
                                  2
         5 I
               10 |
                       1 |
   4 |
                                  2
        11 |
               12 |
                       1 |
   5 I
         2 |
                4 |
                       1 |
                                  3
   6 |
         6 I
                8 I
                       1 |
                                  3
   7 |
         8 |
                7 |
                                  3
                       1 |
   8 |
         12 |
               13 |
                       1 |
                                  3
(8 rows)
SELECT * FROM pgr_drivingDistance(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3
      );
 seq | from_v | node | edge | cost | agg_cost
----+-----
   1 l
            2 |
                  2 |
                       -1 l
                                0 |
                                           0
                  1 |
                                1 |
   2 |
            2 |
                         1 |
                                           1
   3 |
            2 |
                  5 |
                         4 |
                                1 |
                                           1
   4 |
            2 |
                  6 |
                         8 |
                                1 |
                                           2
   5 I
            2 |
                  8 |
                         7 |
                                1 |
                                           2
   6 |
           2 |
                 10 |
                       10 |
                                1 |
                                           2
   7 |
            2 |
                         6 I
                                1 |
                                           3
                 7 |
   8 |
            2 |
                  9 I
                         9 I
                                1 |
                                           3
           2 |
   9 |
                 11 |
                        12 |
                                1 |
                                           3
           2 |
                                1 |
  10 |
                 13 |
                        14 l
                                           3
  11 |
           13 |
                 13 |
                        -1 |
                                0 |
                                           0
  12 |
           13 |
                 10 |
                        14 l
                                1 |
                                           1
  13 l
           13 |
                  5 |
                        10 |
                                           2
                                1 |
                                           2
  14 |
           13 |
                 11 |
                        12 |
                                1 |
```

```
15 |
           13 |
                   2 |
                          4 |
  16 |
           13 |
                          8 |
                                 1 |
                                            3
                   6 |
  17 |
                   8 |
                          7 |
                                 1 |
                                            3
           13 |
  18 l
           13 |
                  12 |
                         13 |
                                 1 |
                                            3
(18 rows)
SELECT * FROM pgr_drivingDistance(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
 ---+----
                                 0 |
   1 |
            2 |
                   2 |
                         -1 |
            2 |
                         1 |
   2 |
                   1 |
                                 1 |
                                            1
           2 |
   3 |
                   5 I
                          4 |
                                 1 |
                                            1
   4 |
            2 |
                   6 |
                          8 |
                                 1 |
                                            2
   5 |
           2 |
                   8 |
                          7 |
                                 1 |
   6 |
           2 |
                   7 |
                          6 |
                                 1 |
                                            3
                                 1 |
   7 |
           2 |
                                            3
                  9 |
                         9 |
   8 |
           13 |
                  13 |
                         -1 |
                                 0 |
                                            0
   9 |
           13 |
                  10 I
                         14 l
                                 1 |
                                            1
  10 I
           13 l
                         12 |
                                            2
                  11 |
                                 1 |
                                 1 |
           13 |
                  12 |
                         13 |
                                            3
  11 |
(11 rows)
```

Examples for queries marked as ${\tt undirected}$ with ${\tt cost}$ and ${\tt reverse_cost}$ ${\tt columns}\P$

The examples in this section use the following $Network\ for\ queries\ marked\ as\ undirected\ and\ cost\ and\ reverse_cost\ columns\ are\ used$

```
SELECT * FROM pgr_drivingDistance(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        2, 3, false
      );
 seq | node | edge | cost | agg_cost
----+----
   1 |
              -1 |
                                  0
         2 |
                       0 |
   2 |
                1 |
          1 |
                       1 |
                                  1
   3 |
          3 |
                 2 |
                       1 |
                                  1
   4 I
          5 I
                 4 |
                       1 |
                                  1
   5 I
                                  2
          4 |
                 3 |
                       1 |
                                  2
   6 I
          6 |
                8 |
                       1 |
   7 |
         8 |
                7 |
                                  2
                       1 |
               10 |
                                  2
   8 |
         10 |
                       1 |
   9 |
         7 |
                6 |
                       1 |
                                  3
```

```
10 |
          9 |
                 16 |
                          1 |
                 12 |
                          1 |
                                      3
  11 |
         11 |
  12 |
         13 |
                 14 |
                          1 |
                                      3
(12 rows)
SELECT * FROM pgr_drivingDistance(
      'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        13, 3, false
      );
 seq | node | edge | cost | agg_cost
   1 |
         13 |
                 -1 |
                          0 |
   2 |
         10 |
                 14 |
                          1 |
                                      1
                 10 |
   3 |
          5 |
                          1 |
                                      2
   4 |
                 12 |
                                      2
         11 |
                          1 |
   5 |
          2 |
                  4 |
                          1 |
                                      3
   6 |
          6 |
                  8 |
                                      3
                          1 |
   7 |
          8 |
                  7 |
                                      3
                          1 |
                                      3
   8 |
         12 |
                 13 |
                          1 |
(8 rows)
SELECT * FROM pgr_drivingDistance(
      'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3, false
      );
 seq | from_v | node | edge | cost | agg_cost
                                    0 |
   1 |
             2 |
                    2 |
                           -1 |
                                                0
   2 |
             2 |
                            1 |
                    1 |
                                    1 |
                                                1
   3 |
             2 |
                    3 |
                            2 |
                                    1 l
                                                1
             2 |
                                                1
   4 |
                    5 |
                            4 |
                                    1 |
   5 |
             2 |
                    4 |
                            3 |
                                                2
   6 I
             2 |
                                                2
                    6 |
                            8 I
                                    1 |
   7 |
             2 |
                                                2
                    8 |
                            7 |
                                    1 |
                                                2
             2 |
   8 |
                   10 |
                           10 |
                                    1 |
   9 |
             2 |
                    7 |
                            6 |
                                    1 |
                                                3
  10 I
             2 |
                    9 |
                           16 |
                                    1 |
                                                3
             2 |
                           12 |
                                                3
  11 |
                   11 |
                                    1 |
             2 |
                                                3
  12 |
                   13 |
                           14 |
                                    1 |
  13 |
            13 |
                   13 |
                           -1 l
                                    0 |
                                                0
  14 |
            13 |
                   10 I
                           14 l
                                                1
                                    1 |
                                                2
  15 |
            13 |
                    5 l
                           10 |
                                    1 |
                                                2
  16 |
            13 |
                   11 |
                           12 |
                                    1 |
  17 |
            13 |
                    2 |
                            4 |
                                                3
                                    1 |
  18 I
            13 |
                            8 |
                                                3
                    6 |
                                    1 |
                    8 |
                                                3
  19 l
            13 |
                            7 |
                                    1 |
```

```
20 |
           13 |
                   12 |
                          13 |
                                   1 |
                                               3
(20 rows)
SELECT * FROM pgr_drivingDistance(
      'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3, false, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
   1 |
            2 |
                    2 |
                          -1 l
                                   0 |
                                   1 |
   2 |
            2 |
                    1 |
                           1 |
                                               1
            2 |
                            2 |
   3 |
                    3 |
                                   1 |
                                               1
   4 |
            2 |
                    5 |
                            4 |
                                   1 |
                                               1
            2 |
                           3 |
   5 I
                    4 I
                                   1 |
                                               2
            2 |
   6 |
                    6 I
                                               2
                           8 |
                                   1 |
   7 |
            2 |
                    8 |
                           7 |
                                   1 |
                                               2
   8 |
            2 |
                    7 |
                           6 |
                                               3
                                   1 |
                                   1 l
   9 |
            2 |
                    9 |
                          16 |
                                               3
                                               0
  10 |
           13 |
                   13 |
                          -1 |
                                   0 |
  11 l
           13 |
                   10 |
                          14 |
                                               1
  12 |
           13 |
                   11 |
                          12 |
                                   1 |
                                               2
  13 |
           13 |
                   12 |
                          13 l
                                               3
                                   1 |
(13 rows)
```

Examples for queries marked as directed with cost column¶

The examples in this section use the following Network for queries marked as directed and only cost column is used

```
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        2, 3
     );
 seq | node | edge | cost | agg_cost
----+----
   1 |
         2 |
               -1 |
                       0 |
   2 |
         5 I
                4 |
                       1 |
                                  1
         6 I
                8 |
                       1 |
                                  2
   3 |
                                  2
        10 l
              10 |
   4 |
                       1 |
   5 I
         9 |
                9 I
                       1 |
   6 I
        11 |
               11 |
                       1 |
                                  3
   7 |
         13 |
               14 |
                       1 |
                                  3
(7 rows)
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
```

```
13, 3
     );
 seq | node | edge | cost | agg_cost
----+----
        13 | -1 | 0 |
(1 row)
SELECT * FROM pgr_drivingDistance(
       'SELECT id, source, target, cost FROM edge_table',
       array[2,13], 3
     );
 seq | from_v | node | edge | cost | agg_cost
----+-----
  1 |
          2 |
              2 | -1 |
                           0 |
              5 | 4 |
  2 |
          2 |
                           1 |
                                      1
         2 |
  3 |
              6 I
                      8 |
                           1 |
  4 |
          2 | 10 |
                    10 |
  5 I
          2 |
                9 |
                      9 |
                            1 |
                                      3
  6 I
          2 |
              11 |
                            1 l
                                      3
                    11 |
  7 |
          2 |
               13 |
                      14 |
                             1 |
                                      3
  8 |
         13 |
               13 |
                     -1 l
                                      0
(8 rows)
SELECT * FROM pgr_drivingDistance(
       'SELECT id, source, target, cost FROM edge_table',
       array[2,13], 3, equicost:=true
     );
 seq | from_v | node | edge | cost | agg_cost
  1 |
          2 |
                2 |
                    -1 l
                             0 I
              5 | 4 | 1 |
  2 |
          2 |
  3 |
          2 |
                6 |
                      8 |
                           1 |
  4 I
          2 |
              10 |
                    10 |
                           1 |
                                      2
          2 |
                9 |
                      9 |
                            1 l
                                      3
  5 |
               11 |
  6 |
          2 |
                             1 |
                                      3
                      11 |
  7 |
         13 |
               13 |
                     -1 |
                                      0
(7 rows)
```

Examples for queries marked as undirected with cost column¶

The examples in this section use the following Network for queries marked as undirected and only cost column is used

```
seq | node | edge | cost | agg_cost
  1 |
        2 |
            -1 |
                    0 |
       1 |
  2 |
            1 |
                    1 |
  3 |
       5 I
              4 l
                    1 |
                              1
  4 |
       6 | 8 |
                    1 |
                              2
  5 |
       8 | 7 |
                              2
                    1 |
  6 I
      10 | 10 |
                    1 |
  7 |
       3 | 5 |
                    1 |
  8 |
       7 |
             6 |
                    1 |
                              3
  9 |
       9 |
             9 |
                    1 |
                              3
  10 |
      11 |
            12 |
                    1 |
  11 |
      13 | 14 |
(11 rows)
SELECT * FROM pgr_drivingDistance(
       'SELECT id, source, target, cost FROM edge_table',
       13, 3, false
     );
 seq | node | edge | cost | agg_cost
  1 | 13 | -1 | 0 |
  2 | 10 | 14 | 1 |
  3 I
       5 | 10 |
                  1 |
      11 | 12 |
  4 |
                  1 |
                              2
  5 l
       2 | 4 |
                   1 |
  6 |
       6 I
             8 |
                    1 |
  7 |
       8 I
             7 |
                              3
                    1 |
  8 I
       12 | 13 |
                    1 l
                              3
(8 rows)
SELECT * FROM pgr_drivingDistance(
       'SELECT id, source, target, cost FROM edge_table',
       array[2,13], 3, false
     );
 seq | from_v | node | edge | cost | agg_cost
----+-----
          2 |
              2 | -1 | 0 |
  2 |
          2 |
              1 | 1 |
                          1 |
  3 I
          2 |
              5 I
                      4 |
                          1 |
                                     1
                      8 |
  4 |
          2 |
              6 l
                          1 |
                                     2
              8 |
  5 I
          2 |
                     7 |
                            1 |
                                     2
  6 |
          2 |
              10 |
                    10 |
                            1 |
                                     2
              3 | 5 |
                          1 |
  7 |
          2 |
                                     3
  8 |
          2 |
              7 | 6 |
                          1 |
```

```
9 |
             2 |
                            9 |
                                                 3
  10 |
             2 |
                            12 |
                                    1 |
                                                 3
                    11 |
             2 |
                    13 |
                            14 |
                                                 3
  11 |
                                    1 |
  12 |
            13 |
                    13 |
                            -1 |
                                    0 |
                                                 0
  13 |
            13 |
                    10 |
                            14
                                                 1
  14 |
            13 |
                     5 I
                           10
                                    1 |
                                                 2
                           12 |
                                                 2
  15 |
            13 |
                                    1 |
                    11 |
                                                 3
                             4 |
  16 |
            13 |
                     2 |
                                    1 |
  17 |
            13 |
                     6 I
                             8 |
                                    1 |
                                                 3
                            7 |
                                                 3
  18 I
            13 |
                     8 |
                                    1 |
  19 |
            13 |
                    12 |
                           13 |
                                    1 |
                                                 3
(19 rows)
SELECT * FROM pgr_drivingDistance(
         'SELECT id, source, target, cost FROM edge_table',
         array[2,13], 3, false, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
----+----+----+----+-----+-----
                                                 0
   1 |
             2 |
                     2 |
                            -1 |
                                    0 |
   2 |
             2 |
                     1 |
                             1 |
                                    1 |
                                                 1
   3 |
             2 |
                     5 I
                             4 |
                                    1 |
                                                 1
                                                 2
             2 |
                             8 |
   4 |
                     6 |
                                    1 |
                                                 2
   5 |
             2 |
                     8 |
                             7 |
                                    1 |
   6 I
             2 |
                     3 |
                             5 I
                                                 3
                                    1 |
   7 |
             2 |
                     7 |
                             6 |
                                    1 |
                                                 3
   8 |
             2 |
                     9 |
                            9 |
                                    1 |
                                                 3
   9 |
            13 |
                    13 |
                           -1 |
                                    0 |
                                                 0
  10 |
            13 |
                           14 l
                    10 |
                                    1 |
                                                 1
  11 l
            13 I
                           12 |
                                                 2
                    11 |
                                    1 |
                                                 3
  12 |
            13 |
                    12 |
                           13 |
                                    1 |
(12 rows)
```

See Also \P

- $pgr_alphaShape$ Alpha shape computation
- $pgr_pointsAsPolygon$ Polygon around set of points
- \bullet Sample Data network.

Indices and tables

- \bullet Index
- Search Page

pgr_KSP¶ Name¶

pgr_KSP — Returns the "K" shortest paths.



Boost Graph Inside Availability: 2.0.0

• Signature change 2.1.0

Synopsis¶

The K shortest path routing algorithm based on Yen's algorithm. "K" is the number of shortest paths desired.

Signature Summary¶

```
pgr_KSP(edges_sql, start_vid, end_vid, K);
pgr_KSP(edges_sql, start_vid, end_vid, k, directed, heap_paths)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Signatures

Minimal Signature¶

```
pgr_ksp(edges_sql, start_vid, end_vid, K);
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Complete Signature¶

```
pgr_KSP(edges_sql, start_vid, end_vid, k, directed, heap_paths)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Description of the Signatures¶

Description of the edges_sql query for dijkstra like functions ¶

edges sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
$\overline{\mathbf{id}}$	ANY-INTEG	ER	Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex edge.
target	ANY-INTEG	ER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMER	ICAL	Weight of the edge (source, target)
			• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	cant-numer:	ICAL	Weight of the edge (target, source),
			• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Column	Type
edges_sql	TEXT
$\operatorname{start_vid}$	BIGINT
end _vid	BIGINT
k	INTEGER
directed	BOOLEAN
heap_paths	BOOLEAN

Roughly, if the shortest path has N edges, the heap will contain about than N $\,\star\,$

k paths for small value of k and k > 1.

Description of the return values¶

Returns set of (seq, path_seq, path_id, node, edge, cost, agg_cost)

Column	Туре
seq	INTEGER
path _seq	INTEGER
$\operatorname{path_id}$	BIGINT
node	BIGINT
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

Warning

During the transition to 3.0, because pgr_ksp version 2.0 doesn't have defined a directed flag nor a heap_path flag, when pgr_ksp is used with only one flag version 2.0 signature will be used.

$Additional\ Examples \P$

Examples to handle the one flag to choose signatures¶

The examples in this section use the following $Network\ for\ queries\ marked\ as\ directed\ and\ cost\ and\ reverse_cost\ columns\ are\ used$

```
SELECT * FROM pgr_KSP(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 12, 2,
    directed:=true
  );
seq | path_id | path_seq | node | edge | cost | agg_cost
4 |
         1 |
                 1 |
                     2 |
                                 1 |
                                          0
  1 |
  2 |
         1 |
                 2 | 5 | 8 |
                                1 |
                                          1
  3 I
         1 |
                 3 | 6 | 9 |
                                1 |
                                          2
  4 |
         1 |
                 4 | 9 |
                           15 |
                                1 |
                                          3
         1 |
                5 l
  5 |
                      12 | -1 |
                                 0 |
```

```
6 I
          2 |
                   1 |
                         2 |
                               4 |
  7 |
          2 |
                   2 |
                         5 I
                               8 I
                                     1 |
                                              1
  8 |
          2 |
                   3 |
                         6 |
                                    1 |
                                              2
                              11 |
          2 |
                   4 | 11 |
  9 |
                              13 |
                                     1 |
                                              3
  10 |
          2 |
                   5 |
                        12 |
                              -1 |
                                     0 |
                                              4
(10 rows)
SELECT * FROM pgr_KSP(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     2, 12, 2
  );
 seq | path_id | path_seq | node | edge | cost | agg_cost
1 | 2 |
                             4 |
  1 |
          1 |
                                   1 |
                               8 |
                   2 |
  2 |
          1 |
                        5 l
                                    1 |
                                              1
  3 |
          1 |
                  3 |
                        6 |
                              9 |
                                   1 |
  4 |
          1 |
                  4 |
                        9 |
                             15 |
                                    1 |
  5 |
          1 |
                  5 |
                              -1 |
                                     0 |
                                              4
                         12 |
          2 |
                                     1 |
  6 |
                   1 |
                         2 |
                               4 |
                                              0
          2 |
                   2 |
  7 |
                         5 I
                               8 |
                                     1 l
  8 I
          2 |
                   3 l
                         6 I
                              11 |
                                     1 |
  9 |
          2 |
                   4 | 11 |
                              13 |
                                     1 |
                                              3
          2 |
                   5 |
                              -1 |
                                     0 |
  10 |
                        12 |
(10 rows)
```

Examples for queries marked as $\mathtt{directed}$ with \mathtt{cost} and $\mathtt{reverse_cost}$ $\mathtt{columns}\P$

The examples in this section use the following $Network\ for\ queries\ marked\ as\ directed\ and\ cost\ and\ reverse_cost\ columns\ are\ used$

```
SELECT * FROM pgr_KSP(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 12, 2
  );
 seq | path_id | path_seq | node | edge | cost | agg_cost
1 | 1 |
                      2 |
                            4 |
                                 1 |
                  2 |
                             8 |
  2 |
         1 |
                       5 l
                                   1 |
                                            1
  3 |
          1 |
                  3 I
                        6 I
                              9 |
                                   1 |
                                   1 |
  4 |
         1 |
                  4 |
                       9 |
                             15 |
         1 |
                  5 |
  5 I
                       12 |
                             -1 l
                                   0 |
                                            4
          2 |
                  1 |
                       2 |
                              4 l
  6 I
                                   1 |
                                            0
                  2 |
  7 |
          2 |
                        5 I
                             8 I
                                   1 |
                                            1
          2 |
                 3 |
                       6 l
                                            2
  8 |
                            11 |
                                 1 |
         2 |
                  4 |
  9 |
                       11 | 13 | 1 |
```

```
10 |
            2 |
                      5 | 12 | -1 | 0 |
(10 rows)
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     2, 12, 2, heap_paths:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
            1 | 1 | 2 | 4 |
                                          1 |
   2 |
            1 |
                      2 |
                           5 |
                                   8 |
                                          1 |
                                                    1
   3 |
            1 |
                      3 |
                            6 |
                                   9 |
                                          1 |
                                                    2
   4 |
            1 |
                      4 |
                            9 |
                                  15 |
                                          1 |
   5 |
            1 |
                      5 I
                           12 |
                                  -1 l
                                          0 I
                                                    4
                                   4 |
  6 I
            2 |
                     1 |
                           2 |
                                          1 |
                                                    0
   7 |
            2 |
                     2 |
                           5 l
                                   8 |
                                          1 |
   8 |
            2 |
                     3 |
                            6 I
                                  11 |
  9 |
            2 |
                     4 |
                            11 |
                                  13 |
                                          1 |
                                                    3
            2 |
                     5 |
  10 |
                            12 |
                                  -1 |
                                          0 |
                                                    4
  11 |
            3 |
                      1 |
                             2 |
                                   4 |
                                          1 |
                                                    0
  12 |
            3 I
                      2 |
                            5 I
                                  10 l
                                          1 |
                                                    1
            3 I
                     3 l
                           10 |
                                 12 |
                                                    2
  13 |
                                          1 |
  14 |
            3 |
                      4 |
                            11 |
                                  13 |
                                                    3
                                          1 |
  15 |
            3 |
                      5 l
                            12 |
                                  -1 |
                                          0 |
                                                    4
(15 rows)
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     2, 12, 2, true, true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
4 I
                                          1 l
   1 |
            1 l
                      1 |
                             2 |
   2 |
            1 |
                      2 |
                           5 l
                                   8 |
                                          1 |
                                                    1
   3 |
            1 |
                      3 |
                            6 |
                                   9 |
                                          1 |
                                                    2
   4 |
            1 |
                      4 |
                            9 |
                                  15 |
                                          1 |
                                                    3
                           12 |
   5 I
            1 |
                      5 I
                                  -1 |
                                          0 |
                                                    4
            2 |
                                   4 |
                                                    0
   6 |
                      1 |
                           2 |
                                          1 |
            2 |
                     2 |
   7 |
                           5 I
                                  8 |
                                          1 |
                                                    1
   8 |
            2 |
                     3 I
                           6 I
                                  11 |
                                          1 |
            2 |
                                          1 |
   9 |
                     4 |
                           11 |
                                  13 |
                                                    3
            2 |
                     5 |
  10 |
                            12 |
                                  -1 l
                                          0 |
                                                    4
            3 |
                      1 |
                            2 |
  11 |
                                   4 |
                                          1 |
                                                    0
  12 |
            3 I
                      2 |
                            5 I
                                  10 |
                                          1 |
                                                    1
            3 |
                      3 |
                                          1 |
                                                    2
  13 |
                            10 |
                                  12 |
```

11 |

13 |

1 |

3

4 |

14 |

3 |

```
15 | 3 | 5 | 12 | -1 | 0 | 4 (15 rows)
```

Examples for queries marked as undirected with cost and $reverse_cost$ columns \P

The examples in this section use the following Network for queries marked as undirected and cost and reverse_cost columns are used

```
SELECT * FROM pgr_KSP(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     2, 12, 2, directed:=false
  );
 seq | path_id | path_seq | node | edge | cost | agg_cost
1 |
  1 |
           1 |
                        2 |
                                2 |
  2 |
           1 |
                   2 |
                         3 |
                                3 |
                                       1 |
  3 |
           1 |
                   3 |
                          4 |
                                       1 |
                               16 |
                                                2
                                      1 |
  4 |
           1 |
                    4 |
                          9 |
                               15 |
                                                3
                    5 I
  5 |
           1 |
                          12 |
                               -1 |
                                      0 |
  6 I
           2 |
                    1 |
                          2 |
                                4 |
                                      1 |
  7 |
           2 |
                    2 |
                         5 |
                                8 I
                                      1 |
                                                1
           2 |
                    3 |
  8 |
                         6 | 11 |
                                      1 |
                                                2
  9 |
           2 |
                    4 |
                        11 |
                               13 |
                                      1 |
                                                3
                    5 | 12 |
           2 |
  10 |
                               -1 |
                                       0 |
(10 rows)
SELECT * FROM pgr_KSP(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     2, 12, 2, false, true
 seq | path_id | path_seq | node | edge | cost | agg_cost
1 |
                    1 |
                          2 |
                                2 |
                                       1 |
           1 |
                         3 |
           1 |
                    2 |
                                3 |
  2 |
                                      1 |
                                                1
  3 |
           1 |
                    3 |
                          4 |
                               16 |
                                      1 |
                                                2
  4 |
           1 |
                    4 |
                          9 |
                               15 |
                                      1 |
                                                3
                    5 |
                                      0 |
  5 |
           1 |
                         12 |
                               -1 |
           2 |
                                4 |
  6 |
                    1 |
                         2 |
                                      1 |
  7 |
           2 |
                    2 |
                         5 I
                                8 I
                                      1 |
                    3 I
                                      1 |
  8 |
           2 |
                         6 I
                               11 |
           2 |
                                      1 |
  9 |
                    4 |
                          11 |
                                13 |
                                                3
           2 |
                    5 I
  10 |
                          12 |
                                -1 |
                                      0 |
                                                4
           3 I
                    1 |
                          2 |
                                4 |
  11 |
                                      1 |
                                                0
          3 I
                    2 |
                         5 I
                               10 |
  12 |
                                     1 |
                                                1
```

10 |

12 |

1 |

3 |

13 |

3 |

```
14 |
           3 |
                     4 |
                           11 |
                                  13 |
 15 l
           3 l
                    5 I
                           12 |
                                  -1 l
                                         0 |
 16 l
           4 |
                     1 |
                            2 |
                                  4 |
                                         1 |
                                                    0
           4 |
4 |
4 |
4 |
                     2 |
 17 |
                           5 l
                                  10 |
                                         1 |
                                                    1
 18 |
                     3 |
                          10 |
                                  12 |
 19 |
                     4 | 11 |
                                  11 |
                                         1 |
                                                    3
                    5 | 6 |
                                  9 |
                                                    4
 20 |
                                         1 |
           4 |
                     6 I
                          9 |
                                  15 |
                                         1 |
                                                    5
 21 |
 22 |
           4 |
                     7 |
                           12 |
                                  -1 l
(22 rows)
```

Examples for queries marked as directed with cost column¶

The examples in this section use the following Network for queries marked as directed and only cost column is used

```
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost FROM edge_table',
 seq | path_id | path_seq | node | edge | cost | agg_cost
(0 rows)
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost FROM edge_table',
     2, 12, 2
  );
 seq | path_id | path_seq | node | edge | cost | agg_cost
____+
          1 | 1 | 2 |
                              4 |
                                    1 |
  1 |
  2 |
          1 |
                   2 | 5 | 8 |
                                    1 |
          1 l
                   3 l
                        6 |
                               9 I
                                     1 l
  3 I
                                               2
  4 |
          1 |
                   4 |
                                     1 |
                        9 |
                              15 |
                                               3
          1 | 1 | 2 | 2 | 2 | 2 | 2 |
                   5 | 12 |
  5 |
                               -1 |
                                      0 |
                        2 |
  6 |
                    1 |
                               4 |
                                      1 |
  7 |
                   2 | 5 |
                               8 |
                                     1 |
                                               1
                   3 | 6 | 11 |
                                               2
  8 |
                                    1 |
          2 |
                   4 | 11 | 13 |
  9 |
                                     1 |
                                               3
  10 |
           2 |
                   5 | 12 | -1 |
(10 rows)
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost FROM edge_table',
     2, 12, 2, heap_paths:=true
  );
```

+		+-	+	+	+	
1	1	1	2	4	1	0
2	1	2	5	8	1	1
3	1	3	6	9	1	2
4	1	4	9	15	1	3
5	1	5	12	-1	0	4
6	2	1	2	4	1	0
7	2	2	5	8	1	1
8	2	3	6	11	1	2
9	2	4	11	13	1	3
10	2	5	12	-1	0	4
11	3	1	2	4	1	0
12	3	2	5	10	1	1
13	3	3	10	12	1	2
14	3	4	11	13	1	3
15	3	5	12	-1	0	4
(15 rd	ows)					
	T * FROM pgr_KS		caot (and+ EDC	NM odgo	+25121
,	'SELECT id, sour		iget, (JUSU FRU	m eage_	table,
'	2, 12, 2, true,		iget, (JOST FAC	m eage_	table,
);	2, 12, 2, true,	true				
);		true				
); seq	2, 12, 2, true, path_id path_	true _seq	node +	edge	cost	agg_cost
); seq + 1	2, 12, 2, true, path_id path	true _seq + 1	node + 2	edge + 4	cost + 1	agg_cost 0
); seq + 1 2	2, 12, 2, true, path_id path 1 1	true _seq +- 1 2	node + 2 5	edge + 4 8	cost + 1 1	agg_cost 0 1
); seq + 1 2 3	2, 12, 2, true, path_id path 1 1 1	true _seq +- 1 2 3	node + 2 5 6	edge + 4 8 9	cost + 1 1 1	agg_cost 0 0 1 2
); seq + 1 2 3 4	2, 12, 2, true, path_id path 1 1 1 1	true _seq + 1 2 3 4	node + 2 5 6	edge + 4 8 9	cost + 1 1 1	agg_cost 0 1 2 3
); seq + 1 2 3 4 5	2, 12, 2, true, path_id path	true _seq 1 2 3 4 5	node + 2 5 6 9	edge + 4 8 9 15	cost 1 1 1 1 0	agg_cost 0 1 2 3 4
); seq + 1 2 3 4 5 6	2, 12, 2, true, path_id path	true _seq 1 2 3 4 5 1	node+ 2 5 6 9 12 2	edge 	cost 1	agg_cost 0 1 2 3 4
); seq + 1 2 3 4 5 6 7	2, 12, 2, true, path_id path	true _seq 1 2 3 4 5 1 2	node+ 2 5 6 9 12 2 5	edge 	cost + 1 1 1 0 1 1	agg_cost 0 1 2 3 4 0 1
); seq + 1 2 3 4 5 6 7 8	2, 12, 2, true, path_id path 1 1 1 1 1 2 2	true _seq +- 1 2 3 4 5 1 2 3	node+ 2 5 6 9 12 2 5 6	edge + 4 8 9 15 -1 4 8	cost + 1 1 1 0 1 1 1	agg_cost 0 1 2 3 4 0 1 2
); seq + 1 2 3 4 5 6 7 8 9	2, 12, 2, true, path_id path 1 1 1 1 2 2 2	true _seq 1 2 3 4 5 1 2 3 4	node	edge	cost 1 1 1 1 0 1 1 1	agg_cost 0 1 2 3 4 0 1 2 3
); seq + 1 2 3 4 5 6 7 8	2, 12, 2, true, path_id path	true _seq 1 2 3 4 5 1 2 3 4 5 5	node	edge	cost + 1 1 1 0 1 1 1	agg_cost 0 1 2 3 4 0 1 2
); seq + 1 2 3 4 5 6 7 8 9 10 11	2, 12, 2, true, path_id path 1 1 1 1 2 2 2	true _seq 1 2 3 4 5 1 2 3 4 5 1 1 1	node	edge	cost 1 1 1 1 0 1 1 1	agg_cost 0 1 2 3 4 0 1 2 3
); seq + 1 2 3 4 5 6 7 8 9 10 11 12	2, 12, 2, true, path_id path	true _seq 1 2 3 4 5 1 2 3 4 5 5	node	edge	cost 1 1 1 1 1 1 1 1 1 1	agg_cost 0 1 2 3 4 0 1 2 3 4 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
); seq + 1 2 3 4 5 6 7 8 9 10 11 12 13	2, 12, 2, true, path_id path	true _seq 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 3	node	edge	cost 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1	agg_cost 0 1 2 3 4 0 1 2 3 4 0 1 2 3 4 0 1 2
); seq + 1 2 3 4 5 6 7 8 9 10 11 12 13 14	2, 12, 2, true, path_id path	true _seq 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 4	node+ 2 5 6 9 12 2 5 6 11 12 2 5 10 11	edge	cost 1 1 1 1 1 1 1 1 1 1	agg_cost 0 1 2 3 4 0 1 2 3 4 0 1 2 3 4 0 1 2 3 3
); seq + 1 2 3 4 5 6 7 8 9 10 11 12 13	2, 12, 2, true, path_id path	true _seq 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 3	node+ 2 5 6 9 12 2 5 6 11 12 2 5 10	edge	cost 1 1 1 1 1 1 1 1 1 1	agg_cost 0 1 2 3 4 0 1 2 3 4 0 1 2 3 3 4 0 1 2 3

seq | path_id | path_seq | node | edge | cost | agg_cost

Examples for queries marked as $\mathtt{undirected}$ with \mathtt{cost} $\mathtt{column}\P$

The examples in this section use the following $Network\ for\ queries\ marked\ as\ undirected\ and\ only\ cost\ column\ is\ used$

```
SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, directed:=false
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
                                                            0
   1 |
             1 |
                         1 |
                                 2 |
                                        4 |
                                                1 |
                         2 |
                                 5 I
                                        8 I
   2 |
             1 |
                                                1 |
                                                            1
   3 |
             1 |
                         3 I
                                 6 I
                                        9 |
                                                1 |
   4 |
             1 |
                         4 |
                                 9 |
                                        15 |
                                                1 |
                                                            3
   5 |
             1 |
                         5 |
                                12 |
                                       -1 |
                                                0 |
                                                            4
             2 |
                         1 |
                                2 |
                                        4 |
                                                            0
   6 |
                                                1 |
   7 |
             2 |
                         2 |
                                 5 |
                                        8 |
                                                1 |
                                                            1
             2 |
   8 |
                         3 I
                                6 I
                                       11 |
                                                1 |
                                                            2
             2 |
                         4 |
   9 |
                                11 |
                                       13 |
                                                1 |
                                                            3
  10 |
             2 |
                         5 |
                                12 |
                                       -1 |
                                                            4
(10 rows)
SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, directed:=false, heap_paths:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
   1 |
             1 |
                         1 |
                                 2 |
                                        4 |
                                                1 |
   2 |
             1 |
                         2 |
                                 5 I
                                        8 |
                                                1 |
                                                            1
   3 |
             1 |
                         3 |
                                 6 I
                                        9 |
                                                            2
                                                1 |
   4 |
             1 |
                         4 |
                                9 |
                                       15 |
                                                1 |
                                                            3
   5 I
                         5 I
                                       -1 l
             1 |
                                12 |
                                                0 |
                                                            4
             2 |
   6 I
                         1 |
                               2 I
                                        4 I
                                                            0
                                                1 |
             2 |
                         2 |
                                                1 |
   7 |
                                 5 |
                                        8 |
                                                            1
   8 |
             2 |
                        3 |
                                 6 |
                                       11 |
             2 I
                         4 I
   9 |
                                11 |
                                       13 l
                                                1 |
                                                            3
             2 |
                         5 |
                                12 |
                                                0 |
                                                            4
  10 |
                                       -1 |
             3 |
                         1 |
                                 2 |
                                        4 |
  11 |
                                                1 |
                                                            0
  12 |
             3 |
                         2 |
                                 5 |
                                       10 |
                                                1 |
                                                            1
  13 l
             3 |
                         3 I
                                10 |
                                       12 |
                                                1 |
                                                            2
             3 |
                         4 |
                                                            3
  14 |
                                11 |
                                       13 |
                                                1 |
             3 |
                         5 I
                                                0 |
  15 |
                                12 |
                                       -1 |
                                                            4
(15 rows)
```

See Also \P

- http://en.wikipedia.org/wiki/K_shortest_path_routing
- Sample Data network.

Indices and tables

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pgr_dijkstraVia - Proposed¶ Name¶

pgr_dijkstraVia — Using dijkstra algorithm, it finds the route that goes through a list of vertices.



Boost Graph Inside

Availability: 2.2.0

Synopsis

Given a list of vertices and a graph, this function is equivalent to finding the shortest path between $\langle (\text{vertex_i}) \rangle$ and $\langle (\text{vertex_i+1}) \rangle$ for all $\langle (\text{i} < \text{size}_of(\text{vertex_via}))$.

The paths represents the sections of the route.

Note

This is a proposed function

Signatrue Summary¶

```
pgr_dijkstraVia(edges_sql, via_vertices)
pgr_dijkstraVia(edges_sql, via_vertices, directed, strict, U_turn_on_edge)
```

RETURNS SET OF (seq, path_pid, path_seq, start_vid, end_vid, node, edge, cost, agg_cost, route_agg_cost) or EMPTY SET

Signatures¶

Minimal Signature¶

Example:

Find the route that visits the vertices $1\ 3\ 9$ in that order

```
SELECT * FROM pgr_dijkstraVia(
  'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
   ARRAY[1, 3, 9]
);
seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost | route_agg_cost
       1 |
                                               0 |
 1 |
              1 |
                            3 |
                               1 |
                                     1 | 1 |
                                                          0
              2 |
                           3 |
                                2 |
                                     4 | 1 |
                                                1 |
                                                          1
 2 |
       1 |
                    1 |
 3 |
              3 |
                           3 l
                                5 I
                                     8 | 1 |
                                               2 |
                                                          2
       1 |
                     1 |
 4 I
       1 |
              4 |
                     1 |
                           3 |
                                6 |
                                     9 |
                                          1 |
                                               3 |
                                                          3
 5 I
              5 |
                     1 |
                            3 |
                                9 |
                                    16 | 1 |
                                                4 |
       1 |
                            3 |
                                               5 |
                                                          5
 6 |
       1 |
              6 |
                     1 |
                                4 |
                                     3 |
                                          1 |
 7 |
       1 |
              7 |
                     1 |
                            3 |
                                3 |
                                    -1 |
                                         0 |
                                                6 |
              1 |
       2 |
                     3 |
                            9 | 3 | 5 |
                                          1 |
                                               0 |
                                                          6
 8 I
 9 |
       2 |
              2 |
                     3 |
                            9 |
                                6 | 9 |
                                          1 |
                                               1 |
                                                          7
 10 |
        2 |
             3 |
                      3 |
                           9 | 9 | -2 | 0 |
                                                2 |
(10 rows)
```

Complete Signature¶

Example:

6 |

(6 rows)

Find the route that visits the vertices 1 3 9 in that order on an undirected graph, avoiding U-turns when possible

```
SELECT * FROM pgr_dijkstraVia(
  'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
   ARRAY[1, 3, 9], false, strict:=true, U_turn_on_edge:=false
);
seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost | route_agg_cost
1 | 1 | 3 | 1 | 1 | 1 | 0 |
 1 |
       1 |
                                                       0
 2 |
       1 |
             2 |
                    1 |
                          3 | 2 |
                                   2 |
                                        1 |
                                              1 |
                                                        1
 3 I
       1 |
              3 |
                    1 |
                          3 |
                               3 | -1 | 0 |
                                              2 I
                          9 |
                     3 I
                               3 |
                                   5 | 1 |
                                              0 |
 4 |
       2 |
              1 |
                                                        2
                          9 |
                               6 | 9 |
                                             1 |
       2 |
              2 |
                     3 I
                                                        3
 5 I
                                        1 |
```

9 | -2 | 0 |

2 |

Description of the Signature¶

2 |

3 |

Description of the edges_sql query for dijkstra like functions¶

3 |

9 |

edges_sql: an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
reverse_	c ant -numerical	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source), When negative: edge (target, so
		not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter	Туре
edges_sql	TEXT
via_vertices	ARRAY[ANY-INTEGER]
directed	BOOLEAN
strict	BOOLEAN

Parameter	Туре
U_turn_on_edge	BOOLEAN

Description of the parameters of the signatures \P

Parameter	Type	Description
edges_sql	TEXT	SQL query as described above
via_vertices	ARRAY [A	N A:rnyegerd rtices identifiers
directed	BOOLEAN	(optional) Default is true (is the graph is considered as U
strict	BOOLEAN	(optional) ignores if a subsec and returns everything it fou directed). When set to false Undirected
$U_turn_on_edge$	BOOLEAN	(optional) Default is true (is the graph is considered as U

Description of the return values \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
seq BIGINT	Sequential value starting from 1.
path_pidlGINT	Identifier of the path.
path_se&IGINT	Sequential value starting from 1 for the par
start_vi d IGINT	Identifier of the starting vertex of the path
${f end_vid}{f BIGINT}$	Identifier of the ending vertex of the path.
node BIGINT	Identifier of the node in the path from star end_vid.

Column	Туре	Description
edge	BIGINT	Identifier of the edge used to go from node in the path sequence1 for the last node of the last node of the route.
cost	FLOAT	Cost to traverse from node using edge to t the route sequence.
agg_co	stFLOAT	Total cost from start_vid to end_vid of t
route_a	a <u>gy.04</u> Tost	Total cost from start_vid of path_pid = the current path_pid .

$Examples \P$

Example 1:

Find the route that visits the vertices 1 5 3 9 4 in that order

```
SELECT * FROM pgr_dijkstraVia(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
);
seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost | route_agg_cost
                                                                  0 |
                                                                                0
  1 |
           1 |
                    1 |
                              1 |
                                      5 |
                                             1 |
                                                   1 |
                                                         1 |
                                      5 |
                                             2 |
                                                                                1
  2 |
           1 |
                    2 |
                              1 |
                                                   4 |
                                                         1 |
                                                                   1 |
  3 |
          1 |
                    3 |
                              1 |
                                      5 |
                                             5 |
                                                  -1 |
                                                          0 |
                                                                   2 |
                                                                                 2
  4 I
          2 |
                    1 |
                                      3 |
                                             5 I
                                                   8 |
                                                         1 |
                                                                  0 |
                                                                                2
                              5 |
                                                                                3
  5 |
          2 |
                    2 |
                              5 |
                                      3 |
                                             6 |
                                                   9 |
                                                         1 |
                                                                   1 |
          2 |
                                      3 |
                                                                   2 |
  6 |
                    3 |
                              5 I
                                             9 |
                                                  16 |
                                                          1 |
  7 |
          2 |
                    4 |
                              5 I
                                      3 |
                                             4 |
                                                   3 |
                                                         1 |
                                                                  3 |
                                                                                 5
          2 |
                    5 I
                              5 I
                                      3 |
                                             3 |
                                                  -1 |
                                                          0 |
                                                                   4 I
                                                                                 6
  8 1
                                                   5 |
                                                                                6
  9 |
          3 |
                    1 |
                              3 |
                                      9 |
                                             3 |
                                                         1 |
                                                                  0 |
 10 l
           3 |
                    2 |
                              3 |
                                       9 |
                                             6 |
                                                   9 |
                                                          1 |
                                                                   1 |
                                                                                 7
           3 |
                    3 |
                              3 |
                                       9 |
                                                  -1 |
                                                                   2 |
                                                                                 8
 11 |
                                             9 |
                                                          0 |
                              9 |
                    1 |
                                       4 |
                                                          1 |
                                                                                 8
 12 |
           4 |
                                             9 |
                                                  16 |
                                                                   0 |
                                                                                 9
                    2 |
                              9 |
                                       4 |
                                             4 |
                                                  -2 |
 13 |
           4 |
                                                          0 |
                                                                   1 |
```

Example 2:

(13 rows)

What's the aggregate cost of the third path?

```
SELECT agg_cost FROM pgr_dijkstraVia(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
```

```
ARRAY[1, 5, 3, 9, 4]
)
WHERE path_id = 3 AND edge <0;
agg_cost
(1 row)
Example 3:
What's the route's aggregate cost of the route at the end of the third path?
SELECT route_agg_cost FROM pgr_dijkstraVia(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
WHERE path_id = 3 AND edge < 0;
route_agg_cost
              8
(1 row)
Example 4:
How are the nodes visited in the route?
SELECT row_number() over () as node_seq, node
FROM pgr_dijkstraVia(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
)
WHERE edge <> -1 ORDER BY seq;
node_seq | node
        1 |
               1
        2 |
               2
        3 |
               5
        4 |
               6
        5 I
               9
        6 |
               4
        7 |
               3
        8 |
               6
        9 |
               9
               4
       10 |
(10 rows)
```

Example 5:

```
What are the aggregate costs of the route when the visited vertices are reached?
```

```
SELECT path_id, route_agg_cost FROM pgr_dijkstraVia(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
WHERE edge < 0;
 path_id | route_agg_cost
      1 |
       2 |
       3 |
                        8
       4 |
(4 rows)
Example 6:
show the route's seq and aggregate cost and a status of "passes in front" or
"visits" node 9
```

```
SELECT seq, route_agg_cost, node, agg_cost ,
CASE WHEN edge = -1 THEN 'visits'
ELSE 'passes in front'
END as status
FROM pgr dijkstraVia(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
   ARRAY[1, 5, 3, 9, 4])
WHERE node = 9 and (agg_cost <> 0 or seq = 1);
 seq | route_agg_cost | node | agg_cost | status
----+-----
                 4 | 9 | 2 | passes in front
8 | 9 | 2 | visits
  6 I
  11 |
(2 rows)
```

ROLLBACK;

ROLLBACK

See Also¶

- http://en.wikipedia.org/wiki/Dijkstra%27s_algorithm
- Sample Data network.

Indices and tables

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The problem definition (Advanced documentation) \P Given the following query:

```
\label{eq:cost_invariant} $$  \operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}_{\operatorname{vid}}}}}}}}}}}}}}}}}}}}} } } } } } }
```

- $\setminus (\text{source} = \text{bigcup source}_i \setminus),$
- $\t(target = \bigcup target_i\),$

The graphs are defined as follows:

Directed graph

The weighted directed graph, $\backslash (G_d(V,E)\backslash)$, is definied by:

- the set of vertices $\langle (V \rangle)$
- the set of edges $\backslash (E \backslash)$

Undirected graph

The weighted undirected graph, $\langle G_u(V,E) \rangle$, is definied by:

- the set of vertices $\setminus (V \setminus)$
 - $(V = source \cup target \cup \{start_v\{vid\}\} \cup \{end_{vid}\}\})$
- the set of edges $\backslash (E \backslash)$

The problem

Given:

- $\sl(start_{vid} \in V)$ a starting vertex
- \(end_{vid} \in V\) an ending vertex

Then:

where:

- $\\ (node_i \in V)$
- $\\(node_1 = start_{vid})$
- \(\forall i \neq | \pi |, \quad (node_i, node_ $\{i+1\}$, cost_i) \in E\)
- \(\(\text{edge_i} = \begin{\cases} id_{(node_i, node_{i+1}, \cost_i)} & \quad \text{\when } i \neq | \pi | \\ -1 & \quad \text{\when } i = | \pi | \\ \end{\cases}\\)
- $\langle \text{cost}_i = \text{cost}_{\{\text{node}_i, \text{node}_{\{i+1\}}\}} \rangle$
- \(\agg_cost_i = \begin{cases} 0 &\quad \text{when } i = 1 \\ \displaystyle\sum_{k=1}^{i} \cost_{(node_{k-1}, node_k)} &\quad \text{when } i \neq 1 \\ \end{cases}\)

In other words: The algorithm returns a the shortest path between $\sl(start_{vid})$ and $\sl(path_seq)$ indicates the relative position in the path of the $\sl(path_seq)$.

- \(cost\) is the cost of the edge to be used to go to the next node.
- $\langle (agg \setminus cost) \rangle$ is the cost from the $\langle (start_{vid}) \rangle$ up to the node.

If there is no path, the resulting set is empty.

See Also¶ Indices and tables

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pgr_KSP¶

Name¶ pgr_KSP — Returns the "K" shortest paths.



Boost Graph Inside Availability: 2.0.0

• Signature change 2.1.0

Synopsis¶ The K shortest path routing algorithm based on Yen's algorithm. "K" is the number of shortest paths desired.

Signature Summary¶

```
pgr_KSP(edges_sql, start_vid, end_vid, K);
pgr_KSP(edges_sql, start_vid, end_vid, k, directed, heap_paths)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Signatures¶ Minimal Signature¶

```
pgr_ksp(edges_sql, start_vid, end_vid, K);
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Complete Signature¶

```
pgr_KSP(edges_sql, start_vid, end_vid, k, directed, heap_paths)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Description of the Signatures ¶ Description of the edges_sql query for dijkstra like functions ¶

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.

Column	Type	Default	Description
target	ANY-INTEG	ER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMER	ICAL	Weight of the edge (source, target)
reverse_	_c any -numer	I <i>C</i> IAL	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source),
			• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

 $SMALLINT,\,INTEGER,\,BIGINT,\,REAL,\,FLOAT$

Description of the parameters of the signatures \P

Column	Туре
edges_sql	TEXT
$start_vid$	BIGINT
${ m end_vid}$	BIGINT
k	INTEGER
directed	BOOLEAN
heap_paths	BOOLEAN

Roughly, if the shortest path has N edges, the heap will contain about than N * k paths for small value of k and k > 1.

Description of the return values¶

Returns set of (seq, path_seq, path_id, node, edge, cost, agg_cost)

Column	Type
\overline{seq}	INTEGER
$path_seq$	INTEGER
$\operatorname{path}_{-\operatorname{id}}$	BIGINT
node	BIGINT
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

Warning

During the transition to 3.0, because pgr_ksp version 2.0 doesn't have defined a directed flag nor a heap_path flag, when pgr_ksp is used with only one flag version 2.0 signature will be used.

 $\begin{tabular}{ll} \bf Additional \ \bf Examples \ \bf I \ \bf I$

The examples in this section use the following Network for queries marked as directed and cost and reverse_cost columns are used

```
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2,
      directed:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
   1 |
              1 |
                          1 |
                                  2 |
                                         4 |
                                                 1 |
                                                             0
   2 |
              1 |
                          2 |
                                  5 I
                                         8 |
                                                 1 |
                                                             1
   3 |
              1 |
                          3 |
                                  6 |
                                         9 |
                                                 1 |
                                                             2
   4 |
              1 |
                          4 |
                                  9 |
                                        15 |
                                                 1 |
                                                             3
                          5 I
   5 I
              1 |
                                12 |
                                        -1 |
                                                 0 |
                                                             4
              2 |
   6 |
                          1 |
                                  2 |
                                         4 |
                                                 1 |
                                                             0
   7 |
              2 |
                          2 |
                                  5 I
                                         8 |
                                                 1 |
                                                             1
   8 |
              2 |
                          3 |
                                  6 I
                                                 1 |
                                                             2
                                        11 |
   9 |
              2 |
                          4 |
                                        13 |
                                                 1 |
                                                             3
                                11 |
  10 |
              2 |
                                                             4
                          5 |
                                12 |
                                        -1 |
                                                 0 |
```

```
(10 rows)
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     2, 12, 2
  );
seq | path_id | path_seq | node | edge | cost | agg_cost
                      1 |
                             2 |
                                    4 I
                      2 |
                                    8 |
  2 |
            1 |
                             5 I
                                           1 |
  3 |
            1 |
                      3 l
                             6 |
                                    9 |
                                           1 |
            1 |
                      4 |
  4 |
                            9 |
                                   15 |
                                           1 |
                                                      3
  5 |
            1 |
                      5 I
                             12 |
                                   -1 |
                                           0 |
                           2 |
  6 |
            2 |
                      1 |
                                    4 I
                                           1 |
            2 |
                      2 |
                            5 I
                                    8 I
  7 |
                                           1 |
  8 |
           2 |
                      3 |
                            6 l
                                   11 |
                                           1 |
  9 |
            2 |
                      4 |
                                          1 |
                             11 |
                                   13 |
 10 |
            2 |
                      5 |
                             12 |
                                           0 |
                                                      4
                                   -1 |
(10 rows)
```

Examples for queries marked as $\mathtt{directed}$ with \mathtt{cost} and $\mathtt{reverse_cost}$ $\mathtt{columns}\P$

The examples in this section use the following Network for queries marked as directed and cost and reverse_cost columns are used

```
SELECT * FROM pgr_KSP(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     2, 12, 2
  );
seq | path_id | path_seq | node | edge | cost | agg_cost
---+----+-----
                                4 I
                                      1 |
  1 |
           1 |
                    1 |
                          2 |
                   2 |
                        5 |
                                8 |
  2 |
           1 |
                                      1 |
                                                1
           1 |
                    3 |
  3 |
                          6 |
                                9 |
                                      1 |
                                                2
  4 |
          1 |
                    4 |
                          9 |
                               15 |
                                      1 |
  5 I
          1 |
                   5 I
                        12 |
                               -1 |
                                      0 |
                                                4
           2 |
                   1 |
                                4 |
  6 |
                        2 |
                                      1 |
          2 |
  7 |
                   2 |
                        5 l
                                8 |
                                      1 |
  8 |
          2 |
                   3 l
                         6 l
                               11 |
                                      1 |
           2 |
  9 |
                    4 |
                         11 |
                               13 |
                                      1 |
                                                3
 10 |
           2 |
                   5 |
                         12 |
                               -1 l
                                      0 |
(10 rows)
```

SELECT * FROM pgr_KSP(

'SELECT id, source, target, cost, reverse_cost FROM edge_table',

2, 12, 2, heap_paths:=true); seq | path_id | path_seq | node | edge | cost | agg_cost _____ 1 | 1 | 1 | 2 | 1 | 2 | 1 | 2 | 5 I 8 I 1 | 1 3 | 2 3 | 1 | 6 I 9 | 1 | 4 | 3 4 | 1 | 9 | 15 I 1 | 5 I 1 | 5 I 12 | -1 | 6 | 2 | 1 | 2 | 4 | 1 | 0 7 | 2 | 2 | 5 | 8 | 1 | 1 2 | 8 | 3 | 2 6 | 11 | 1 | 9 | 2 | 4 | 11 | 13 | 1 | 2 | 10 | 5 I 12 | -1 l 0 | 4 3 | 11 | 1 | 2 | 4 | 1 | 0 12 | 3 | 2 | 5 I 10 | 13 | 3 | 3 | 10 | 2 12 | 3 | 4 | 3 14 | 11 | 13 | 1 | 15 | 3 | 5 | 12 | 0 | 4 -1 | (15 rows) SELECT * FROM pgr_KSP('SELECT id, source, target, cost, reverse_cost FROM edge_table', 2, 12, 2, true, true); seq | path_id | path_seq | node | edge | cost | agg_cost 1 | 1 | 1 | 1 | 2 | 4 | 2 | 2 | 8 I 1 | 5 I 1 | 1 3 | 1 | 3 I 9 I 2 6 | 1 | 4 | 3 4 | 1 | 9 | 15 | 1 | 5 | 1 | 5 | 12 | -1 | 6 I 2 | 1 | 2 | 4 I 1 | 0 7 | 2 | 2 | 5 I 8 | 1 | 1 2 | 3 | 2 8 | 6 | 11 | 1 | 9 | 2 | 4 | 11 | 13 | 3 10 | 2 | 5 | 12 | -1 l 0 | 4 3 | 0 11 | 1 | 2 | 4 | 1 | 3 | 2 | 5 I 12 | 10 | 1 | 1 13 | 3 | 3 I 10 | 12 | 1 | 2 11 l 14 | 3 | 4 | 13 | 3 1 | 15 | 3 | 5 I 12 | -1 l 4 (15 rows)

Examples for queries marked as $undirected\ with\ cost\ and\ reverse_cost\ columns\P$

The examples in this section use the following Network for queries marked as undirected and cost and reverse_cost columns are used

```
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     2, 12, 2, directed:=false
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
                                    2 |
            1 | 1 | 2 |
                                          1 |
   2 |
            1 |
                      2 |
                             3 |
                                   3 |
                                          1 |
                                                     1
                      3 |
   3 |
            1 |
                             4 |
                                   16 |
                                          1 |
                                                     2
   4 |
            1 |
                      4 |
                             9 |
                                   15 |
                                          1 |
   5 |
            1 |
                      5 |
                            12 |
                                   -1 l
                                          0 |
                                                     4
  6 |
                           2 |
                                   4 |
            2 |
                     1 |
                                          1 |
                                                     0
                     2 |
   7 |
            2 |
                             5 |
                                   8 |
                                          1 |
            2 |
                      3 |
                             6 |
   8 |
                                   11 |
                                          1 l
   9 |
            2 |
                      4 |
                                                     3
                            11 |
                                   13 |
            2 |
                      5 |
                                   -1 |
                                           0 |
  10 |
                            12 |
                                                     4
(10 rows)
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     2, 12, 2, false, true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
____+
            1 |
                      1 |
                             2 |
                                    2 |
                                           1 |
   1 |
                      2 |
                             3 |
                                    3 |
   2 |
            1 |
                                           1 |
                                                     1
   3 I
                      3 |
                             4 I
                                   16 l
                                          1 l
                                                     2
            1 |
                      4 |
                                   15 |
                                                     3
   4 |
            1 |
                           9 |
                                          1 |
   5 |
            1 |
                     5 | 12 |
                                   -1 |
                     1 l
   6 I
            2 |
                             2 I
                                   4 I
                                           1 l
                                                     0
   7 |
            2 |
                      2 |
                             5 |
                                    8 |
                                           1 |
                                                     1
            2 |
                      3 |
                             6 |
                                                     2
   8 |
                                   11 |
                                           1 |
   9 |
            2 |
                      4 |
                            11 |
                                   13 |
                                                     3
  10 l
            2 |
                     5 |
                            12 |
                                   -1 l
                                           0 |
                                                     4
            3 |
                     1 |
                            2 |
                                   4 |
                                                     0
  11 |
                                          1 |
                      2 |
            3 |
                             5 I
  12 |
                                   10 |
                                           1 |
                                                     1
  13 l
            3 l
                      3 l
                            10 l
                                   12 |
                                           1 |
                                           1 |
  14 I
            3 l
                      4 |
                            11 |
                                   13 |
                                                     3
                      5 |
  15 |
            3 |
                            12 |
                                   -1 |
                                           0 |
                                                     4
            4 |
                      1 |
                             2 |
  16 |
                                   4 |
                                           1 |
                                                     0
  17 |
            4 |
                      2 |
                             5 I
                                   10 |
                                           1 |
                                                     1
            4 |
                      3 I
                                                     2
  18 |
                            10 |
                                   12 |
                                           1 |
  19 |
            4 |
                      4 |
                            11 |
                                   11 |
                                           1 |
                                                     3
```

```
20 |
            4 |
                      5 I
                             6 |
                                   9 |
                                           1 |
            4 |
                       6 I
                             9 |
                                   15 |
                                         1 |
                                                      5
 21 |
 22 |
            4 |
                       7 |
                                           0 |
                                                      6
                             12 |
                                   -1 |
(22 rows)
```

Examples for queries marked as directed with cost column¶

The examples in this section use the following Network for queries marked as directed and only cost column is used

```
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost FROM edge_table',
     2, 3, 2
  );
 seq | path_id | path_seq | node | edge | cost | agg_cost
(0 rows)
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost FROM edge_table',
     2, 12, 2
  );
 seq | path_id | path_seq | node | edge | cost | agg_cost
                                4 |
           1 |
                    1 |
                         2 |
                                     1 |
           1 |
                                       1 |
  2 |
                    2 |
                         5 I
                                 8 |
                                                 1
  3 |
           1 |
                    3 |
                          6 |
                                9 |
                                       1 |
                                                 2
  4 |
           1 |
                    4 |
                          9 |
                                15 |
                                       1 |
                    5 |
  5 I
          1 |
                         12 |
                                -1 l
                                       0 |
          2 |
                                4 |
  6 |
                    1 |
                                      1 l
                         2 |
          2 |
                    2 |
  7 |
                         5 |
                                8 |
                                      1 |
                                                 1
  8 |
          2 |
                    3 |
                          6 I
                                11 |
                                      1 |
           2 |
                    4 | 11 |
  9 I
                                13 |
                                      1 l
                                                 3
           2 |
                    5 | 12 |
                                -1 |
                                       0 |
                                                 4
  10 |
(10 rows)
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost FROM edge_table',
     2, 12, 2, heap_paths:=true
  );
 seq | path_id | path_seq | node | edge | cost | agg_cost
           1 |
                    1 |
                           2 |
                                 4 |
                                       1 |
  2 |
           1 |
                    2 |
                           5 I
                                 8 I
                                       1 |
                                                 1
  3 |
                    3 |
                          6 I
                                9 |
                                     1 |
          1 |
                                                 2
                    4 |
  4 |
          1 |
                           9 | 15 | 1 |
```

```
5 |
                        5 I
                               12 |
                                      -1 |
   6 I
                                2 |
                                       4 |
             2 |
                        1 |
                                               1 |
                                                          0
   7 |
             2 |
                        2 |
                                5 I
                                       8 |
                                               1 |
                                                          1
             2 |
   8 |
                        3 |
                                6 |
                                      11 |
                                               1 |
                                                          2
   9 |
             2 |
                         4 |
                               11 |
                                      13 |
                                                          3
  10 |
             2 |
                        5 I
                               12 |
                                      -1 l
                                               0 |
                                                          4
             3 |
                                                          0
                        1 |
                                2 |
                                       4 |
  11 |
                                               1 |
                        2 |
             3 |
                                5 I
                                      10 |
  12 |
                                               1 |
                                                          1
  13 |
             3 |
                        3 I
                               10 |
                                      12 |
                                               1 |
                                                          2
  14 l
             3 |
                        4 |
                               11 |
                                      13 |
                                               1 |
                                                          3
  15 |
             3 I
                        5 I
                               12 |
                                      -1 |
                                               0 |
                                                          4
(15 rows)
SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, true, true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
____+
   1 |
             1 |
                        1 |
                                2 |
                                       4 |
   2 |
             1 |
                         2 |
                                5 I
                                       8 |
                                               1 |
                                                          1
   3 |
             1 |
                        3 |
                                6 I
                                       9 |
                                               1 |
                                                          2
                                                          3
   4 |
             1 |
                        4 |
                                9 |
                                      15 |
                                               1 |
   5 |
             1 |
                        5 I
                               12 |
                                      -1 |
                                               0 |
                                                          4
             2 |
                                2 |
                                       4 |
   6 I
                        1 |
                                               1 |
                                                          0
   7 |
             2 |
                        2 |
                                5 I
                                       8 |
                                               1 |
                                                          1
   8 |
             2 |
                        3 |
                                6 |
                                                          2
                                      11 |
                                               1 |
   9 |
             2 |
                        4 |
                               11 |
                                      13 |
                                               1 |
                                                          3
             2 |
                        5 I
  10 |
                               12 |
                                      -1 |
                                               0 |
                                                          4
             3 I
                                       4 I
                                                          0
  11 l
                        1 |
                                2 |
                                               1 |
                        2 |
  12 |
             3 |
                                5 |
                                      10 |
                                               1 |
                                                          1
  13 |
             3 |
                        3 |
                               10 |
                                      12 |
                                                          2
                         4 I
                                                          3
  14 l
             3 I
                               11 l
                                      13 l
                                               1 |
             3 |
                        5 I
                               12 |
                                               0 |
                                                          4
  15 |
                                      -1 |
(15 rows)
```

Examples for queries marked as undirected with cost column¶

The examples in this section use the following Network for queries marked as undirected and only cost column is used

```
SELECT * FROM pgr_KSP(
    'SELECT id, source, target, cost FROM edge_table',
    2, 12, 2, directed:=false
   );
seq | path_id | path_seq | node | edge | cost | agg_cost
```

```
1 |
             1 |
                         1 |
                                2 |
                                       4 |
                                               1 |
                                                          0
   2 |
             1 |
                         2 |
                                               1 |
                                5 I
                                       8 |
                                                          1
   3 |
             1 |
                         3 |
                                6 |
                                       9 |
                                               1 |
                                                          2
   4 |
             1 |
                         4 |
                                9 |
                                      15 |
                                                          3
   5 I
             1 |
                         5 I
                               12 |
                                      -1 l
                                                          4
             2 |
                                       4 |
                                                          0
   6 |
                         1 |
                                2 |
                                               1 |
             2 |
                         2 |
   7 |
                                5 I
                                       8 |
                                               1 |
                                                          1
   8 |
             2 |
                         3 I
                                6 I
                                      11 |
                                               1 |
             2 |
   9 |
                         4 |
                               11 |
                                       13 |
                                               1 |
                                                          3
  10 |
             2 |
                         5 I
                               12 |
                                      -1 |
                                               0 |
                                                          4
(10 rows)
SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, directed:=false, heap_paths:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
____+
   1 |
             1 |
                         1 |
                                2 |
                                       4 |
   2 |
             1 |
                         2 |
                                5 I
                                       8 |
                                               1 |
                                                          1
   3 |
             1 |
                         3 I
                                6 I
                                       9 |
                                               1 |
                                                          2
                         4 |
                                      15 |
                                                          3
   4 |
             1 |
                                9 |
                                               1 |
   5 |
             1 |
                         5 |
                               12 |
                                      -1 |
   6 |
             2 |
                                2 |
                                       4 |
                                                          0
                         1 |
                                               1 |
   7 |
             2 |
                         2 |
                                5 I
                                       8 |
                                               1 |
                                                          1
   8 |
             2 |
                         3 |
                                6 |
                                               1 |
                                                          2
                                       11 |
             2 |
   9 |
                         4 |
                               11 |
                                      13 |
                                               1 |
                                                          3
             2 |
                         5 I
  10 |
                               12 |
                                      -1 l
                                               0 |
                                                          4
             3 |
                         1 |
                                2 |
                                       4 I
                                                          0
  11 l
                                               1 |
             3 |
                        2 |
  12 |
                                5 |
                                      10 |
                                               1 |
                                                          1
  13 |
             3 |
                         3 |
                               10 |
                                      12 |
             3 |
                         4 |
                                                          3
  14 l
                               11 |
                                      13 |
                                               1 |
                                                          4
             3 |
                         5 |
                               12 |
  15 |
                                      -1 |
(15 rows)
```

See Also¶

- $\bullet \ \, http://en.wikipedia.org/wiki/K_shortest_path_routing\\$
- $\bullet \quad Sample \ Data \ {\rm network}.$

Indices and tables

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- Search Page

```
pgr_trsp - Turn Restriction Shortest Path (TRSP)¶
```

Name¶ pgr_trsp — Returns the shortest path with support for turn restrictions.

Synopsis¶ The turn restricted shorthest path (TRSP) is a shortest path algorithm that can optionally take into account complicated turn restrictions like those found in real world navigable road networks. Performamnce wise it is nearly as fast as the A* search but has many additional features like it works with edges rather than the nodes of the network. Returns a set of *pgr_costResult* (seq, id1, id2, cost) rows, that make up a path.

Description¶ The Turn Restricted Shortest Path algorithm (TRSP) is similar to the shooting star in that you can specify turn restrictions.

The TRSP setup is mostly the same as *Dijkstra shortest path* with the addition of an optional turn restriction table. This provides an easy way of adding turn restrictions to a road network by placing them in a separate table.

sql:

a SQL query, which should return a set of rows with the following columns:

```
SELECT id, source, target, cost, [,reverse_cost] FROM edge_table
```

id:

int4 identifier of the edge

source:

int4 identifier of the source vertex

target:

int4 identifier of the target vertex

cost:

float8 value, of the edge traversal cost. A negative cost will prevent the edge from being inserted in the graph.

reverse cost:

(optional) the cost for the reverse traversal of the edge. This is only used when the directed and has_rcost parameters are true (see the above remark about negative costs).

source:

int4 NODE id of the start point

target:

int4 NODE id of the end point

directed:

true if the graph is directed

has rcost:

if true, the reverse_cost column of the SQL generated set of rows will be used for the cost of the traversal of the edge in the opposite direction.

restrict_sql:

(optional) a SQL query, which should return a set of rows with the following columns:

SELECT to_cost, target_id, via_path FROM restrictions

to_cost:

float8 turn restriction cost

 $target_id:$

int4 target id

via_path:

text comma separated list of edges in the reverse order of rule

Another variant of TRSP allows to specify **EDGE id** of source and target together with a fraction to interpolate the position:

source_edge:

```
int4 EDGE id of the start edge
source_pos:
float8 fraction of 1 defines the position on the start edge
target\_edge:
int4 EDGE id of the end edge
target_pos:
float8 fraction of 1 defines the position on the end edge
Returns set of pgr_costResult[]:
seq:
row sequence
id1:
node ID
id2:
edge ID (-1 for the last row)
cost:
cost to traverse from id1 using id2
History
```

• New in version 2.0.0

Support for Vias¶ Warning

The Support for Vias functions are prototypes. Not all corner cases are being considered.

We also have support for vias where you can say generate a from A to B to C, etc. We support both methods above only you pass an array of vertices or and array of edges and percentage position along the edge in two arrays.

sql:

a SQL query, which should return a set of rows with the following columns:

```
SELECT id, source, target, cost, [,reverse_cost] FROM edge_table
```

id:

int4 identifier of the edge

source:

int4 identifier of the source vertex

target:

int4 identifier of the target vertex

cost:

float8 value, of the edge traversal cost. A negative cost will prevent the edge from being inserted in the graph.

 $reverse_cost:$

(optional) the cost for the reverse traversal of the edge. This is only used when the directed and has_rcost parameters are true (see the above remark about negative costs).

vids:

int4[] An ordered array of NODE id the path will go through from start to end.

directed:

true if the graph is directed

has rcost:

if true, the reverse_cost column of the SQL generated set of rows will be used for the cost of the traversal of the edge in the opposite direction.

 $restrict_sql:$

(optional) a SQL query, which should return a set of rows with the following columns:

SELECT to_cost, target_id, via_path FROM restrictions

 $to_cost:$

float8 turn restriction cost

target id:

int4 target id

via path:

text commar separated list of edges in the reverse order of rule

Another variant of TRSP allows to specify **EDGE id** together with a fraction to interpolate the position:

eids:

```
int4 An ordered array of EDGE id that the path has to traverse pcts:

float8 An array of fractional positions along the respective edges in eids, where 0.0 is the start of the edge and 1.0 is the end of the eadge.

Returns set of pgr_costResult[]:
seq:
row sequence
id1:
route ID
id2:
node ID
id3:
edge ID (-1 for the last row)
cost:
cost to traverse from id2 using id3
History
```

• Via Support prototypes new in version 2.1.0

Examples¶ Without turn restrictions

```
SELECT * FROM pgr_trsp(
      'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table',
        7, 12, false, false
    );
 seq \mid id1 \mid id2 \mid cost
               6 I
   0 |
         7 |
   1 |
         8 |
               7 |
   2 |
         5 |
               8 I
                       1
   3 |
         6 |
               9 |
                       1
   4 |
         9 |
               15 |
              -1 l
   5 |
       12 |
(6 rows)
```

With turn restrictions

Then a query with turn restrictions is created as:

```
2, 7, false, false,
       'SELECT to_cost, target_id::int4,
       from_edge || coalesce('','' || via_path, '''') AS via_path
       FROM restrictions'
   );
seq | id1 | id2 | cost
  0 |
       2 |
            4 |
                     1
       5 |
  1 |
             10 |
                     1
  2 | 10 |
             12 |
                     1
  3 | 11 | 11 |
                     1
  4 |
       6 |
             8 |
                     1
  5 | 5 |
             7 |
                    1
  6 | 8 |
            6 |
                    1
  7 | 7 | -1 |
(8 rows)
SELECT * FROM pgr_trsp(
     'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table',
       7, 11, false, false,
       'SELECT to_cost, target_id::int4,
       from_edge || coalesce('','' || via_path, '''') AS via_path
       FROM restrictions'
   );
seq | id1 | id2 | cost
        7 |
  0 |
              6 I
                     1
  1 |
        8 |
             7 |
                     1
  2 | 5 |
            8 |
  3 | 6 |
            9 |
  4 | 9 | 15 |
                    1
  5 | 12 | 13 |
                    1
  6 | 11 | -1 |
(7 rows)
An example query using vertex ids and via points:
SELECT * FROM pgr_trspViaVertices(
     'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table',
       ARRAY[2,7,11]::INTEGER[],
       false, false,
       'SELECT to_cost, target_id::int4, from_edge ||
     coalesce('',''||via_path,'''') AS via_path FROM restrictions');
```

'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table',

SELECT * FROM pgr_trsp(

```
seq | id1 | id2 | id3 | cost
        1 |
  1 |
                   4 |
              2 |
                           1
  2 |
        1 |
              5 l
                   10 |
                           1
  3 |
        1 |
             10 |
                   12 |
                           1
  4 |
        1 |
             11 |
                   11 |
                          1
  5 I
              6 |
                    8 |
        1 |
                          1
              5 I
                    7 |
  6 I
        1 |
  7 |
        1 |
              8 I
                    6 I
                          1
        2 |
              7 I
  8 I
                    6 I
                           1
  9 |
        2 |
              8 I
                    7 |
                           1
        2 |
  10 |
              5 I
                    8 |
                           1
  11 |
        2 |
              6 |
                    9 |
 12 |
        2 |
              9 |
                   15 |
                           1
 13 |
        2 | 12 | 13 |
                           1
 14 |
        2 | 11 | -1 |
(14 rows)
```

An example query using edge ids and vias:

```
SELECT * FROM pgr_trspViaEdges(
       'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost,
        reverse_cost FROM edge_table',
        ARRAY[2,7,11]::INTEGER[],
        ARRAY[0.5, 0.5, 0.5]::FLOAT[],
        true,
        true,
        'SELECT to_cost, target_id::int4, FROM_edge ||
     coalesce('',''||via_path,'''') AS via_path FROM restrictions');
 seq | id1 | id2 | id3 | cost
   1 |
        1 | -1 |
                     2 | 0.5
               2 |
   2 |
         1 |
                     4 |
                          1
              5 I
   3 |
        1 |
                     8 I
                            1
              6 I
                    9 I
   4 |
        1 |
                            1
  5 |
         1 |
               9 |
                    16 |
   6 |
               4 |
                     3 I
         1 |
                            1
         1 |
   7 |
               3 |
                     5 I
                            1
   8 |
         1 |
               6 l
                     8 |
                            1
   9 |
         1 |
               5 l
                     7 |
                            1
  10 I
         2 |
               5 I
                     8 |
                            1
  11 |
         2 |
               6 |
                     9 |
                           1
  12 |
         2 |
               9 | 16 |
                           1
  13 |
         2 |
               4 |
                    3 |
         2 |
               3 |
                     5 I
  14 l
```

```
15 | 2 | 6 | 11 | 0.5 (15 rows)
```

The queries use the Sample Data network.

See Also¶

• pgr_costResult//

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Traveling Sales Person - Family of functions \P

- pgr_TSP When input is given as matrix cell information.
- pgr eucledian TSP When input are coordinates.

pgr_TSP¶ Name¶

• pgr_TSP - Returns a route that visits all the nodes exactly once.

Availability: 2.0.0

• Signature changed 2.3.0

Synopsis¶

The travelling salesman problem (TSP) or travelling salesperson problem asks the following question:

• Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

This implementation uses simulated annealing to return the approximate solution when the input is given in the form of matrix cell contents. The matrix information must be symmetrical.

Signature Summary¶

```
pgr_TSP(matrix_cell_sql)
pgr_TSP(matrix_cell_sql,
    start_id, end_id,
    max_processing_time,
  tries_per_temperature, max_changes_per_temperature, max_consecutive_non_changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
Signatures¶
Basic Use¶
pgr_TSP(matrix_cell_sql)
RETURNS SETOF (seq, node, cost, agg_cost)
Example:
Because the documentation examples are auto generated and tested for non
changing results, and the default is to have random execution, the example is
wrapping the actual call.
WITH
query AS (
    SELECT * FROM pgr_TSP(
        SELECT * FROM pgr_dijkstraCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 14),
            directed := false
        $$
    )
)
SELECT agg_cost < 20 AS under_20 FROM query WHERE seq = 14;
under_20
 t
(1 row)
Complete\ Signature \P
pgr_TSP(matrix_cell_sql,
    start_id, end_id,
    max_processing_time,
```

```
tries_per_temperature, max_changes_per_temperature, max_consecutive_non_changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
Example:
SELECT * FROM pgr_TSP(
    SELECT * FROM pgr_dijkstraCostMatrix(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 14),
        directed := false
    )
    $$,
    start_id := 7,
    randomize := false
);
 seq | node | cost | agg_cost
----+----
   1 |
          7 |
                1 |
                1 |
   2 |
        8 I
                           1
                 1 |
   3 I
         5 |
                           2
                1 |
   4 |
          2 |
                           3
   5 |
          1 |
                2 |
                           4
   6 I
          3 |
                1 |
                           6
         4 |
                           7
   7 |
                1 |
   8 |
         9 |
                1 |
                           8
   9 |
        12 |
                1 |
                           9
  10 |
        11 |
                1 |
                          10
  11 |
                 1 |
         10 |
                          11
  12 |
         13 |
                3 |
                          12
  13 |
          6 I
                3 I
                          15
  14 |
          7 |
                0 |
                          18
(14 rows)
```

Description of the Signatures \P

Description of the Matrix Cell SQL query¶

Column	Type	Description
start_vid	BIGINT	Identifier of the starting vertex.
${\bf end_vid}$	BIGINT	Identifier of the ending vertex.
agg_cost	FLOAT	Cost for going from start_vid to end_vid

Can be Used with:

- pgr_dijkstraCostMatrix proposed
- $\bullet \hspace{0.2cm} pgr_withPointsCostMatrix proposed$
- $\bullet \quad pgr_floydWarshall$
- \bullet $pgr_johnson$

To generate a symmetric matrix

• directed := false.

If using directed := true, the resulting non symmetric matrix must be converted to symmetric by fixing the non symmetric values according to your application needs.

Description Of the Control parameters

The control parameters are optional, and have a default value.

Parameter	Туре	Default	Description
<u>-</u>		`0` La	The greedy part of the implementation will ast visiting vertex before returning to state of the implementation will be as the state of
max_processing_time **tries_per_temperature**		R`` `500`	Maximum number of times a neighbor(s)
max_changes_per_tempera **max_consecutive_non_cha	nges ``INTE	GER`` `100`	
initial_temperature **final_temperature**	``FLOAT``	0.1	Starting temperature. Ending temperature.
cooling_factor **randomize**	``FLOAT`` ``BOOLEAN``	`0.9` `true`	Value between between 0 and 1 (not inclu Choose the random seed

- false: Use `1` as seed. Using this value will get t

- true: Use current time as seed

```
Description of the return columns
```

Returns set of ``(seq, node, cost, agg_cost)``

```
Description
**seq**
     ``INTEGER`` Row sequence.
```

``BIGINT`` Identifier of the node/coordinate/point. **node**

``FLOAT`` Cost to traverse from the current ``node`` ito the next ``node`` in the current in the **cost** - ``O`` for the last row in the path sequence.

agg_cost ``FLOAT`` Aggregate cost from the ``node`` at ``seq = 1`` to the current node. - ``O`` for the first row in the path sequence.

Examples

Example:

Using with points of interest.

To generate a symmetric matrix:

- the side information of pointsOfInterset is ignored by not including it in the query
- and directed := false

```
SELECT * FROM pgr_TSP(
    $$
   SELECT * FROM pgr_withPointsCostMatrix(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
        'SELECT pid, edge_id, fraction from pointsOfInterest',
       array[-1, 3, 5, 6, -6], directed := false);
    $$,
    start_id := 5,
    randomize := false
);
seq | node | cost | agg_cost
----+-----
  1 |
         5 |
             1 |
                          1
  2 |
        6 |
              1 |
        3 | 1.6 |
                          2
  3 |
        ... | 3.6
-6 | 0.3 | 4.9
5 | 0 | 5 ^
        -1 | 1.3 |
  4 |
       -6 | 0.3 |
  6 |
(6 rows)
```

The queries use the *Sample Data* network.

See Also¶

- Traveling Sales Person Family of functions
- http://en.wikipedia.org/wiki/Traveling_salesman_problem
- http://en.wikipedia.org/wiki/Simulated_annealing

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$pgr_eucledianTSP\P$ Name¶

 ${\tt pgr_eucledianTSP}$ - Returns a route that visits all the coordinates pairs exactly once.

Availability: 2.3.0

Synopsis¶

The travelling salesman problem (TSP) or travelling salesperson problem asks the following question:

• Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

This implementation uses simulated annealing to return the approximate solution when the input is given in the form of coordinates.

```
Signature Summary¶
```

```
pgr_eucledianTSP(coordinates_sql)
pgr_eucledianTSP(coordinates_sql,
    start_id, end_id,
    max_processing_time,
    tries_per_temperature, max_changes_per_temperature, max_consecutive_non_changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
```

Signatures¶

Minimal Signature¶

```
pgr_eucledianTSP(coordinates_sql)
RETURNS SETOF (seq, node, cost, agg_cost)
Example:
Because the documentation examples are auto generated and tested for non
changing results, and the default is to have random execution, the example is
wrapping the actual call.
WITH
query AS (
    SELECT * FROM pgr_eucledianTSP(
     SELECT id, st_X(the_geom) AS x, st_Y(the_geom) AS y FROM edge_table_vertices_pgr
    )
)
SELECT agg_cost < 20 AS under_20 FROM query WHERE seq = 18;</pre>
under_20
_____
 t
(1 row)
Complete Signature¶
pgr_eucledianTSP(coordinates_sql,
    start_id, end_id,
    max_processing_time,
  tries_per_temperature, max_changes_per_temperature, max_consecutive_non_changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
Example:
SELECT* from pgr_eucledianTSP(
  SELECT id, st_X(the_geom) AS x, st_Y(the_geom) AS y FROM edge_table_vertices_pgr
    tries_per_temperature := 3,
    cooling_factor := 0.5,
    randomize := false
```

| agg_cost

);

seq | node | cost

```
1 | 1.4142135623731 |
                  1 | 1.4142135623731
  2 |
         3 I
  3 |
                          1 | 2.41421356237309
         4 |
  4 |
         9 | 0.58309518948453 | 3.41421356237309
  5 I
        16 | 0.58309518948453 | 3.99730875185762
  6 |
         6 |
                            1 | 4.58040394134215
  7 |
         5 I
                            1 | 5.58040394134215
         8 I
  8 |
                            1 | 6.58040394134215
  9 |
         7 | 1.58113883008419 | 7.58040394134215
 10 l
        14 | 1.49999999999 | 9.16154277142634
 11 |
        15 |
                          0.5 | 10.6615427714253
                          1.5 | 11.1615427714253
 12 |
        13 |
 13 |
        17 | 1.11803398874989 | 12.6615427714253
 14 |
                           1 | 13.7795767601752
        12 |
 15 |
        11 |
                           1 | 14.7795767601752
 16 |
        10 |
                           2 | 15.7795767601752
         2 |
                           1 | 17.7795767601752
 17 |
                           0 | 18.7795767601752
 18 |
         1 |
(18 rows)
```

Description of the Signatures¶

Description of the coordinates SQL query¶

Column	Type	Description
id	BIGINT	Identifier of the coordinate. (optional)
x	FLOAT	X value of the coordinate.
y	FLOAT	Y value of the coordinate.

When the value of **id** is not given then the coordinates will receive an **id** starting from 1, in the order given.

Description Of the Control parameters

The control parameters are optional, and have a default value.

Parameter Type Default Description

start_vid ``BIGINT`` 'O` The greedy part of the implementation will

end_vid ``BIGINT`` 'O` Last visiting vertex before returning to st

max_processing_time ``FLOAT`` '+infinity` Stop the annealing processing when the

```
**tries_per_temperature**
                                                            ``INTEGER`` `500`
                                                                                                            Maximum number of times a neighbor(s)
**max_changes_per_temperature** ``INTEGER`` `60`
                                                                                                                Maximum number of times the solution
**max_consecutive_non_changes** ``INTEGER`` `100`
                                                                                                                Maximum number of consecutive times
**initial_temperature** ``FLOAT``
                                                                                  `100`
                                                                                                         Starting temperature.
                                                    ``FLOAT`` `0.1`
``FLOAT`` `0.9`
**final_temperature**
                                                                                                        Ending temperature.
                                                                                                      Value between between 0 and 1 (not include
**cooling_factor**
                                                    ``BOOLEAN`` `true`
**randomize**
                                                                                                     Choose the random seed
                                                                                 - true: Use current time as seed
                                                                                 - false: Use `1` as seed. Using this value will get t
Description of the return columns
......
Returns set of ``(seq, node, cost, agg_cost)``
______
                                                                        Description
___________
**seq**
                         ``INTEGER`` Row sequence.
                         ``BIGINT`` Identifier of the node/coordinate/point.
**node**
                        ``FLOAT`` Cost to traverse from the current ``node`` ito the next ``node`` in the current in the
**cost**
                                              - ``O`` for the last row in the path sequence.
**agg_cost** ``FLOAT`` Aggregate cost from the ``node`` at ``seq = 1`` to the current node.
                                            - ``0`` for the first row in the path sequence.
Examples
Example:
Skipping the Simulated Annealing & showing some process information
SET client_min_messages TO DEBUG1;
SET
SELECT* from pgr_eucledianTSP(
     SELECT id, st_X(the_geom) AS x, st_Y(the_geom) AS y FROM edge_table_vertices_pgr
        tries_per_temperature := 0,
        randomize := false
```

```
);
DEBUG: pgr_eucledianTSP Processing Information
Initializing tsp class ---> tsp.greedyInitial ---> tsp.annealing ---> OK
Cycle(100) total changes =0 0 were because delta energy < 0
Total swaps: 3
Total slides: 0
Total reverses: 0
Times best tour changed: 4
Best cost reached = 18.7796
seq | node | cost
                            agg_cost
____+__
         1 | 1.4142135623731 |
         3 | 1 | 1.4142135623731
4 | 1 | 2.41421356237309
  2 |
  3 |
         4 |
                           1 | 2.41421356237309
  4 |
         9 | 0.58309518948453 | 3.41421356237309
        16 | 0.58309518948453 | 3.99730875185762
  5 |
         6 I
  6 I
                           1 | 4.58040394134215
  7 |
         5 I
                           1 | 5.58040394134215
  8 |
         8 I
                           1 | 6.58040394134215
  9 |
         7 | 1.58113883008419 | 7.58040394134215
  10 l
        14 | 1.49999999999 | 9.16154277142634
        15 |
                         0.5 | 10.6615427714253
  11 |
  12 |
        13 |
                         1.5 | 11.1615427714253
        17 | 1.11803398874989 | 12.6615427714253
  13 |
        12 l
                          1 | 13.7795767601752
  14 |
        11 |
                           1 | 14.7795767601752
  15 |
  16 |
        10 |
                          2 | 15.7795767601752
                          1 | 17.7795767601752
  17 |
         2 |
  18 |
                          0 | 18.7795767601752
         1 |
(18 rows)
```

The queries use the Sample Data network.

History

• New in version 2.3.0

See Also¶

- Traveling Sales Person Family of functions
- http://en.wikipedia.org/wiki/Traveling_salesman_problem
- http://en.wikipedia.org/wiki/Simulated_annealing

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General Information¶ Origin¶

The traveling sales person problem was studied in the 18th century by mathematicians Sir William Rowam Hamilton and Thomas Penyngton Kirkman.

A discussion about the work of Hamilton & Kirkman can be found in the book Graph Theory (Biggs et al. 1976).

ISBN-13: 978-0198539162ISBN-10: 0198539169

It is believed that the general form of the TSP have been first studied by Kalr Menger in Vienna and Harvard. The problem was later promoted by Hassler, Whitney & Merrill at Princeton. A detailed description about the connection between Menger & Whitney, and the development of the TSP can be found in On the history of combinatorial optimization (till 1960)

Problem Definition¶

Given a collection of cities and travel cost between each pair, find the cheapest way for visiting all of the cities and returning to the starting point.

Characteristics¶

- The travel costs are symmetric:
 - traveling costs from city A to city B are just as much as traveling from B to A.
- This problem is an NP-hard optimization problem.
- To calculate the number of different tours through (n) cities:
 - Given a starting city,
 - There are (n-1) choices for the second city,
 - And (n-2) choices for the third city, etc.
 - Multiplying these together we get ((n-1)! = (n-1) (n-2) . . 1).
 - Now since our travel costs do not depend on the direction we take around the tour:
 - * this number by 2
 - * $\langle (n-1)!/2 \rangle$.

TSP & Simulated Annealing¶

The simulated annealing algorithm was originally inspired from the process of annealing in metal work.

Annealing involves heating and cooling a material to alter its physical properties due to the changes in its internal structure. As the metal cools its new structure becomes fixed, consequently causing the metal to retain its newly obtained properties. [C001]

Pseudocode

Given an initial solution, the simulated annealing process, will start with a high temperature and gradually cool down until the desired temperature is reached.

For each temperature, a neighbouring new solution **snew** is calculated. The higher the temperature the higher the probability of accepting the new solution as a possible bester solution.

Once the desired temperature is reached, the best solution found is returned

```
Solution = initial_solution;

temperature = initial_temperature;
while (temperature > final_temperature) {

   do tries_per_temperature times {
      snew = neighbour(solution);
      If P(E(solution), E(snew), T) >= random(0, 1)
            solution = snew;
   }

   temperature = temperature * cooling factor;
}
Output: the best solution
```

pg Routing Implementation ¶

pgRouting's implementation adds some extra parameters to allow some exit controls within the simulated annealing process.

To cool down faster to the next temperature:

- max_changes_per_temperature: limits the number of changes in the solution per temperature
- max_consecutive_non_changes: limits the number of consecutive non changes per temperature

This is done by doing some book keeping on the times **solution** = **snew**; is executed.

- max_changes_per_temperature: Increases by one when **solution** changes
- max_consecutive_non_changes: Reset to 0 when **solution** changes, and increased each **try**

Additionally to stop the algorithm at a higher temperature than the desired one:

- max_processing_time: limits the time the simulated annealing is performed.
- book keeping is done to see if there was a change in **solution** on the last temperature

Note that, if no change was found in the first max_consecutive_non_changes tries, then the simulated annealing will stop.

```
Solution = initial_solution;
temperature = initial_temperature;
while (temperature > final_temperature) {
    do tries_per_temperature times {
        snew = neighbour(solution);
        If P(E(solution), E(snew), T) >= random(0, 1)
            solution = snew;
        when max_changes_per_temperature is reached
            or max_consecutive_non_changes is reached
            BREAK;
   }
    temperature = temperature * cooling factor;
    when no changes were done in the current temperature
        or max_processing_time has being reached
        BREAK;
}
Output: the best solution
```

Choosing parameters¶

There is no exact rule on how the parameters have to be chose, it will depend on the special characteristics of the problem.

- Your computational time is crucial, then put your time limit to max_processing_time.
- Make the **tries_per_temperture** depending on the number of cities, for example:
 - $-\,$ Useful to estimate the time it takes to do one cycle: use 1
 - * this will help to set a reasonable max_processing_time
 - \(n * (n-1)\)
 - (500 * n)
- For a faster decreasing the temperature set **cooling_factor** to a smaller number, and set to a higher number for a slower decrease.
- When for the same given data the same results are needed, set **randomize** to false.
 - When estimating how long it takes to do one cycle: use false

A recommendation is to play with the values and see what fits to the particular data.

Description Of the Control parameters¶

The control parameters are optional, and have a default value.

Parameter	Type	Default
start_vid	BIGINT	0
end_vid	BIGINT	0
max_process	sifig <u>OA</u> ffime	+infinity
tries_per_te	em perarn re	500
max_change	s <u>IMMGE</u> Remperature	60
max_consec	utive <u>EG</u> Ran_changes	100
initial_temp	e ratût ie	100
final_temper	rablicat .	0.1
cooling_fact	OFLOAT	0.9

Parameter	Туре	Default
randomize	BOOLEAN	true

Description of the return columns \P

Returns set of (seq, node, cost, agg_cost)

Column	Type	Description
\overline{seq}	INTEGER	Row sequence.
\mathbf{node}	BIGINT	Identifier of the node/coordinate/point.
cost	FLOAT	Cost to traverse from the current node ito the next node in the path sequence.
agg_cost	FLOAT	Aggregate cost from the node at seq = 1 to the current node. • 0 for the first row in the path sequence.

See Also \P

References

 $[{\rm C001}] \quad {\rm Simulated~annaeling~algorithm~for~beginners}$

- $\bullet \ \ http://en.wikipedia.org/wiki/Traveling_salesman_problem$
- http://en.wikipedia.org/wiki/Simulated_annealing

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Driving Distance - Category \P

- pgr_drivingDistance Driving Distance based on pgr_dijkstra
- • $pgr_withPointsDD$ - Proposed - Driving Distance based on pgr_withPoints

```
- pgr_alphaShape - Alpha shape computation
       - pgr_pointsAsPolygon - Polygon around a set of points
pgr_alphaShape¶ Name¶
pgr_alphaShape — Core function for alpha shape computation.
Synopsis
Returns a table with (x, y) rows that describe the vertices of an alpha shape.
table() pgr_alphaShape(text sql [, float8 alpha]);
Description\P
sql:
text a SQL query, which should return a set of rows with the following columns:
SELECT id, x, y FROM vertex_table
id:
int4 identifier of the vertex
float8 x-coordinate
float8 y-coordinate
alpha:
(optional) float8 alpha value. If specified alpha value equals 0 (default), then
optimal alpha value is used. For more information, see CGAL - 2D Alpha
Shapes.
Returns a vertex record for each row:
x-coordinate
y-coordinate
If a result includes multiple outer/inner rings, return those with separator row
(x=NULL \text{ and } y=NULL).
```

· Post pocessing

History

- Renamed in version 2.0.0
- Added alpha argument with default 0 (use optimal value) in version 2.1.0
- Supported to return multiple outer/inner ring coordinates with separator row (x=NULL and y=NULL) in version 2.1.0

$Examples \P$

PgRouting's alpha shape implementation has no way to control the order of the output points, so the actual output might different for the same input data. The first query, has the output ordered, he second query shows an example usage:

Example: the (ordered) results

```
SELECT * FROM pgr_alphaShape(
   'SELECT id::integer, ST_X(the_geom)::float AS x, ST_Y(the_geom)::float AS y
   FROM edge_table_vertices_pgr') ORDER BY x, y;
 x | y
   0 |
         2
 0.5 | 3.5
   2 1
         0
   2 |
         4
 3.5 l
        4
   4 I
        1
   4 |
   4 |
         3
(8 rows)
```

Example: calculating the area

 ${\bf Steps:}$

- Calculates the alpha shape the ORDER BY clause is not used.
- constructs a polygon
- $\bullet\,$ and computes the area

```
) AS a
   ) AS foo;
 st_area
   11.75
(1 row)
The queries use the Sample Data network.
See Also\P
  • pgr\_pointsAsPolygon - Polygon around set of points
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pgr_pointsAsPolygon¶ Name¶
pgr_pointsAsPolygon — Draws an alpha shape around given set of points.
Synopsis¶
Returns the alpha shape as (multi)polygon geometry.
geometry pgr_pointsAsPolygon(text sql [, float8 alpha]);
Description
sql:
text a SQL query, which should return a set of rows with the following columns:
SELECT id, x, y FROM vertex_result;
id:
int4 identifier of the vertex
float8 x-coordinate
y:
```

float8 y-coordinate

alpha:

(optional) ${\tt float8}$ alpha value. If specified alpha value equals 0 (default), then optimal alpha value is used. For more information, see CGAL - 2D Alpha Shapes.

Returns a (multi)polygon geometry (with holes).

History

- Renamed in version 2.0.0
- Added alpha argument with default 0 (use optimal value) in version 2.1.0
- Supported to return a (multi)polygon geometry (with holes) in version 2.1.0

Examples¶

In the following query there is no way to control which point in the polygon is the first in the list, so you may get similar but different results than the following which are also correct.

The query use the *Sample Data* network.

See Also¶

- \bullet $pgr_drivingDistance$ Driving Distance
- pgr_alphaShape Alpha shape computation

Indices and tables

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See Also¶ Indices and tables

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See Also¶ Indices and tables

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All Pairs - Family of Functions

- $\bullet \ pgr_floydWarshall$ Floyd-Warshall's Algorithm
- pgr_johnson- Johnson's Algorithm

```
pgr\_aStar - Shortest Path A* pgr\_bdAstar - Bi-directional A* Shortest Path pgr\_bdDijkstra - Bi-directional Dijkstra Shortest Path Dijkstra - Family of functions
```

- pgr_dijkstra Dijkstra's algorithm for the shortest paths.
- pgr_dijkstraCost Get the aggregate cost of the shortest paths.
- $pgr_dijkstraCostMatrix$ proposed Use pgr_dijkstra to create a costs matrix.
- pgr_drivingDistance Use pgr_dijkstra to calculate catchament information
- pgr_KSP Use Yen algorithm with pgr_dijkstra to get the K shortest paths.
- pgr_dijkstraVia Proposed Get a route of a seuence of vertices.

```
pgr_KSP - K-Shortest Path
pgr_trsp - Turn Restriction Shortest Path (TRSP)
Traveling Sales Person - Family of functions
```

- pgr_TSP When input is given as matrix cell information.
- $pgr_eucledianTSP$ When input are coordinates.

Driving Distance - Category

- pgr_drivingDistance Driving Distance based on pgr_dijkstra
- pgr_withPointsDD Proposed Driving Distance based on pgr_withPoints
- · Post pocessing
 - $-\ pgr_alphaShape$ Alpha shape computation
 - pgr_pointsAsPolygon Polygon around a set of points

Available Functions but not official pgRouting functions¶

- Stable Proposed Functions
- Experimental Functions

Stable Proposed Functions¶

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.

As part of the Dijkstra - Family of functions

- pgr_dijkstraCostMatrix proposed Use pgr_dijkstra to calculate a cost matrix.
- pgr_dijkstraVia Proposed Use pgr_dijkstra to make a route via vertices.

Families

aStar - Family of functions

- pgr_aStar A* algorithm for the shortest path.
- $pgr_aStarCost proposed$ Get the aggregate cost of the shortest paths.
- $\bullet \ pgr_aStarCostMatrix proposed$ Get the cost matrix of the shortest paths.

Bidirectional A^* - Family of functions

- pgr_bdAstar Bidirectional A* algorithm for obtaining paths.
- $pgr_bdAstarCost$ Proposed Bidirectional A* algorithm to calculate the cost of the paths.
- $pgr_bdAstarCostMatrix$ proposed Bidirectional A* algorithm to calculate a cost matrix of paths.

Bidirectional Dijkstra - Family of functions

- pgr_bdDijkstra Bidirectional Dijkstra algorithm for the shortest paths.
- • $pgr_bdDijkstraCost$ - Proposed - Bidirectional Dijkstra to calculate the cost of the shortest paths
- pgr_bdDijkstraCostMatrix proposed Bidirectional Dijkstra algorithm to create a matrix of costs of the shortest paths.

Flow - Family of functions

- pgr_maxFlow Proposed Only the Max flow calculation using Push and Relabel algorithm.
- pgr_boykovKolmogorov Proposed Boykov and Kolmogorov with details of flow on edges.
- pgr_edmondsKarp Proposed Edmonds and Karp algorithm with details of flow on edges.
- pgr_pushRelabel Proposed Push and relabel algorithm with details of flow on edges.
- Applications
 - $-\ pgr_edgeDisjointPaths$ Proposed Calculates edge disjoint paths between two groups of vertices.
 - $-\ pgr_maxCardinalityMatch$ Proposed Calculates a maximum cardinality matching in a graph.

with Points - Family of functions

- pgr withPoints Proposed Route from/to points anywhere on the graph.
- pgr_withPointsCost Proposed Costs of the shortest paths.
- $pgr_withPointsCostMatrix$ proposed Costs of the shortest paths.
- pgr_withPointsKSP Proposed K shortest paths.
- pgr_withPointsDD Proposed Driving distance.

categories

Cost - Category

- $pgr_aStarCost proposed$
- $\bullet pgr_bdAstarCost Proposed$
- $\bullet \ pgr_bdDijkstraCost Proposed$
- $\bullet \quad pgr_dijkstraCost$
- pqr withPointsCost Proposed

Cost Matrix - Category

- $pgr_aStarCostMatrix$ proposed
- $\bullet \hspace{0.2cm} pgr_bdAstarCostMatrix \hspace{0.1cm} \hbox{-} \hspace{0.1cm} proposed$
- $\bullet \ pgr_bdDijkstraCostMatrix proposed$
- pgr_dijkstraCostMatrix proposed
- pgr_withPointsCostMatrix proposed

KSP Category

- pgr_KSP Driving Distance based on pgr_dijkstra
- pgr_withPointsKSP Proposed Driving Distance based on pgr_dijkstra

aStar - Family of functions¶ The A* (pronounced "A Star") algorithm is based on Dijkstra's algorithm with a heuristic that allow it to solve most shortest path problems by evaluation only a sub-set of the overall graph.

- pgr_aStar A* algorithm for the shortest path.
- $pgr_aStarCost proposed$ Get the aggregate cost of the shortest paths.
- pgr_aStarCostMatrix proposed Get the cost matrix of the shortest paths.

pgr_aStar¶ Name¶

pgr_aStar — Returns the shortest path using A* algorithm.



Boost Graph Inside

Availability:

- pgr_astar(one to one) 2.0.0, Signature changed 2.3.0
- pgr_astar(other signatures) 2.4.0

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- Vertices of the graph are:
 - ${\bf positive}$ when it belongs to the edges_sql

- Values are returned when there is a path.
 - When the starting vertex and ending vertex are the same, there is no path.
 - * The agg_cost the non included values (v, v) is 0
 - When the starting vertex and ending vertex are the different and there is no path:
 - * The agg_cost the non included values (u, v) is ∞
- \bullet When (x,y) coordinates for the same vertex identifier differ:
 - A random selection of the vertex's (x,y) coordinates is used.
- Running time: $\langle O((E + V) * \log V) \rangle$

Signature Summary¶

```
pgr_aStar(edges_sql, start_vid, end_vid)
pgr_aStar(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon)
```

Warning

Proposed functions for next mayor release.

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- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.

```
pgr_aStar(edges_sql, start_vid, end_vids, directed, heuristic, factor, epsilon) -- proposed
pgr_aStar(edges_sql, starts_vid, end_vid, directed, heuristic, factor, epsilon) -- proposed
pgr_aStar(edges_sql, starts_vid, end_vids, directed, heuristic, factor, epsilon) -- proposed
RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)
OR EMPTY SET
```

Signatures

Minimal Signature¶

```
pgr_aStar(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

Example:

Using the defaults

```
SELECT * FROM pgr_astar(
  'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
seq | path_seq | node | edge | cost | agg_cost
----+-----
                 2 |
  1 |
            1 |
                       4 |
                              1 |
                5 I
  2 |
            2 |
                      10 |
                             1 |
  3 |
            3 |
                 10 |
                      12 |
                           1 |
  4 |
            4 |
                 11 | 13 |
                             1 |
                                       3
            5 | 12 | -1 |
  5 I
                              0 |
(5 rows)
```

One to One¶

pgr_aStar(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)

Example:

Undirected using Heuristic 2

```
SELECT * FROM pgr_astar(
  'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   2, 12,
   directed := false, heuristic := 2);
seq | path_seq | node | edge | cost | agg_cost
  1 |
            1 | 2 | 4 | 1 |
            2 |
                   5 |
                         8 |
  2 |
                                 1 |
                                            1
             3 |
                   6 I
                         11 |
                                           2
  3 |
                                 1 |
  4 |
             4 |
                   11 |
                         13 |
                                 1 |
  5 I
             5 I
                   12 |
                         -1 l
                                 0 |
(5 rows)
```

One to many¶

pgr_aStar(edges_sql, start_vid, end_vids, directed, heuristic, factor, epsilon) -- Proposed RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET

This signature finds the shortest path from one start_vid to each end_vid in end_vids:

on a directed graph when directed flag is missing or is set to
true.

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform a one to one pgr_astar where the starting vertex is fixed, and stop when all end_vids are reached.

- The result is equivalent to the union of the results of the one to one per astar.
- The extra end_vid in the result is used to distinguish to which path it belongs.

Example:

```
SELECT * FROM pgr_astar(
   'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    2, ARRAY[3, 12], heuristic := 2);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
   1 |
                           3 |
                                   2 |
                                          4 |
                                                               0
               1 |
                                                   1 |
   2 |
               2 |
                           3 |
                                   5 I
                                          8 |
                                                   1 |
                                                               1
   3 |
               3 |
                           3 |
                                   6 I
                                          9 |
                                                   1 |
                                                               2
                                          16 |
                                                               3
   4 |
               4 |
                           3 1
                                   9 |
                                                  1 I
                                                               4
   5 |
               5 |
                           3 I
                                   4 |
                                          3 |
                                                   1 l
   6 I
               6 I
                           3 |
                                   3 |
                                          -1 l
                                                               5
   7 |
               1 |
                          12 I
                                   2 |
                                          4 |
                                                               0
   8 |
               2 |
                          12 |
                                   5 |
                                          8 |
                                                  1 I
                                                               1
                                                               2
   9 |
               3 |
                          12 |
                                   6 |
                                          11 |
                                                  1 |
  10 |
               4 |
                          12 |
                                  11 |
                                          13 |
                                                               3
               5 |
                          12 |
                                 12 |
  11 |
                                          -1 l
(11 rows)
```

Many to One¶

pgr_aStar(edges_sql, starts_vid, end_vid, directed, heuristic, factor, epsilon) -- Proposed RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET

This signature finds the shortest path from each start_vid in start_vids to one end_vid:

on a directed graph when directed flag is missing or is set to
true.

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to one pgr_aStar where the ending vertex is fixed.

- The result is the union of the results of the one to one pgr_aStar.
- The extra start_vid in the result is used to distinguish to which path it belongs.

Example:

```
SELECT * FROM pgr_astar(
  'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   ARRAY[7, 2], 12, heuristic := 0);
seq | path_seq | start_vid | node | edge | cost | agg_cost
       --------
                         2 |
                                2 |
  1 |
             1 |
                                       4 |
                                              1 |
                                                         0
                         2 |
  2 |
             2 |
                                5 I
                                      10 |
                                              1 |
                                                         1
  3 |
             3 |
                         2 |
                               10 |
                                      12 |
                                              1 |
  4 I
             4 I
                         2 |
                               11 |
                                      13 |
                                                         3
                                              1 I
  5 I
             5 |
                         2 |
                               12 |
                                      -1 l
                                              0 |
  6 I
             1 |
                         7 |
                                7 |
                                       6 I
                                              1 |
  7 |
             2 |
                         7 |
                                8 |
                                       7 |
                                              1 |
                                                         1
  8 I
             3 I
                         7 I
                                5 I
                                      10 |
                                              1 |
                                                         2
             4 |
                         7 |
                                                         3
  9 |
                               10 |
                                      12 |
                                              1 |
                         7 |
  10 |
             5 I
                               11 |
                                      13 |
                                              1 |
                                                         4
 11 |
             6 |
                         7 |
                               12 |
                                      -1 |
                                              0 |
(11 rows)
```

Many to Many¶

pgr_aStar(edges_sql, starts_vid, end_vids, directed, heuristic, factor, epsilon) -- Propose RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET

This signature finds the shortest path from each start_vid in start_vids to each end_vid in on a directed graph when directed flag is missing or is set to

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to Many pgr_dijkstra for all start_vids.

- The result is the union of the results of the one to one pgr dijkstra.
- The extra start_vid in the result is used to distinguish to which path it belongs.

The extra start_vid and end_vid in the result is used to distinguish to which path it belongs.

Example:

SELECT * FROM pgr_astar(

'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table', ARRAY[7, 2], ARRAY[3, 12], heuristic := 2);

seq]	path_seq	start_vid	end_vid	no	de	edge	cost	agg_cost
1	1	2	3	+-· 	2	-+ 4	+	+ 0
2	2	2	3	1	5	8	1	1
3	3	2	3	1	6	9	1	1 2
4	4	2	3		9	16	1	J 3
5	5 I	2	3	1	4	3	1	4
6	6 I	2	3	1	3	-1	1 0	J 5
7	1	7	3	1	7	l 6	1	0
8	2	7	3	1	8	7	1	1
9	3	7	3	1	5	8	1	1 2
10	4	7	3	1	6	9	1	3
11	5	7	3	1	9	16	1	l 4
12	6	7	3	1	4	3	1	J 5
13	7	7	3		3	-1	0	l 6
14	1	2	12	1	2	l 4	1	0
15	2	2	12	1	5	10	1	1
16	3	2	12	1	10	12	1	1 2
17	4	2	12		11	13	1	3
18	5 I	2	12	1	12	l -1	0	l 4
19	1	7	12	1	7	l 6	1	0
20	2	7	12	1	8	7	1	1
21	3	7	12	1	5	10	1	1 2
22	4	7	12	1	10	12	1	3
23	5 I	7	12	1	11	13	1	4
24	6	7	12	1	12	l -1	0	J 5
(24 row	rs)							

Description of the Signatures \P

Description of the edges_sql query for a star like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
$\overline{\mathbf{id}}$	ANY-INTEG	JER	Identifier of the edge.
source	ANY-INTEG	ER	Identifier of the first end point vertex edge.
target	ANY-INTEG	IER	Identifier of the second end point vert edge.

Column	Type Default	Description
cost	ANY-NUMERICAL	Weight of the edge (source, target)
reverse_	c ant -numeri€al	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source), When negative: edge (target, so not exist, therefore it's not part graph.
x1	ANY-NUMERICAL	X coordinate of source vertex.
y1	ANY-NUMERICAL	Y coordinate of source vertex.
x2	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

 $SMALLINT,\,INTEGER,\,BIGINT,\,REAL,\,FLOAT$

Description of the parameters of the signatures \P

Parameter Type	Description		
edges_sqlTEXT	Edges SQL query as described above.		
${f start_vid}$ ANY-INTEGER	Starting vertex identifier.		
${f end_vid}$ ANY-INTEGER	Ending vertex identifier.		
directed BOOLEAN	• Optional.		
	When false the graph is considered as Undirected.Default is true which considers the		

graph as Directed.

Parameter Type		Description
heuristic INTEGER		(optional). Heuristic number. Current valid values $0\sim5$. Default 5
		 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) 1: h(v) abs(max(dx, dy)) 2: h(v) abs(min(dx, dy)) 3: h(v) = dx * dx + dy * dy 4: h(v) = sqrt(dx * dx + dy * dy) 5: h(v) = abs(dx) + abs(dy)
factor	FLOAT	(optional). For units manipulation. \backslash (factor $> 0 \backslash$). Default 1. see <i>Factor</i>
epsilon	FLOAT	(optional). For less restricted results. \land (epsilon >= $1 \land$). Default 1.

Description of the return values for a path \P

Column	Туре
\overline{seq}	INT
$\operatorname{path_id}$	INT
path _seq	INT
start_vid	BIGINT
${ m end_vid}$	BIGINT
node	BIGINT
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

See Also¶

- aStar Family of functions
- Sample Data
- http://www.boost.org/libs/graph/doc/astar_search.html
- http://en.wikipedia.org/wiki/A* search algorithm

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pgr_aStarCost - proposed¶ Name¶

 $pgr_aStarCost$ — Returns the aggregate cost shortest path using aStar - Family of functions algorithm.



Boost Graph Inside

Availability: 2.4.0

Signature Summary¶

Warning

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 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.

```
pgr_aStarCost(edges_sql, start_vid, end_vid) -- Proposed
pgr_aStarCost(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon) -- Prop
pgr_aStarCost(edges_sql, start_vid, end_vids, directed, heuristic, factor, epsilon) -- Prop
pgr_aStarCost(edges_sql, starts_vid, end_vid, directed, heuristic, factor, epsilon) -- Prop
pgr_aStarCost(edges_sql, starts_vid, end_vids, directed, heuristic, factor, epsilon) -- Prop
```

RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET

```
Signatures¶
Minimal Signature¶
pgr_aStarCost(edges_sql, start_vid, end_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
Example:
Using the defaults
SELECT * FROM pgr_aStarCost(
  'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   2, 12);
start_vid | end_vid | agg_cost
                12 |
        2 |
(1 row)
One to One¶
pgr_aStarCost(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
Example:
Setting a Heuristic
SELECT * FROM pgr_aStarCost(
   'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   2, 12,
   directed := false, heuristic := 2);
start_vid | end_vid | agg_cost
-----
                12 | 4
        2 |
(1 row)
One to many¶
```

pgr_aStarCost(edges_sql, start_vid, end_vids, directed, heuristic, factor, epsilon) -- Prop
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET

This signature finds a path from one start_vid to each end_vid in end_vids:

on a directed graph when directed flag is missing or is set to
true.

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform a one to one pgr_astar where the starting vertex is fixed, and stop when all end_vids are reached.

- The result is equivalent to the union of the results of the one to one per astar.
- The extra end_vid column in the result is used to distinguish to which path it belongs.

Example:

Many to One¶

pgr_aStarCost(edges_sql, starts_vid, end_vid, directed, heuristic, factor, epsilon) -- Prop
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET

This signature finds the shortest path from each start_vid in start_vids to one end_vid:

on a directed graph when directed flag is missing or is set to

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to one pgr_aStar where the ending vertex is fixed.

- The result is the union of the results of the one to one pgr_aStar.
- The extra start_vid column in the result is used to distinguish to which path it belongs.

Example:

Many to Many¶

pgr_aStarCost(edges_sql, starts_vid, end_vids, directed, heuristic, factor, epsilon) -- Pro
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET

This signature finds the shortest path from each start_vid in start_vids to each end_vid in on a directed graph when directed flag is missing or is set to

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to Many pgr_dijkstra for all start_vids.

- The result is the union of the results of the one to one pgr_dijkstra.
- The extra start_vid in the result is used to distinguish to which path it belongs.

The extra start_vid and end_vid in the result is used to distinguish to which path it belongs.

Example:

Description of the Signatures¶

Description of the edges_sql query for astar like functions¶

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
reverse_	_c ant -numerical	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source), When negative: edge (target, so not exist, therefore it's not part
x 1	ANY-NUMERICAL	graph. X coordinate of source vertex.
y 1	ANY-NUMERICAL	Y coordinate of source vertex.
$\mathbf{x}2$	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter Type	Description
edges_sqlTEXT	Edges SQL query as described above.
${f start_vid}$ ANY-INTEGER	Starting vertex identifier.
${f end_vid}$ ANY-INTEGER	Ending vertex identifier.
directed BOOLEAN	• Optional.
	 When false the graph is considered as Undirected. Default is true which considers the graph as Directed.

Parameter Type		Description	
heuristic	INTEGER	(optional). Heuristic number. Current valid values $0{\sim}5$. Default 5	
		 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) 1: h(v) abs(max(dx, dy)) 2: h(v) abs(min(dx, dy)) 3: h(v) = dx * dx + dy * dy 4: h(v) = sqrt(dx * dx + dy * dy) 5: h(v) = abs(dx) + abs(dy) 	
factor	FLOAT	(optional). For units manipulation. \setminus (factor $> 0 \setminus$). Default 1. See <i>Factor</i>	
epsilon	FLOAT	(optional). For less restricted results. \(epsilon>= 1 \). Default 1.	

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vid BIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending ver- ending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

See Also \P

- aStar Family of functions.
- $\bullet \ \ \textit{Sample Data} \ \text{network}.$
- $\bullet \ \, \rm http://www.boost.org/libs/graph/doc/astar_search.html$
- $\bullet \ \, \rm http://en.wikipedia.org/wiki/A*_search_algorithm$

Indices and tables

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$pgr_aStarCostMatrix - proposed$ Name

pgr_aStarCostMatrix - Calculates the a cost matrix using pgr_aStar.

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.



Boost Graph Inside

Availability: 2.4.0

Synopsis

Using aStar algorithm, calculate and return a cost matrix.

Signature Summary¶

pgr_aStarCostMatrix(edges_sql, vids)
pgr_aStarCostMatrix(edges_sql, vids, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost)

Signatures¶

Minimal Signature¶

The minimal signature: • Is for a directed graph.

pgr_aStarCostMatrix(edges_sql, vids)
RETURNS SET OF (start_vid, end_vid, agg_cost)

Example:

Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_aStarCostMatrix(
   'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
         2 |
                 1 |
                            2
         3 |
                  1 |
         4 |
                 1 |
                  2 |
         1 |
                            1
         3 I
                 2 |
                 2 |
         4 |
                            2
         1 |
                  3 |
                            5
         2 |
                  3 |
                            1
        4 |
                 3 |
         1 |
                 4 |
                            5
         2 |
                 4 |
         3 |
                 4 |
                            3
(12 rows)
Complete\ Signature \P
pgr_aStarCostMatrix(edges_sql, vids, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost)
Example:
Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.
This example returns a symmetric cost matrix.
SELECT * FROM pgr_aStarCostMatrix(
   'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    directed := false, heuristic := 2
);
 start_vid | end_vid | agg_cost
-----
                 1 |
         2 |
                           1
         3 |
                  1 |
         4 |
                  1 |
                            3
         1 |
                  2 |
                            1
                  2 |
         3 |
         4 |
                  2 |
                             2
```

2

1

3 |

3 |

1 |

2 |

(12 rows)

Description of the Signatures \P

Description of the edges_sql query for a star like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
reverse_	_c ant -numerical	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source),
		 When negative: edge (target, so not exist, therefore it's not part graph.
x1	ANY-NUMERICAL	X coordinate of source vertex.
y1	ANY-NUMERICAL	Y coordinate of source vertex.
x2	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter	Type	Description	
$\overline{\mathrm{edges_sql}}$	TEXT	Edges SQL query as described above.	
\mathbf{vids}	ARRAY[ANY-IN	TECHNY of vertices_identifiers.	
$\mathbf{directed}$	BOOLEAN	• Optional.	
		 When false the graph is considered as Undirected. Default is true which considers the graph as Directed. 	
heuristic	INTEGER	(optional). Heuristic number. Current valid values 0~5. Default 5	
		 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) 1: h(v) abs(max(dx, dy)) 2: h(v) abs(min(dx, dy)) 3: h(v) = dx * dx + dy * dy 4: h(v) = sqrt(dx * dx + dy * dy) 5: h(v) = abs(dx) + abs(dy) 	
factor	FLOAT	(optional). For units manipulation. \((factor > 0\)). Default 1.	
epsilon	FLOAT	(optional). For less restricted results. \(epsilon >= 1\). Default 1.	

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vidBIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vertending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

$\mathrm{Examples} \P$

```
Example:
Use with tsp
SELECT * FROM pgr_TSP(
    SELECT * FROM pgr_aStarCostMatrix(
      'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
      (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        directed:= false, heuristic := 2
    )
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
   1 |
          1 |
                  1 |
                             0
          2 |
   2 |
                  1 |
                             1
   3 |
          3 |
                  1 |
                             2
   4 |
          4 |
                  3 |
                             3
   5 I
          1 |
                 0 |
(5 rows)
```

See Also¶

- aStar Family of functions
- Cost Matrix Category
- Traveling Sales Person Family of functions
- ullet The queries use the $Sample\ Data$ network.

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The problem definition (Advanced documentation)¶ The A* (pronounced "A Star") algorithm is based on Dijkstra's algorithm with a heuristic, that is an estimation of the remaining cost from the vertex to the goal, that allows to solve most shortest path problems by evaluation only a sub-set of the overall graph. Running time: $\langle O((E+V) * \log V) \rangle$

Heuristic¶

Currently the heuristic functions available are:

- 0: $\langle h(v) = 0 \rangle$ (Use this value to compare with pgr_dijkstra)
- 1: $\langle (h(v) = abs(max(\Delta v, \Delta v)) \rangle)$
- 2: $\langle (h(v) = abs(min(\Delta v, \Delta v)) \rangle$
- 3: $\langle (h(v)) = \langle Delta x * \langle Delta x + \langle Delta y * \langle Delta y \rangle \rangle$
- 4: $\langle (h(v) = \operatorname{sqrt}(\Delta x * \Delta x + \Delta y * \Delta y) \rangle)$
- 5: $\langle (h(v) = abs(\Delta v) + abs(\Delta v) \rangle$

where $\(\Delta x = x_1 - x_0\)$ and $\(\Delta y = y_1 - y_0\)$

Factor¶ Analysis 1

Working with $cost/reverse_cost$ as length in degrees, x/y in lat/lon: Factor = 1 (no need to change units)

Analysis 2

Working with $cost/reverse_cost$ as length in meters, x/y in lat/lon: Factor = would depend on the location of the points:

latitude	conversion	Factor
45	1 longitude degree is 78846.81 m	78846
0	1 longitude degree is 111319.46 m $$	111319

Analysis 3

Working with cost/reverse_cost as time in seconds, x/y in lat/lon: Factor: would depend on the location of the points and on the average speed say 25 m/s is the speed.

latitude	conversion	Factor
45	1 longitude degree is $(78846.81\text{m})/(25\text{m/s})$	3153 s
0	1 longitude degree is (111319.46 m)/(25m/s)	$4452~\mathrm{s}$

See Also¶

- \bullet pgr_aStar
- $pgr_aStarCost proposed$
- $\bullet \quad pgr_aStarCostMatrix \ \ proposed$
- $\bullet \ \ http://www.boost.org/libs/graph/doc/astar_search.html$
- http://en.wikipedia.org/wiki/A* search algorithm

Indices and tables

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Bidirectional A^* - Family of functions

- pgr_bdAstar Bidirectional A* algorithm for obtaining paths.
- $pgr_bdAstarCost$ Proposed Bidirectional A* algorithm to calculate the cost of the paths.
- pgr_bdAstarCostMatrix proposed Bidirectional A* algorithm to calculate a cost matrix of paths.

$\mathbf{pgr_bdAstar}\P\quad \mathrm{Name}\P$

pgr_bdAstar — Returns the shortest path using A* algorithm.



Boost Graph Inside

Availability:

- pgr_bdAstar(one to one) 2.0.0, Signature change on 2.5.0
- pgr_bdAstar(other signatures) 2.5.0

Signature Summary¶

```
pgr_bdAstar(edges_sql, start_vid, end_vid)
pgr_bdAstar(edges_sql, start_vid, end_vid, directed [, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq , node, edge, cost, agg_cost)
    OR EMPTY SET
```

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL

- Name might change.
- Signature might change.
- Functionality might change.
- pgTap tests might be missing.
- Might need c/c++ coding.
- May lack documentation.
- Documentation if any might need to be rewritten.
- Documentation examples might need to be automatically generated.
- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

```
pgr_bdAstar(edges_sql, start_vid, end_vids [, directed, heuristic, factor, epsilon])
pgr_bdAstar(edges_sql, start_vids, end_vid [, directed, heuristic, factor, epsilon])
pgr_bdAstar(edges_sql, start_vids, end_vids [, directed, heuristic, factor, epsilon])
```

RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost) OR EMPTY SET

Using these signatures, will load once the graph and perform several one to one pgr_bdAstar

- The result is the union of the results of the one to one pgr bdAStar.
- The extra start_vid and/or end_vid in the result is used to distinguish to which path it belongs.

Avaliability

- pgr_bdAstar(one to one) 2.0, signature change on 2.5
- pgr_bdAstar(other signatures) 2.5

$Signatures \P$

Minimal Signature¶

```
pgr_bdAstar(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

This usage finds the shortest path from the start_vid to the end_vid on a directed graph

- with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example:

Using the defaults

```
SELECT * FROM pgr_bdAstar(
   'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
   FROM edge_table',
   2, 3
);
seq | path_seq | node | edge | cost | agg_cost
_____
         1 | 2 | 4 | 1 |
         2 | 5 | 8 | 1 |
  2 |
                   9 |
  3 |
          3 | 6 |
                        1 |
                        1 |
         4 | 9 | 16 |
  4 |
                   3 |
  5 |
          5 I
               4 |
                          1 |
  6 |
         6 l
              3 |
                    -1 |
                          0 |
                                 10
(6 rows)
```

pgr_bdAstar One to One \P

```
pgr_bdAstar(edges_sql, start_vid, end_vid, directed [, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

This usage finds the shortest path from the start_vid to the end_vid allowing the user to ch

- heuristic,and/or factor
- and/or epsilon.

Note

In the One to One signature, because of the deprecated signature existence, it is compulsory to indicate if the graph is **directed** or **undirected**.

Example:

Directed using Heuristic 2

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, 3,
    true, heuristic := 2
);
seq | path_seq | node | edge | cost | agg_cost
```

```
1 |
                               4 |
   2 |
               2 |
                       5 I
                               8 |
                                       1 |
                                                    2
   3 |
               3 |
                       6 |
                               9 |
                                                    3
                                       1 |
   4 |
               4 |
                       9 |
                              16 |
                                       1 |
                                                    4
   5 I
               5 I
                       4 |
                               3 |
                                       1 |
                                                    5
   6 I
               6 I
                       3 |
                              -1 |
(6 rows)
```

pgr_bdAstar One to many¶

pgr_bdAstar(edges_sql, start_vid, end_vids [, directed, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET

This usage finds the shortest path from the start_vid to each end_vid in end_vids allowing t if the graph is directed or undirected

- and/or heuristic,
- and/or factor
- and/or epsilon.

Example:

Directed using Heuristic 3 and a factor of 3.5

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
   FROM edge_table',
    2, ARRAY[3, 11],
   heuristic := 3, factor := 3.5
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
____+
   1 |
             1 |
                              2 |
                                     4 |
                                            1 |
                                                      0
   2 |
             2 |
                       3 |
                              5 I
                                     8 |
                                            1 |
                                                   25.5
             3 |
   3 |
                       3 |
                              6 |
                                     9 |
                                            1 |
                                                  38.75
                                    16 |
   4 |
             4 |
                       3 |
                              9 |
                                            1 |
                                                  64.25
   5 I
             5 I
                       3 |
                              4 |
                                     3 |
                                            1 |
                                                  65.25
   6 |
             6 I
                       3 |
                              3 |
                                            0 |
                                                  66.25
                                    -1 l
                                     4 |
   7 |
             1 |
                      11 |
                              2 |
                                            1 |
                                                      0
   8 |
             2 |
                      11 |
                              5 |
                                     8 |
                                            1 |
                                                      1
                                                      2
   9 |
             3 I
                      11 |
                              6 I
                                    11 |
                                            1 |
  10 |
             4 |
                      11 |
                             11 |
                                    -1 |
                                           0 |
                                                      3
(10 rows)
```

pgr_bdAstar Many to One¶

pgr_bdAstar(edges_sql, start_vids, end_vid [, directed, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET

This usage finds the shortest path from each start_vid in start_vids to the end_vid allowing if the graph is directed or undirected

- and/or **heuristic**,
- and/or factor
- and/or epsilon.

Example:

Undirected graph with Heuristic 4

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], 3,
    false, heuristic := 4
);
```

seq	path_seq	start_vid	node	edge	cost	agg_cost
+		+	++	+-	+	
1	1	1 2	2	2	1	0
2	2	1 2	3	-1	0	1
3	1	7	7	6	1	0
4	2	7	8	7	1 3	.23606797749979
5	3	7	5	8	1 5	.65028153987288
6	4	7	6	5	1 6	.65028153987288
7	5	7	3	-1 l	0 7	.65028153987288
(7 rov	ıs)					

pgr_bdAstar Many to Many¶

pgr_bdAstar(edges_sql, start_vids, end_vids [, directed, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET

This usage finds the shortest path from each start_vid in start_vids to each end_vid in end_ if the graph is directed or undirected

- and/or heuristic,
- and/or factor
- and/or epsilon.

Example:

Directed graph with a factor of 0.5

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], ARRAY[3, 11],
    factor := 0.5
);
seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
   1 |
                            2 |
                                       3 |
                                              2 |
                                                      4 |
                                                              1 |
                                                                          0
   2 |
               2 |
                            2 |
                                              5 I
                                                                          2
                                       3 |
                                                      8 |
                                                              1 |
   3 I
               3 |
                            2 |
                                       3 |
                                              6 |
                                                      9 |
                                                              1 |
                                                                        3.5
   4 |
               4 |
                            2 |
                                       3 |
                                              9
                                                                        4.5
                                                     16 |
                                                              1 |
   5 |
               5 |
                            2 |
                                       3 |
                                              4
                                                      3 |
                                                              1 |
                                                                        5.5
                            2 |
                                                                        6.5
   6 I
               6 |
                                       3 |
                                              3 |
                                                     -1 I
                                                              0 |
                                              2 |
   7 |
               1 |
                            2 |
                                      11 |
                                                      4 |
                                                              1 |
                                                                          0
   8 |
               2 |
                            2 |
                                      11 |
                                              5 |
                                                      8 |
                                                                          1
   9 |
               3 |
                            2 |
                                      11 |
                                              6 |
                                                                          2
                                                     11 |
                                                              1 |
  10 |
               4 |
                            2 |
                                                              0 |
                                                                          3
                                      11 |
                                              11 |
                                                     -1 |
                            7 |
                                                                          0
  11 |
               1 |
                                       3 |
                                              7
                                                      6 |
                                                              1 |
  12 |
               2 |
                            7 |
                                       3 |
                                              8
                                                      7 |
                                                                        2.5
  13 |
               3 |
                            7 |
                                       3 |
                                              5
                                                      8 |
                                                              1 |
                                                                        4.5
  14 |
               4 |
                            7 |
                                       3 |
                                              6
                                                              1 |
                                                                          6
                                                      9 |
                                                                          7
               5 I
                            7 |
                                       3 |
  15 |
                                              9 |
                                                     16 |
                                                              1 |
                                                                          8
  16 |
               6 |
                            7 |
                                       3 |
                                               4 |
                                                      3 |
                                                              1 |
               7 |
                            7 |
                                       3 |
                                              3 |
                                                              0 |
                                                                          9
  17 |
                                                     -1 l
  18 |
               1 |
                            7 |
                                      11 |
                                              7 |
                                                      6 |
                                                              1 |
                                                                          0
               2 |
                            7 |
                                              8 |
                                                      7 |
                                                                          1
  19 |
                                      11 |
                                                              1 |
                                                                          2
  20 |
               3 |
                            7 |
                                      11 |
                                              5 I
                                                      8 |
                                                              1 |
               4 |
                            7 |
                                                                          3
  21 |
                                      11 |
                                              6 I
                                                     11 |
                                                              1 |
  22 |
               5 I
                                      11 |
                                                                          4
                            7 |
                                                     -1 |
                                                              0 |
                                              11 |
(22 rows)
```

Description of the Signatures¶

Description of the edges_sql query for a tar like functions¶ edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
$\overline{\mathbf{id}}$	ANY-INTEG	JER	Identifier of the edge.
source	ANY-INTEG	ER	Identifier of the first end point vertex edge.
target	ANY-INTEG	ER	Identifier of the second end point vert edge.

Column	Type Default	Description
cost	ANY-NUMERICAL	Weight of the edge (source, target)
reverse_	_c ant -numerical	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source), When negative: edge (target, so not exist, therefore it's not part graph.
x1	ANY-NUMERICAL	X coordinate of source vertex.
y1	ANY-NUMERICAL	Y coordinate of source vertex.
x2	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter Type	Description
edges_sqlTEXT	Edges SQL query as described above.
${f start_vid}$ ANY-INTEGER	Starting vertex identifier.
start_vidsARRAY[ANY-IN	TECERIng vertices identifierers.
${f end_vid}$ ANY-INTEGER	Ending vertex identifier.
${f end_vids}$ ARRAY[ANY-IN	TEGERIG vertices identifiers.
directed BOOLEAN	• Optional.

- When false the graph is considered as Undirected.
- Default is true which considers the graph as Directed.

Parameter Type		Description
heuristic INTEGER		(optional). Heuristic number. Current valid values $0\sim5$. Default 5
		 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) 1: h(v) abs(max(dx, dy)) 2: h(v) abs(min(dx, dy)) 3: h(v) = dx * dx + dy * dy 4: h(v) = sqrt(dx * dx + dy * dy) 5: h(v) = abs(dx) + abs(dy)
factor	FLOAT	(optional). For units manipulation. \backslash (factor $> 0 \backslash$). Default 1. see <i>Factor</i>
epsilon	FLOAT	(optional). For less restricted results. \land (epsilon >= $1 \land$). Default 1.

Description of the return values for a path \P

Returns set of (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)

Column	Туре
\overline{seq}	INT
$\operatorname{path_id}$	INT
path_seq	INT
start_vid	BIGINT
start_vid	DIGINI
${f end_vid}$	BIGINT
node	BIGINT
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

See Also¶

- Bidirectional A* Family of functions
- Sample Data network.
- http://www.boost.org/libs/graph/doc/astar_search.html
- http://en.wikipedia.org/wiki/A*_search_algorithm

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pgr_bdAstarCost - Proposed¶ Name¶

pgr_bdAstarCost — Returns the shortest path using A* algorithm.



Boost Graph Inside

Availability: 2.5.0

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Signature Summary¶

```
pgr_bdAstarCost(edges_sql, start_vid, end_vid)
pgr_bdAstarCost(edges_sql, start_vid, end_vid [, directed , heuristic, factor, epsilon])
pgr_bdAstarCost(edges_sql, start_vid, end_vids [, directed, heuristic, factor, epsilon])
pgr_bdAstarCost(edges_sql, start_vids, end_vid [, directed, heuristic, factor, epsilon])
pgr_bdAstarCost(edges_sql, start_vids, end_vids [, directed, heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

Using these signatures, will load once the graph and perform several one to one pgr_bdAstarCost

- The result is the union of the results of the one to one pgr_bdAstarCost.
- The extra start_vid and/or end_vid in the result is used to distinguish to which path it belongs.

Signatures¶

Minimal Signature¶

```
pgr_bdAstarCost(edges_sql, start_vid, end_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from the start_vid to the end_vid

- on a **directed** graph
 with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example:

Using the defaults

```
pgr_bdAstarCost One to One¶
```

```
pgr_bdAstarCost(edges_sql, start_vid, end_vid [, directed, heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from the start_vid to each end_vid in end_vids allowing t if the graph is directed or undirected

- heuristic,
- and/or factor
- and/or epsilon.

Note

In the One to One signature, because of the deprecated signature existence, it is compulsory to indicate if the graph is **directed** or **undirected**.

Example:

Directed using Heuristic 2

pgr_bdAstarCost One to many¶

```
pgr_bdAstarCost(edges_sql, start_vid, end_vids [, directed, heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from the start_vid to each end_vid in end_vids allowing t

- and/or heuristic,
- and/or factor
- and/or epsilon.

Example:

Directed using Heuristic 3 and a factor of 3.5

if the graph is directed or undirected

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
   FROM edge_table',
    2, ARRAY[3, 11],
   heuristic := 3, factor := 3.5
);
 start_vid | end_vid | agg_cost
         2 |
              3 l
                          66.25
         2 |
                 11 |
                          27.5
(2 rows)
pgr_bdAstarCost Many to One¶
pgr_bdAstarCost(edges_sql, start_vids, end_vid [, directed, heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
This usage finds the shortest path from each start_vid in start_vids to the end_vid allowing
         if the graph is directed or undirected
       • and/or heuristic,
       • and/or factor
       • and/or epsilon.
Example:
Undirected graph with Heuristic 4
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
   FROM edge_table',
   ARRAY[2, 7], 3,
   false, heuristic := 4
);
 start_vid | end_vid |
                         agg_cost
         2 |
                  3 l
        7 |
                 3 | 7.65028153987288
(2 rows)
pgr_bdAstarCost Many to Many¶
pgr_bdAstarCost(edges_sql, start_vids, end_vids [, directed, heuristic, factor, epsilon])
```

RETURNS SET OF (start_vid, end_vid, agg_cost)

This usage finds the shortest path from each start_vid in start_vids to each end_vid in end_ if the graph is directed or undirected

- and/or heuristic,
- and/or factor
- and/or epsilon.

Example:

Directed graph with a factor of 0.5

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], ARRAY[3, 11],
    factor := 0.5
);
 start_vid | end_vid | agg_cost
         2 |
                   3 |
                            7.5
                  11 |
         2 |
         7 |
                  3 |
                           11.5
         7 |
                  11 |
                            6.5
(4 rows)
```

Description of the Signatures¶

Description of the edges_sql query for a star like functions \P

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point ver edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)

• When negative: edge (source, to not exist, therefore it's not part graph.

Column	Type Default	Description
reverse_	cant-numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.
x 1	ANY-NUMERICAL	X coordinate of source vertex.
y1	ANY-NUMERICAL	Y coordinate of source vertex.
x2	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter Type	Description
edges_sqlTEXT	Edges SQL query as described above.
${f start_vid}$ ANY-INTEGER	Starting vertex identifier.
start_vidsARRAY[ANY-IN	TEGERIng vertices identifierers.
end_vid ANY-INTEGER	Ending vertex identifier.
end_vids ARRAY[ANY-IN	TEGERIG vertices identifiers.
directed BOOLEAN	• Optional.

- When false the graph is considered as Undirected.
- Default is true which considers the graph as Directed.

Parameter	Type	Description
heuristic INTEGER		(optional). Heuristic number. Current valid values 0~5. Default 5
		 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) 1: h(v) abs(max(dx, dy)) 2: h(v) abs(min(dx, dy)) 3: h(v) = dx * dx + dy * dy 4: h(v) = sqrt(dx * dx + dy * dy) 5: h(v) = abs(dx) + abs(dy)
factor	FLOAT	(optional). For units manipulation. \setminus (factor $> 0 \setminus$). Default 1. see <i>Factor</i>
epsilon	FLOAT	(optional). For less restricted results. \(epsilon>= 1 \). Default 1.

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vid BIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending ver ending vertices are in the o
agg_cost FLOAT	Aggregate cost from start

See Also \P

- ullet Bidirectional A^* Family of functions
- Sample Data network.
- Migration Guide
- $\bullet \ \, http://www.boost.org/libs/graph/doc/astar_search.html$
- $\bullet \ \, \rm http://en.wikipedia.org/wiki/A*_search_algorithm$

Indices and tables

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- Search Page

$pgr_bdAstarCostMatrix - proposed\P$ Name¶

pgr_bdAstarCostMatrix - Calculates the a cost matrix using pgr_bdAstar.



Boost Graph Inside

Availability: 2.5.0

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary¶

```
pgr_bdAstarCostMatrix(edges_sql, start_vids)
pgr_bdAstarCostMatrix(edges_sql, start_vids, [, directed , heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

Signatures

Minimal Signature¶

```
pgr_bdAstarCostMatrix(edges_sql, start_vids)
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

This usage calculates the cost from the each start_vid in start_vids to each start_vid in st on a directed graph

- with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example:

Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_bdAstarCostMatrix(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);</pre>
```

start_vid	١	end_vid	١	agg_cost
+			+-	
1	l	2	1	1
1	l	3		12
1	l	4		12
2	l	1		1
2	١	3		10
2	l	4		9
3	l	1		2
3	l	2		1
3	١	4		5
4	l	1		5
4	l	2		2
4	l	3		1
(12 rows)				

Complete Signature \P

```
pgr_bdAstarCostMatrix(edges_sql, start_vids, [, directed , heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

This usage calculates the cost from the each start_vid in start_vids to each start_vid in st if the graph is directed or undirected

- heuristic,
- and/or factor

• and/or epsilon.

Example:

Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

2 |

4 |

1 |

2 |

3 |

```
SELECT * FROM pgr_bdAstarCostMatrix(
   'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
);
 start_vid | end_vid | agg_cost
         1 |
                    2 |
                               1
                    3 |
                               2
         1 |
                               5
         1 |
                    4 |
         2 |
                    1 |
         2 |
                    3 |
                               1
                               2
         2 |
                    4 |
         3 |
                    1 |
```

(12 rows)

Description of the Signatures¶

3 |

3 |

4 | 4 |

4 |

Description of the edges_sql query for a star like functions¶

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

1

1

5

2

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point verte edge.
target	ANY-INTEGER	Identifier of the second end point ve edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)

• When negative: edge (source, to not exist, therefore it's not part graph.

Column	Type Default	Description
reverse_	cant-numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.
x 1	ANY-NUMERICAL	X coordinate of source vertex.
y1	ANY-NUMERICAL	Y coordinate of source vertex.
x2	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter Type	Description	
edges_sqlTEXT	Edges SQL query as described above.	
${f start_vid}$ ANY-INTEGER	Starting vertex identifier.	
start_vidsARRAY[ANY-INTSCHR]ng vertices identifierers.		
end_vid ANY-INTEGER	Ending vertex identifier.	
end_vids ARRAY[ANY-INTEGERN]g vertices identifiers.		
directed BOOLEAN	• Optional.	

- When ${\tt false}$ the graph is considered as Undirected.
- Default is true which considers the graph as Directed.

Parameter	Type	Description	
heuristic	INTEGER	(optional). Heuristic number. Current valid values $0\sim5$. Default 5	
		 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) 1: h(v) abs(max(dx, dy)) 2: h(v) abs(min(dx, dy)) 3: h(v) = dx * dx + dy * dy 4: h(v) = sqrt(dx * dx + dy * dy) 5: h(v) = abs(dx) + abs(dy) 	
factor	FLOAT	(optional). For units manipulation. \setminus (factor $> 0 \setminus$). Default 1. see <i>Factor</i>	
epsilon	FLOAT	(optional). For less restricted results. \(epsilon >= 1 \). Default 1.	

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vidBIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vertending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

```
{\bf Examples}\P
```

Example:

Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_bdAstarCostMatrix(
        'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    )
    $$,</pre>
```

```
randomize := false
);
 seq | node | cost | agg_cost
   1 |
           1 |
                   2 |
   2 |
           3 |
                   1 |
                               2
           4 |
   3 |
                   2 |
                               3
           2 |
                               5
   4 |
                   1 |
   5 I
                   0 |
(5 rows)
```

See Also¶

- Bidirectional A* Family of functions
- Cost Matrix Category
- Traveling Sales Person Family of functions
- The queries use the Sample Data network.

Indices and tables

- Index
- Search Page

Synopsis¶ Based on A* algorithm, the bidirectional search finds a shortest path from a starting vertex (start_vid) to an ending vertex (end_vid). It runs two simultaneous searches: one forward from the start_vid, and one backward from the end_vid, stopping when the two meet in the middle. This implementation can be used with a directed graph and an undirected graph.

Characteristics¶ The main Characteristics are:

- Process is done only on edges with positive costs.
- Values are returned when there is a path.
- When the starting vertex and ending vertex are the same, there is no path.
 - The agg_cost the non included values (v, v) is 0
- When the starting vertex and ending vertex are the different and there is no path:
 - The agg_cost the non included values (u, v) is $\(\inf y \)$
- Running time (worse case scenario): $\langle O((E + V) * \log V) \rangle$
- For large graphs where there is a path bewtween the starting vertex and ending vertex:
 - It is expected to terminate faster than pgr_astar

Description of the Signatures ¶ Description of the edges_sql query for a star like functions ¶

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
reverse_	_c ant -numerical	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source), When negative: edge (target, so not exist, therefore it's not part graph.
x1	ANY-NUMERICAL	X coordinate of source vertex.
y1	ANY-NUMERICAL	Y coordinate of source vertex.
x2	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

 $SMALLINT,\,INTEGER,\,BIGINT,\,REAL,\,FLOAT$

Description of the parameters of the signatures \P

Parameter Type	Description
edges_sqlTEXT	Edges SQL query as described above.
${f start_vid}$ ANY-INTEGER	Starting vertex identifier.

Parameter	Type	Description			
start_vidsARRAY[ANY-INTSGER]ng vertices identifierers.					
$\mathbf{end}\mathbf{_vid}$	ANY-INTEGER	Ending vertex identifier.			
end_vids	ARRAY[ANY-IN	TEGERIG vertices identifiers.			
directed	BOOLEAN	• Optional.			
heuristic	INTEGER	 When false the graph is considered as Undirected. Default is true which considers the graph as Directed. (optional). Heuristic number. Current valid values 0~5. Default 5 • 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) • 1: h(v) abs(max(dx, dy)) • 2: h(v) abs(min(dx, dy)) • 3: h(v) = dx * dx + dy * dy 			
		 4: h(v) = sqrt(dx * dx + dy * dy) 5: h(v) = abs(dx) + abs(dy) 			
factor	FLOAT	(optional). For units manipulation. $\langle (factor > 0 \rangle)$. Default 1. see <i>Factor</i>			
epsilon	FLOAT	(optional). For less restricted results. \((epsilon >= 1\)). Default 1.			

See Also \P Indices and tables

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- Search Page

Bidirectional Dijkstra - Family of functions \P

- $pgr_bdDijkstra$ Bidirectional Dijkstra algorithm for the shortest paths.
- • $pgr_bdDijkstraCost$ - Proposed - Bidirectional Dijkstra to calculate the cost of the shortest paths
- pgr_bdDijkstraCostMatrix proposed Bidirectional Dijkstra algorithm to create a matrix of costs of the shortest paths.

 $pgr_bdDijkstra\P$ $pgr_bdDijkstra$ — Returns the shortest path(s) using Bidirectional Dijkstra algorithm.



Boost Graph Inside

Availability:

- pgr_bdDijkstra(one to one) 2.0.0, Signature changed 2.4.0
- pgr bdDijkstra(other signatures) 2.5.0

Signature Summary¶

```
pgr_bdDijkstra(edges_sql, start_vid, end_vid)
pgr_bdDijkstra(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
OR EMPTY SET
```

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

```
pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)
```

RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost) OR EMPTY SET

Signatures¶

Minimal signature¶

```
pgr_bdDijkstra(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph from one **start_vid** to one end_vid:

Example:

```
SELECT * FROM pgr_bdDijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   2, 3
);
seq | path_seq | node | edge | cost | agg_cost
              1 |
                     2 |
                          4 |
   1 |
                                   1 |
                                             0
             2 | 5 | 8 | 1 |
3 | 6 | 9 | 1 |
4 | 9 | 16 | 1 |
   2 |
   3 |
   4 I
              5 | 4 | 3 | 1 |
   5 |
              6 |
                     3 | -1 |
                                   0 |
   6 |
(6 rows)
```

pgr_bdDijkstra One to One¶

```
pgr_bdDijkstra(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one start_vid to one end_vid:

- on a ${\bf directed}$ graph when ${\bf directed}$ flag is missing or is set to ${\bf true}.$
- on an undirected graph when directed flag is set to false.

Example:

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    false
);
seq | path_seq | node | edge | cost | agg_cost
```

```
1 | 1 | 2 | 2 | 1 | 0
2 | 2 | 3 | -1 | 0 | 1
(2 rows)
```

pgr_bdDijkstra One to many¶

pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET

This signature finds the shortest path from one start_vid to each end_vid in end_vids:

- on a directed graph when directed flag is missing or is set to true.
- on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform a one to one pgr_dijkstra where the starting vertex is fixed, and stop when all end_vids are reached.

- The result is equivalent to the union of the results of the one to one pgr_dijkstra.
- The extra end_vid in the result is used to distinguish to which path it belongs.

Example:

1	1	3	2	4	1	0
2	2	3	5	8	1	1
3	3	3	6	9	1	2
4	4	3	9	16	1	3
5	5	3	4	3	1	4
6 l	6 l	3	3	-1	0	5
7	1	11	2	4	1	0
8	2	11	5	8	1	1
9	3	11	6	11	1	2
10	4	11	11	-1	0	3
(10 rows)						

pgr_bdDijkstra Many to One¶

pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET

This signature finds the shortest path from each start_vid in start_vids to one end_vid:

- on a directed graph when directed flag is missing or is set to true.
- on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to one pgr_dijkstra where the ending vertex is fixed.

- The result is the union of the results of the one to one pgr_dijkstra.
- The extra start_vid in the result is used to distinguish to which path it belongs.

Example:

13 l

(13 rows)

```
SELECT * FROM pgr bdDijkstra(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   ARRAY[2, 7], 3);
seq | path_seq | start_vid | node | edge | cost | agg_cost
---+----+---+----+-
  1 |
             1 |
                         2 |
                                2 |
                                      4 I
                                              1 |
                                                        0
                         2 |
                                             1 |
             2 |
                                5 I
                                       8 |
  2 |
                                                        1
                         2 |
                                6 |
                                      9 |
  3 |
             3 |
                                              1 |
                                                        2
  4 |
             4 |
                         2 |
                                9 |
                                     16 |
                                              1 |
  5 |
             5 I
                         2 |
                                4 |
                                      3 |
                                              1 |
                                                        4
                         2 |
                                3 |
                                      -1 |
  6 |
             6 |
                                              0 |
                                                        5
  7 |
                         7 |
                                7 |
                                       6 I
             1 |
                                              1 |
                                                        0
  8 |
             2 |
                         7 |
                                8 |
                                      7 |
                                              1 |
                                                        1
  9 I
             3 I
                         7 |
                                5 I
                                      8 I
                                              1 l
                                                        2
                                6 |
  10 |
             4 |
                         7 |
                                                        3
                                      9 |
                                              1 |
  11 |
             5 l
                         7 |
                                9 |
                                     16 |
                                              1 |
                                                        4
                         7 |
                                                        5
  12 |
             6 I
                                4 |
                                      3 l
                                              1 |
```

7 |

pgr_bd Dijkstra Many to Many
¶

7 I

pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET

-1 |

0 |

3 I

This signature finds the shortest path from each start_vid in start_vids to each end_vid in end_vids:

- on a directed graph when directed flag is missing or is set to true.
- on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to Many pgr_dijkstra for all start_vids.

- The result is the union of the results of the one to one pgr_dijkstra.
- The extra start_vid in the result is used to distinguish to which path it belongs.

The extra start_vid and end_vid in the result is used to distinguish to which path it belongs.

Example:

17 |

18 I

19 |

20 |

21 |

22 |

(22 rows)

1 I

2 |

3 |

4 |

5 I

SELECT * FROM pgr_bdDijkstra('SELECT id, source, target, cost, reverse_cost FROM edge_table', ARRAY[2, 7], ARRAY[3, 11]); seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost 1 | 1 | 2 | 3 | 2 | 4 | 1 | 0 2 | 2 | 5 I 8 | 2 | 3 | 1 | 1 3 | 3 | 2 | 3 | 6 9 | 1 | 2 2 | 3 4 | 4 | 3 9 16 1 5 I 5 1 2 | 3 4 3 1 4 5 6 I 6 2 | 3 I 3 0 | -1 4 0 7 | 1 l 2 | 2 | 1 l 11 | 8 | 2 | 2 | 11 | 5 | 8 | 1 | 1 9 | 3 | 2 | 11 | 6 | 11 | 2 10 l 4 | 2 | 11 | -1 I 0 | 3 11 | 11 | 7 | 7 6 | 0 1 | 3 | - 1 1 | 2 3 12 7 | -8 7 1 1 13 | 3 7 | 3 5 8 1 | 2 14 I 4 | 7 | 3 I 6 9 | 1 I 3 5 I 3 | 4 7 | 9 16 | 1 | 15 | - 1 7 | 3 | 5 16 | 6 | 4 | 3 | 1 | 7 | 7 | 3 | 3 | -1 I 0 | 6

246

11 |

11 |

11 |

11 |

11 |

7 |

8 1

5 1

10 I

11 |

6 I

7 |

10

12 |

-1 |

1 I

1 |

1 |

1 |

0

1

2

3

4

7 |

7 |

7 |

7 |

7 |

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
$reverse_$	cANY -NUMERI€IAL	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Column	Type	Default	Description
$\overline{\mathrm{sql}}$	TEXT		SQL query as desc
start_vio	d BIGINT		Identifier of the stapath.
start_vio	${ m ds}$ ARRAY[BIGINT	7]	Array of identifiers
${ m end_vid}$	BIGINT		Identifier of the enepath.
end_vids	s ARRAY[BIGINT	·]	Array of identifiers

Column	Type	Default	Description
directed	BOOLEAN	true	When true (DirectedWhen false considered as

Description of the return values for a path \P

Returns set of (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)

Column	Type
seq	INT
$\operatorname{path_id}$	INT
path _seq	INT
start_vid	BIGINT
342 5 <u></u>	
end_vid	BIGINT
node	BIGINT
_	
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

See Also \P

- ullet The queries use the $Sample\ Data$ network.
- $\bullet \ \ Bidirectional \ Dijkstra \ \ Family \ of \ functions$
- $\bullet \ \ http://www.cs.princeton.edu/courses/archive/spr06/cos423/Handouts/EPP\%20 shortest\%20 path\%21/20 path\%21/20 path\%22/20 path\%2$
- $\bullet \ \ https://en.wikipedia.org/wiki/Bidirectional_search$

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pgr_bdDijkstraCost - Proposed¶ pgr_bdDijkstraCost — Returns the shortest path(s)'s cost using Bidirectional Dijkstra algorithm.



Boost Graph Inside

Availability: 2.5.0

Warning

Experimental functions

- They are not officially of the current release.
 - They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Signature Summary¶

```
pgr_dijkstraCost(edges_sql, start_vid, end_vid)
pgr_bdDijkstraCost(edges_sql, start_vid, end_vid, directed)
pgr_bdDijkstraCost(edges_sql, start_vid, end_vids, directed)
pgr_bdDijkstraCost(edges_sql, start_vids, end_vid, directed)
pgr_bdDijkstraCost(edges_sql, start_vids, end_vids, directed)

RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

```
Signatures¶
```

```
Minimal signature¶
```

```
pgr_bdDijkstraCost(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph from one **start_vid** to one **end_vid**:

Example:

pgr_bdDijkstraCost One to One¶

```
pgr_bdDijkstraCost(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one start_vid to one end_vid:

on a directed graph when directed flag is missing or is set to
true.

• on an undirected graph when directed flag is set to false.

Example:

pgr_bdDijkstraCost One to many¶

```
pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one start_vid to each end_vid in end_vids:

on a directed graph when directed flag is missing or is set to

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform a one to one pgr_dijkstra where the starting vertex is fixed, and stop when all end_vids are reached.

- The result is equivalent to the union of the results of the one to one pgr_dijkstra.
- The extra end_vid in the result is used to distinguish to which path it belongs.

Example:

pgr bdDijkstraCost Many to One¶

```
pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each start_vid in start_vids to one end_vid: on a directed graph when directed flag is missing or is set to true.

• on an undirected graph when directed flag is set to false.

Using this signature, will load once the graph and perform several one to one pgr_dijkstra where the ending vertex is fixed.

• The result is the union of the results of the one to one pgr_dijkstra.

• The extra start_vid in the result is used to distinguish to which path it belongs.

Example:

SELECT * FROM pgr_bdDijkstra(
 'SELECT id, source, target, cost, reverse_cost FROM edge_table',
 ARRAY[2, 7], 3);

seq	path_seq	start_vid	1	node	١	edge	1	cost	1	agg_cost
+	-+		۰-		+-		+-		+-	
1	1	2	ı	2	ı	4	ı	1	١	0
2	2	2	ı	5	ı	8		1	1	1
3	3	2		6		9	-	1	-	2
4	4	2		9		16		1	-	3
5	5	2		4		3		1	-	4
6	6 l	2		3	1	-1	1	0	1	5
7	1	7		7	1	6	1	1	1	0
8	2	7		8	1	7	1	1	-	1
9	3	7		5	1	8	1	1	-	2
10	4	7	1	6	1	9	1	1	-	3
11	5	7		9	I	16	1	1	1	4
12	6 I	7	1	4	1	3	1	1	1	5
13	7	7	1	3	1	-1	1	0	-	6
(13 row	s)									

pgr_bdDijkstraCost Many to Many¶

```
pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each start_vid in start_vids to each end_vid in on a directed graph when directed flag is missing or is set to true.

- on an ${\bf undirected}$ graph when ${\tt directed}$ flag is set to ${\tt false}.$

Using this signature, will load once the graph and perform several one to Many pgr_dijkstra for all start_vids.

- The result is the union of the results of the one to one pgr_dijkstra.
- The extra start_vid in the result is used to distinguish to which path it belongs.

The extra start_vid and end_vid in the result is used to distinguish to which path it belongs.

Example:

SELECT * FROM pgr_bdDijkstra(
 'SELECT id, source, target, cost, reverse_cost FROM edge_table',
 ARRAY[2, 7], ARRAY[3, 11]);

seq	path_seq	start_vid	end_vid	node	edge	cost	agg_cost
1	1	1 2	- 3	- 	4		- 0
2	1 2	2] 3	5	8	1	1
3	J 3	2] 3	6	9	1	1 2
4	l 4	2] 3	9	16	1	J 3
5	J 5	1 2] 3	4	3	1	4
6	l 6	1 2] 3	3	l -1	0	l 5
7	1	1 2	11	2	4	1	l 0
8	1 2	1 2	11	J 5	8	1	1
9	J 3	1 2	11	1 6	11	1	1 2
10	l 4	1 2	11	11	-1	0	J 3
11	1	7] 3	7	6	1	l 0
12	1 2	7	J 3	8	7	1	1
13	J 3	7] 3	5	8	1	1 2
14	l 4	7] 3	6	9	1	J 3
15	J 5	7] 3	9	16	1	1 4
16	l 6	7] 3	4	3	1	l 5
17	7	7] 3	3	l -1	0	l 6
18	1	7	11	7	1 6	1	0
19	1 2	7	11	8	7	1	1
20	J 3	7	11	5	1 10	1	1 2
21	l 4	7	11	10	12	1	J 3
22	J 5	7	11	11	-1	0	l 4
(22 r	ows)						

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions¶ edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
$\overline{\mathbf{id}}$	ANY-INTEG	ER	Identifier of the edge.
source	ANY-INTEG	ER	Identifier of the first end point vertex edge.

Column	Type Default	Description
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
reverse_	_c ant -numerical	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

 $SMALLINT,\,INTEGER,\,BIGINT,\,REAL,\,FLOAT$

Description of the parameters of the signatures \P

Column	Туре	Default	Description
$\overline{\mathrm{sql}}$	TEXT		SQL query as desc
start_vid	BIGINT		Identifier of the stapath.
$start_vid$	sARRAY[BIGINT]		Array of identifiers
end_vid	BIGINT		Identifier of the enepath.
end_vids	ARRAY[BIGINT]		Array of identifiers
directed	BOOLEAN	true	 When true 0 Directed When false considered as

Description of the return values for a Cost function ¶

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vidBIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vert ending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

See Also \P

- The queries use the Sample Data network.
- pgr_bdDijkstra
- $\bullet \ \ http://www.cs.princeton.edu/courses/archive/spr06/cos423/Handouts/EPP\%20 shortest\%20 path\%21/20 path\%2$
- https://en.wikipedia.org/wiki/Bidirectional_search

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$pgr_bdDijkstraCostMatrix - proposed\P \quad \mathrm{Name}\P$

 $\verb|pgr_bdDijkstraCostMatrix-Calculates| the a cost matrix using |\textit{pgr_bdDijkstra}|.$



Boost Graph Inside

Availability: 2.5.0

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.

- Functionality might change.
- pgTap tests might be missing.
- Might need c/c++ coding.
- May lack documentation.
- Documentation if any might need to be rewritten.
- Documentation examples might need to be automatically generated.
- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

Synopsis¶

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary¶

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vids)
pgr_bdDijkstraCostMatrix(edges_sql, start_vids, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Signatures¶

Minimal Signature¶

The minimal signature: • Is for a directed graph.

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example:

Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_bdDijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);</pre>
```

start_vid		end_vid		agg_cost
1		2	+- 	1
1		3	1	6
1		4	1	5
2		1	1	1
2		3	1	5
2	1	4	1	4

```
3 |
                    1 |
         3 |
                    2 |
                               1
         3 |
                   4 |
                               3
                   1 |
                               3
         4 |
         4 |
                    2 |
                               2
         4 |
                    3 |
(12 rows)
```

$Complete\ Signature \P$

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vids, directed:=true)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example:

Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_bdDijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    false
);</pre>
```

start_vid	end_vid	agg_cost
1	 2	1
1	3	1 2
1	4] 3
2	1	1
2	3	1
2	4	1 2
3	1	1 2
3	2	1
3	4	1
4	1	3
4	2	1 2
4	3	1
(40		

(12 rows)

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P

edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
$\overline{\mathbf{id}}$	ANY-INTEG	ER	Identifier of the edge.
source	ANY-INTEG	ER	Identifier of the first end point vertex edge.
target	ANY-INTEG	ER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMER	ICAL	Weight of the edge (source, target)
			• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	_cant-numer	ICAL	Weight of the edge (target, source),
			• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

ParametefType	Description					
edges_sqEXT	Edges SQL query as described above.					
start_videRAY[ANY-INTEGER] identifiers of the vertices.						
directed BOOLEAN	(optional). When false the graph is considered as Undirected. Default is true which considers the graph as Directed.					

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vid BIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vert ending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

$\operatorname{Examples} \P$

Example:

Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_bdDijkstraCostMatrix(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
                 1 |
   1 |
          1 |
                            0
   2 |
          2 |
                 1 |
                            1
   3 I
          3 I
                 1 |
                            2
                            3
   4 |
          4 I
                 3 |
                            6
   5 |
        1 |
                 0 |
(5 rows)
```

See Also¶

- Bidirectional Dijkstra Family of functions
- Cost Matrix Category
- Traveling Sales Person Family of functions
- The queries use the $Sample\ Data$ network.

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Synopsis¶ Based on Dijkstra's algorithm, the bidirectional search finds a shortest path a starting vertex (start_vid) to an ending vertex (end_vid). It runs two simultaneous searches: one forward from the source, and one backward from the target, stopping when the two meet in the middle. This implementation can be used with a directed graph and an undirected graph.

Characteristics The main Characteristics are:

- Process is done only on edges with positive costs.
- Values are returned when there is a path.
- When the starting vertex and ending vertex are the same, there is no path.
 - The agg cost the non included values (v, v) is 0
- When the starting vertex and ending vertex are the different and there is no path:
 - The agg_cost the non included values (u, v) is $\setminus (\setminus infty \setminus)$
- Running time (worse case scenario): $\langle O((V \setminus log V + E)) \rangle$
- For large graphs where there is a path bewtween the starting vertex and ending vertex:
 - It is expected to terminate faster than pgr_dijkstra

See Also¶ Indices and tables

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withPoints - Family of functions¶ When points are also given as input:

- pgr_withPoints Proposed Route from/to points anywhere on the graph.
- pgr withPointsCost Proposed Costs of the shortest paths.
- pgr_withPointsCostMatrix proposed Costs of the shortest paths.
- $pgr_withPointsKSP$ Proposed K shortest paths.
- pgr_withPointsDD Proposed Driving distance.

pgr_withPoints - Proposed¶ Name¶

pgr_withPoints - Returns the shortest path in a graph with additional temporary vertices.

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.



Boost Graph Inside

Availability: 2.2.0

Synopsis

Modify the graph to include points defined by points_sql. Using Dijkstra algorithm, find the shortest path(s)

Characteristics:¶

The main Characteristics are:

- Process is done only on edges with positive costs.
- Vertices of the graph are:
 - **positive** when it belongs to the edges_sql
 - **negative** when it belongs to the points_sql
- Values are returned when there is a path.
 - When the starting vertex and ending vertex are the same, there is no path. The agg_cost the non included values $(v,\,v)$ is 0
 - When the starting vertex and ending vertex are the different and there is no path: The agg_cost the non included values (u, v) is ∞
- For optimization purposes, any duplicated value in the start_vids or end_vids are ignored.
- The returned values are ordered: start_vid ascending end_vid ascending
- Running time: $\langle O(|\text{start}_\text{vids}| \times (V \log V + E)) \rangle$

Signature Summary¶

```
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid)
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid, directed, driving_side, details)
pgr_withPoints(edges_sql, points_sql, start_vid, end_vids, directed, driving_side, details
pgr_withPoints(edges_sql, points_sql, start_vids, end_vid, directed, driving_side, details
pgr_withPoints(edges_sql, points_sql, start_vids, end_vids, directed, driving_side, details
pgr_withPoints(edges_sql, points_sql, start_vids, end_vids, directed, driving_side, detail
RETURNS SET OF (seq, path_seq, [start_vid,] [end_vid,] node, edge, cost, agg_cost)
```

$Signatures \P$

Minimal Use¶

The minimal signature: • Is for a directed graph.

- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.
- No **details** are given about distance of other points of points_sql query.

```
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

Example:

From point 1 to point 3

```
SELECT * FROM pgr_withPoints(
```

'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id', 'SELECT pid, edge_id, fraction, side from pointsOfInterest',

-1, -3);

-					_				agg_cost
1	 1			٠.		٠.	0.6	٠.	0
2	2	1	2	1	4	1	1	١	0.6
3	3	1	5	1	10	1	1		1.6
4	4	1	10	1	12	1	0.6		2.6
5	5	1	-3	1	-1	1	0		3.2
(5 rows)								

One to One¶

Example:

From point 1 to vertex 3

```
SELECT * FROM pgr_withPoints(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
   'SELECT pid, edge_id, fraction, side from pointsOfInterest',
   -1, 3,
   details := true);
 seq | path_seq | node | edge | cost | agg_cost
----+----
            1 | -1 | 1 | 0.6 |
   2 |
           2 |
                2 | 4 | 0.7 |
                                      0.6
   3 I
            3 l
                 -6 I
                       4 | 0.3 |
                                       1.3
   4 |
           4 |
                 5 |
                        8 |
                              1 |
                                        1.6
           5 |
                        9 |
   5 I
                  6 |
                               1 |
                                        2.6
                              1 |
                       16 |
   6 |
            6 |
                  9 |
                                        3.6
   7 |
            7 |
                  4 | 3 |
                               1 |
                                        4.6
   8 |
           8 |
                   3 | -1 |
                             0 |
                                        5.6
(8 rows)
One to Many¶
pgr_withPoints(edges_sql, points_sql, start_vid, end_vids,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost)
Example:
From point 1 to point 3 and vertex 5
SELECT * FROM pgr_withPoints(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
```

-1	, ARRAY[-3	,5]);				•		•		
seq	path_seq	-				_				
1	1	-3	٠.	-1	٠.	1	٠.	0.6		0
2	2	l –3		2		4		1		0.6
3	3	-3	1	5	1	10	1	1	1	1.6
4	4	-3		10	1	12	1	0.6	1	2.6
5	5	-3		-3	1	-1	1	0	1	3.2
6	1	J 5		-1	1	1	1	0.6	1	0
7	2	J 5		2	1	4	1	1	1	0.6
8	3	J 5	1	5	1	-1	1	0	1	1.6

Many to One¶

(8 rows)

Example:

From point 1 and vertex 2 to point 3

SELECT * FROM pgr_withPoints(

'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id', 'SELECT pid, edge_id, fraction, side from pointsOfInterest', ARRAY[-1,2], -3);

seq	path_seq	start_pid		node	1	edge		cost	1	agg_cost
1	 1	-1	+- 	-1	+- 	1	+- 	0.6	+- 	0
2	2	-1	1	2	I	4	1	1	1	0.6
3	3	-1	1	5		10	1	1	1	1.6
4	4	-1	1	10	1	12	1	0.6	1	2.6
5	5	-1	1	-3	1	-1	1	0	1	3.2
6 I	1	2	1	2	1	4	1	1	1	0
7	2	2	1	5	1	10	1	1	1	1
8	3	2	1	10	1	12	1	0.6	1	2
9	4	2	1	-3		-1	1	0	1	2.6
(9 rows)									

Many to Many¶

Example:

From point 1 and vertex 2 to point 3 and vertex 7

SELECT * FROM pgr_withPoints(

'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id', 'SELECT pid, edge_id, fraction, side from pointsOfInterest', ARRAY[-1,2], ARRAY[-3,7]);

-		start_pid +	- -		•		
1				-1			0
2	2	-1 l	-3	2	4	1	0.6
3] 3	-1	-3	5	10	1	1.6
4	l 4 l	-1 l	-3 l	10 l	12 l	0.6	2.6

5 I	5 l	-1 l	-3 l	-3 I	-1 l	0	3.2
6 I	1	-1 İ	7	-1 İ	1 İ	0.6	0
7	2	-1	7	2	4	1	0.6
8	3	-1	7	5	7	1	1.6
9	4	-1 l	7	8	6 I	1	2.6
10	5 I	-1	7	7	-1	0	3.6
11	1	2	-3	2	4	1	0
12	2	2	-3	5	10	1	1
13	3	2	-3	10	12	0.6	2
14	4	2	-3	-3	-1 I	0	2.6
15	1	2	7	2	4	1	0
16	2	2	7	5	7	1	1
17	3	2	7	8	6 I	1	2
18	4	2	7	7	-1	0	3
(18 rows							

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		 When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	cant-numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query \P

points_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	ANY-INT	EGER(optional) Identifier of the point.
		 If column present, it can not be NULL. If column not present, a sequential identifier will be given automatically.
$edge_ic$	ANY-INT	EGERIdentifier of the "closest" edge to the point.
fraction	ANY-NUM	ERICWalue in <0,1> that indicates the relative postition from the first end point of the edge.
\mathbf{side}	CHAR	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is:
		 In the right, left of the edge or If it doesn't matter with 'b' or NULL. If column not present 'b' is considered.

Where:

ANY-INTEGER:

smallint, int, bigint

ANY-NUMERICAL:

smallint, int, bigint, real, float

Description of the parameters of the signatures \P

ParametefType	Description	
edges_sqEXT	Edges SQL query as described above.	
points_staxt	Points SQL query as described above.	
start_vidNY-INTEGERStarting vertex identifier. When negative: is a point's pid.		
	pia.	

ParametefType	Description
end_vidANY-INTEG	EREnding vertex identifier. When negative: is a point's pid.
start_vi dR RAY[ANY	-INTEGER] identifiers of starting vertices. When negative: is a point's pid.
end_vid&RRAY[ANY	-IATEGER] identifiers of ending vertices. When negative: is a point's pid.
directed BOOLEAN	(optional). When false the graph is considered as Undirected. Default is true which considers the graph as Directed.
driving_SMAR	 (optional) Value in ['b', 'r', 'l', NULL] indicating if the driving side is: In the right or left or If it doesn't matter with 'b' or NULL. If column not present 'b' is considered.
details BOOLEAN	(optional). When true the results will include the points in points_sql that are in the path. Default is false which ignores other points of the points_sql.

Description of the return values \P

Type

 ${\bf Description}$

 Column

Returns set of (seq, [path_seq,] [start_vid,] [end_vid,] node, edge, cost, agg_cost)

seq	INTEGER	Row sequence.
path_sec	1 INTEGER	Path sequence that indicates the relative position on the path.
start_vic	BIGINT	Identifier of the starting vertex. When negative: is a point's pid.
end_vid	BIGINT	Identifier of the ending vertex. When negative: is a point's pid.
node	BIGINT	 Identifier of the node: • A positive value indicates the node is a vertex of edges_sql. • A negative value indicates the node is a point of points_sql.
edge	BIGINT	Identifier of the edge used to go from node to the next node in the path s -1 for the last row in the path sequence.
cost	FLOAT	Cost to traverse from node using edge to the next node in the path sequence 0 for the last row in the path sequence.

Column	Type	Description	
agg_cost	FLOAT	Aggregate cost from start_pid to node. the first row in the path sequence.	• 0 for

Examples

Example:

Which path (if any) passes in front of point 6 or vertex 6 with **right** side driving topology.

```
SELECT ('(' || start_pid || ' => ' || end_pid ||') at ' || path_seq || 'th step:')::TEXT AS pat
        CASE WHEN edge = -1 THEN ' visits'
            ELSE ' passes in front of'
        END as status,
        CASE WHEN node < 0 THEN 'Point'
           ELSE 'Vertex'
        END as is_a,
        abs(node) as id
    FROM pgr_withPoints(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
       'SELECT pid, edge_id, fraction, side from pointsOfInterest',
        ARRAY[1,-1], ARRAY[-2,-3,-6,3,6],
        driving_side := 'r',
        details := true)
    WHERE node IN (-6,6);
        path_at |
                               status
                                          | is_a | id
-----
 (-1 => -6) at 4th step: | visits
                                               | Point | 6
 (-1 \Rightarrow -3) at 4th step: | passes in front of | Point | 6
 (-1 \Rightarrow -2) at 4th step: | passes in front of | Point | 6
 (-1 \Rightarrow -2) at 6th step: | passes in front of | Vertex |
 (-1 => 3) at 4th step: | passes in front of | Point |
 (-1 => 3) at 6th step: | passes in front of | Vertex | (-1 => 6) at 4th step: | passes in front of | Point |
 (-1 => 6) at 6th step: | visits
                                              | Vertex |
 (1 \Rightarrow -6) at 3th step: | visits
                                               | Point |
 (1 => -3) at 3th step: | passes in front of | Point |
 (1 \Rightarrow -2) at 3th step: | passes in front of | Point |
 (1 => -2) at 5th step: | passes in front of | Vertex |
 (1 => 3) at 3th step:
                           passes in front of | Point |
 (1 \Rightarrow 3) at 5th step:
                        passes in front of | Vertex |
 (1 => 6) at 3th step:
                           passes in front of | Point | 6
 (1 => 6) at 5th step:
                        - 1
                           visits
                                               | Vertex | 6
```

```
(16 rows)
```

Example:

Which path (if any) passes in front of point 6 or vertex 6 with **left** side driving topology.

```
SELECT ('(' || start_pid || ' => ' || end_pid ||') at ' || path_seq || 'th step:')::TEXT AS pat
        CASE WHEN edge = -1 THEN ' visits'
           ELSE ' passes in front of'
        END as status,
        CASE WHEN node < 0 THEN 'Point'
           ELSE 'Vertex'
        END as is_a,
        abs(node) as id
    FROM pgr_withPoints(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
       'SELECT pid, edge_id, fraction, side from pointsOfInterest',
        ARRAY[1,-1], ARRAY[-2,-3,-6,3,6],
        driving_side := 'l',
        details := true)
    WHERE node IN (-6,6);
        path_at |
                          status
                                       | is_a | id
______
 (-1 => -6) at 3th step: | visits | Point | 6
 (-1 \Rightarrow -3) at 3th step: | passes in front of | Point | 6
 (-1 \Rightarrow -2) at 3th step: | passes in front of | Point | 6
 (-1 \Rightarrow -2) at 5th step: | passes in front of | Vertex | 6
 (-1 \Rightarrow 3) at 3th step: | passes in front of | Point | 6
 (-1 => 3) at 5th step: | passes in front of | Vertex |
 (-1 => 6) at 3th step: | passes in front of | Point |
 (-1 => 6) at 5th step: | visits
(1 => -6) at 4th step: | visits
                                          | Vertex |
                                             | Point |
 (1 => -3) at 4th step: | passes in front of | Point |
 (1 => -2) at 4th step: | passes in front of | Point | 6
 (1 \Rightarrow -2) at 6th step: | passes in front of | Vertex | 6
 (1 => 3) at 4th step: | passes in front of | Point |
 (1 => 3) at 6th step: | passes in front of | Vertex |
 (1 => 6) at 4th step: | passes in front of | Point | 6
 (1 => 6) at 6th step: | visits
                                              | Vertex |
(16 rows)
```

Example:

Many to many example with a twist: on undirected graph and showing details.

```
SELECT * FROM pgr_withPoints(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7],
    directed := false,
    details := true);
seq | path_seq | start_pid | end_pid | node | edge | cost | agg_cost
   1 |
                                     -3 I
                                             -1 |
                                                           0.6 |
                                                                         0
                                                      1 |
   2 |
               2 |
                          -1 |
                                     -3 I
                                              2 |
                                                     4 |
                                                           0.7 |
                                                                       0.6
   3 I
               3 |
                          -1 |
                                     -3 |
                                             -6 |
                                                     4 |
                                                           0.3 |
                                                                       1.3
               4 |
                          -1 |
                                     -3 |
                                                     10 |
   4 |
                                              5
                                                -
                                                             1 |
                                                                       1.6
   5 |
               5 |
                          -1 |
                                     -3 |
                                             10 |
                                                     12 |
                                                           0.6
                                                                       2.6
   6 I
               6 I
                          -1 l
                                     -3 |
                                             -3 |
                                                     -1 l
                                                             0 |
                                                                       3.2
   7 |
               1 |
                          -1 |
                                      7 |
                                             -1 |
                                                     1 |
                                                           0.6 |
                                                                         0
   8 |
               2 |
                          -1 |
                                      7 |
                                              2 |
                                                     4 |
                                                           0.7 |
                                                                       0.6
   9 |
              3 |
                          -1 |
                                      7 |
                                             -6 I
                                                     4 |
                                                           0.3 |
                                                                       1.3
               4 |
                          -1 |
                                      7 |
                                                     7 |
  10 |
                                              5 |
                                                             1 |
                                                                       1.6
                                      7 |
               5 |
                           -1 |
                                              8 |
                                                     6 |
                                                           0.7 |
                                                                       2.6
  11 |
  12 |
               6 |
                           -1 |
                                      7 |
                                             -4
                                                     6 |
                                                           0.3 |
                                                                       3.3
  13 |
               7 |
                           -1 l
                                      7 |
                                              7
                                                             0 |
                                                                       3.6
                            2 |
                                     -3 I
                                              2 |
                                                     4 |
                                                           0.7 |
  14 |
               1 |
                                                                         0
                            2 |
                                             -6 I
                                                                       0.7
               2 |
                                     -3 |
                                                     4 |
                                                           0.3 |
  15 |
  16 |
               3 |
                            2 |
                                     -3 |
                                              5 |
                                                     10 |
                                                                         1
                            2 |
                                             10 l
  17 |
               4 |
                                     -3 |
                                                     12 |
                                                           0.6
                                                                         2
  18 |
               5 I
                           2 |
                                     -3 |
                                             -3 I
                                                     -1 |
                                                             0 |
                                                                       2.6
                            2 |
                                      7 |
                                              2 |
                                                     4 |
                                                           0.7 |
  19 |
               1 |
                                                                         0
  20 |
               2 |
                            2 |
                                      7 |
                                             -6 |
                                                     4 |
                                                           0.3 |
                                                                       0.7
                                              5 I
  21 |
               3 |
                            2 |
                                      7 |
                                                      7 |
                                                             1 |
                                                                         1
  22 I
               4 I
                                      7 |
                                              8 I
                                                                         2
                            2 |
                                                           0.7 |
                                                     6 I
                                             -4 |
  23 |
               5 |
                            2 |
                                      7 |
                                                     6 |
                                                           0.3 |
                                                                       2.7
  24 |
               6 |
                            2 |
                                      7 |
                                              7 |
                                                     -1 |
                                                                         3
(24 rows)
```

The queries use the Sample Data network.

History

• Proposed in version 2.2

See Also¶

• withPoints - Family of functions

Indices and tables

- Index
- Search Page

pgr_withPointsCost - Proposed¶ Name¶

pgr_withPointsCost - Calculates the shortest path and returns only the aggregate cost of the shortest path(s) found, for the combination of points given.

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.



Boost Graph Inside

Availability: 2.2.0

Synopsis

Modify the graph to include points defined by points_sql. Using Dijkstra algorithm, return only the aggregate cost of the shortest path(s) found.

Characteristics:¶

The main Characteristics are: • It does not return a path.

- Returns the sum of the costs of the shortest path for pair combination of vertices in the modified graph.
- Vertices of the graph are:
 - **positive** when it belongs to the edges_sql
 - **negative** when it belongs to the points_sql
- Process is done only on edges with positive costs.
- Values are returned when there is a path.

- The returned values are in the form of a set of (start_vid, end_vid, agg_cost).
- When the starting vertex and ending vertex are the same, there is no path.
 - * The agg_cost in the non included values (v, v) is 0
- When the starting vertex and ending vertex are the different and there is no path.
 - * The agg_cost in the non included values (u, v) is $\setminus (\setminus infty \setminus)$
- If the values returned are stored in a table, the unique index would be the pair: (start_vid, end_vid).
- For undirected graphs, the results are symmetric.
 - The agg_cost of (u, v) is the same as for (v, u).
- For optimization purposes, any duplicated value in the start_vids or end_vids is ignored.
- The returned values are ordered:
 - start_vid ascending
 - end_vid ascending
- Running time: $\langle O(| \text{start} \vee \text{vids} | * (V \setminus \text{log } V + E)) \rangle$

Signature Summary¶

pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vid, directed, driving_side)
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vids, directed, driving_side)
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vid, directed, driving_side)
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vids, directed, driving_side)
RETURNS SET OF (start_vid, end_vid, agg_cost)

Note

There is no **details** flag, unlike the other members of the withPoints family of functions.

 $Signatures \P$

Minimal Use¶

The minimal signature: • Is for a directed graph.

• The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.

pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)

Example:

```
SELECT * FROM pgr_withPointsCost(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -3);
 start_pid | end_pid | agg_cost
-----
       -1 | -3 | 3.2
(1 row)
One to One¶
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vid,
    directed:=true, driving_side:='b')
RETURNS SET OF (seq, node, edge, cost, agg_cost)
Example:
SELECT * FROM pgr_withPointsCost(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3,
    directed := false);
 start_pid | end_pid | agg_cost
   -1 | 3 | 1.6
(1 row)
One to Many¶
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vids,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
Example:
SELECT * FROM pgr_withPointsCost(
  'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, ARRAY[-3,5]);
 start_pid | end_pid | agg_cost
       -1 |
                -3 |
                         3.2
                5 |
                         1.6
       -1 |
(2 rows)
```

```
Many to One¶
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vid,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
Example:
SELECT * FROM pgr_withPointsCost(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
   ARRAY[-1,2], -3);
 start_pid | end_pid | agg_cost
_____
       -1 | -3 | 3.2
               -3 l
        2 |
                        2.6
(2 rows)
Many to Many¶
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vids,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
Example:
SELECT * FROM pgr_withPointsCost(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
   ARRAY[-1,2], ARRAY[-3,7]);
 start_pid | end_pid | agg_cost
-----
       -1 |
            -3 | 3.2
       -1 |
               7 |
                        3.6
                -3 l
                        2.6
        2 |
                7 |
        2 |
(4 rows)
Description of the Signatures¶
```

Description of the edges_sql query for dijkstra like functions¶

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
$reverse_$	c ant -numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query \P

points_sql:

an SQL query, which should return a set of rows with the following columns: $\,$

Column	Type	Description
pid	ANY-INTEGE	R(optional) Identifier of the point.

- If column present, it can not be NULL.
- If column not present, a sequential identifier will be given automatically.

 ${\bf edge_id}$ ANY-INTEGERIdentifier of the "closest" edge to the point.

 $\label{eq:continuous} \textbf{fraction ANY-NUMERICWA} lue in <0.1> that indicates the relative position from the first end point of the edge.$

Column	Type	Description
side	CHAR	 (optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: In the right, left of the edge or If it doesn't matter with 'b' or NULL. If column not present 'b' is considered.

Where:

ANY-INTEGER:

smallint, int, bigint

ANY-NUMERICAL:

smallint, int, bigint, real, float

Description of the parameters of the signatures \P

$Parametef \Gamma ype$	Description				
edges_sqEXT	Edges SQL query as described above.				
points_staxt	Points SQL query as described above.				
start_vidNY-INTEGE	RStarting vertex identifier. When negative: is a point's pid.				
end_vidANY-INTEGE	end_vidANY-INTEGEREnding vertex identifier. When negative: is a point's pid.				
start_vidRRAY[ANY-	INTEGER identifiers of starting vertices. When negative: is a point's pid.				
end_vid&RRAY[ANY-	INTEGER identifiers of ending vertices. When negative: is a point's pid.				
directed BOOLEAN	(optional). When false the graph is considered as Undirected. Default is true which considers the graph as Directed.				
driving_ShAR	 (optional) Value in ['b', 'r', 'l', NULL] indicating if the driving side is: In the right or left or If it doesn't matter with 'b' or NULL. If column not present 'b' is considered. 				

Description of the return values \P

Returns set of (start_vid, end_vid, agg_cost)

Column	Type	Description
start_vid	BIGINT	Identifier of the starting vertex. When negative: is a point's pid.
${ m end_vid}$	BIGINT	Identifier of the ending point. When negative: is a point's pid.
agg_cost	FLOAT	Aggregate cost from start_vid to end_vid.

$\mathrm{Examples}\P$

Example:

With **right** side driving topology.

```
SELECT * FROM pgr_withPointsCost(
  'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
   ARRAY[-1,2], ARRAY[-3,7],
   driving_side := 'l');
 start_pid | end_pid | agg_cost
-----
                       3.2
       -1 | -3 |
               7 |
                       3.6
       -1 |
                       2.6
        2 |
                -3 |
                7 |
        2 |
(4 rows)
```

Example:

With left side driving topology.

```
SELECT * FROM pgr_withPointsCost(
  'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
   'SELECT pid, edge_id, fraction, side from pointsOfInterest',
   ARRAY[-1,2], ARRAY[-3,7],
   driving_side := 'r');
start_pid | end_pid | agg_cost
  -----
              -3 | 4
       -1 |
       -1 |
               7 |
                       4.4
       2 |
              -3 |
                       2.6
                7 |
        2 |
                         3
(4 rows)
```

Example:

Does not matter driving side.

```
SELECT * FROM pgr_withPointsCost(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7],
    driving_side := 'b');
 start_pid | end_pid | agg_cost
               -3 | 3.2
7 | 3.6
        -1 |
        -1 |
         2 |
                  -3 l
                          2.6
         2 |
                  7 |
                              3
(4 rows)
```

The queries use the Sample Data network.

History

• Proposed in version 2.2

See Also \P

• withPoints - Family of functions

Indices and tables

- Index
- Search Page

pgr_withPointsCostMatrix - proposed¶ Name¶

 $pgr_withPointsCostMatrix$ - Calculates the shortest path and returns only the aggregate cost of the shortest path(s) found, for the combination of points given.

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL

- Name might not change. (But still can)
- Signature might not change. (But still can)
- Functionality might not change. (But still can)
- pgTap tests have being done. But might need more.
- Documentation might need refinement.



Boost Graph Inside

Availability: 2.2.0

Signature Summary¶

```
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vids)
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vids, directed, driving_side)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Note

There is no **details** flag, unlike the other members of the withPoints family of functions.

Signatures¶

Minimal Signature¶

The minimal signature: • Is for a directed graph.

• The driving side is set as ${\bf b}$ both. So arriving/departing to/from the point(s) can be in any direction.

```
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example:

```
3 |
-6 |
                  4.3
-6 I
         6 I
                  1.3
        -6 I
-1 |
                  1.3
         3 |
-1 |
                  5.6
-1 |
         6 I
                  2.6
3 |
         -6 I
                  1.7
        -1 |
3 |
                 1.6
3 I
        6 l
                  1
6 |
        -6 I
                 1.3
        -1 l
6 |
                  2.6
6 |
         3 |
                    3
```

(12 rows)

$Complete\ Signature \P$

Example:

returning a symmetrical cost matrix

- Using the default **side** value on the **points_sql** query
- Using an undirected graph
- Using the default driving_side value

-6		-1		1.3
-6	1	3	1	1.7
-6	1	6	1	1.3
-1	1	-6	1	1.3
-1	1	3	1	1.6
-1	1	6	1	2.6
3	1	-6	1	1.7
3	1	-1	1	1.6
3	1	6	1	1
6	1	-6	1	1.3
6	1	-1	1	2.6
6	1	3	1	1
(40				

(12 rows)

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	c4N Y-NUMERI€IAL	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query \P

points_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	ANY-INTEGE	R(optional) Identifier of the point.

- If column present, it can not be NULL.
- If column not present, a sequential identifier will be given automatically.

Column	Type	Description
edge_ic	l ANY-INTEGE	RIdentifier of the "closest" edge to the point.
fraction	ANY-NUMERI	CWalue in $<0,1>$ that indicates the relative postition from the first end point of the edge.
side	CHAR	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is:
		 In the right, left of the edge or If it doesn't matter with 'b' or NULL. If column not present 'b' is considered.

Where:

ANY-INTEGER:

smallint, int, bigint

ANY-NUMERICAL:

smallint, int, bigint, real, float

Description of the parameters of the signatures \P

ParametefType	Description
edges_sqEXT	Edges SQL query as described above.
points_staxt	Points SQL query as described above.
start_vidRRAY[ANY	-INTEGER identifiers of starting vertices. When negative: is a point's pid.
directed BOOLEAN	(optional). When false the graph is considered as Undirected. Default is true which considers the graph as Directed.
driving_ ShAR	 (optional) Value in ['b', 'r', 'l', NULL] indicating if the driving side is In the right or left or If it doesn't matter with 'b' or NULL. If column not present 'b' is considered.

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vidBIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vert ending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

$\operatorname{Examples} \P$

Example:

Use with tsp

```
SELECT * FROM pgr_TSP(
   SELECT * FROM pgr_withPointsCostMatrix(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
       'SELECT pid, edge_id, fraction from pointsOfInterest',
       array[-1, 3, 6, -6], directed := false);
   $$,
   randomize := false
);
 seq | node | cost | agg_cost
----+----
                       0
   1 |
       -6 | 1.3 |
   2 |
       -1 | 1.6 |
                      1.3
   3 |
        3 | 1 |
                      2.9
       6 | 1.3 |
   4 |
                       3.9
        -6 | 0 |
   5 |
                        5.2
(5 rows)
```

See Also \P

- with Points Family of functions
- Cost Matrix Category
- Traveling Sales Person Family of functions
- $\bullet \ \ {\rm sample data} \ {\rm network}.$

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pgr_withPointsKSP - Proposed¶ Name¶

 ${\tt pgr_withPointsKSP}$ - Find the K shortest paths using Yen's algorithm.

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.



Boost Graph Inside

Availability: 2.2.0

Synopsis

Modifies the graph to include the points defined in the points_sql and using Yen algorithm, finds the K shortest paths.

Signature Summary¶

pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K)
pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K, directed, heap_paths, drive
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)

Signatures

Minimal Usage¶

The minimal usage: • Is for a directed graph.

- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.
- No details are given about distance of other points of the query.
- No heap paths are returned.

pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)

Example:

```
SELECT * FROM pgr_withPointsKSP(
```

'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id', 'SELECT pid, edge_id, fraction, side from pointsOfInterest', -1, -2, 2);

seq	path_id	path_seq		node	1	edge .		cost	1	agg_cost
1	1	 1	+-	 -1	 	1	- 	0.6	 	0
2	1	. 2	i	2	i	4		1	i	0.6
3	1] 3	İ	5	İ	8		1	İ	1.6
4	1	4	1	6	I	9	l	1	I	2.6
5 l	1	J 5	-	9	I	15	l	0.4	1	3.6
6	1	1 6	-	-2	1	-1		0	١	4
7	2	1	-	-1	1	1		0.6	١	0
8	2	1 2	-	2	I	4	l	1	1	0.6
9	2	3		5	1	8		1		1.6
10	2	4		6	1	11		1		2.6
11	2	J 5		11	1	13		1		3.6
12	2	1 6	-	12		15		0.6		4.6
13	2	7	-	-2		-1		0		5.2
(13 row	ıs)									

Complete Signature¶

Finds the K shortest paths depending on the optional parameters setup.

```
pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K,
    directed:=true, heap_paths:=false, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
```

Example:

With details.

2 3 4 5 6 7 8 9 10 11 12 13 14 15	1 1 1 2 2 2 2 2 2 2 2 2 2	2 3 4 5 1 2 3 4 5 6 7 8 9 10	2 -6 5 6 -1 2 -6 5 10 -3 11 12 -2 9 6	4 4 8 -1 1 4 4 10 12 13 15 15 9 -1	0.7 0.3 1 0 0.6 0.7 0.3 1 0.6 0.4 1 0.6 0.4 1 0.6	0.6 1.3 1.6 2.6 0 0.6 1.3 1.6 2.6 3.2 3.6 4.6 5.2 5.6
16 16 (16 rows)	2	10	9 6	-1	0	6.6

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns: $\,$

Column	Type Default	Description			
$\overline{\mathrm{id}}$	ANY-INTEGER	Identifier of the edge.			
source	ANY-INTEGER	Identifier of the first end point vertex edge.			
target	ANY-INTEGER	Identifier of the second end point vert edge.			
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)			
		• When negative: edge (source, ta not exist, therefore it's not part graph.			
reverse_	cant-numerical	Weight of the edge (target, source),			
		• When negative: edge (target, so not exist, therefore it's not part graph.			

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query \P

points_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	ANY-INTE	GER(optional) Identifier of the point.
		 If column present, it can not be NULL. If column not present, a sequential identifier will be given automatically.
$edge_ic$	l ANY-INTE	GERIdentifier of the "closest" edge to the point.
fraction	ANY-NUME	RICWalue in $<0.1>$ that indicates the relative postition from the first end point of the edge.
\mathbf{side}	CHAR	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is:
		 In the right, left of the edge or If it doesn't matter with 'b' or NULL. If column not present 'b' is considered.

Where:

ANY-INTEGER:

smallint, int, bigint

ANY-NUMERICAL:

smallint, int, bigint, real, float

Description of the parameters of the signatures \P

Parameter	Type	
edges_sql	TEXT	
$points_sql$	TEXT	
$\operatorname{start_pid}$	ANY-INTEGER	
end_pid	ANY-INTEGER	

Parameter	Type		
K	INTEGER		
directed	BOOLEAN		
heap_paths	BOOLEAN		
duining aids	CHAD		
$driving_side$	CHAR		
details	BOOLEAN		
Description of the return values¶			
Returns set of (seq, path_id, path_seq, node, edge, cost, agg_cost)			
Column	Type		

Column	Type
$\overline{\mathrm{seq}}$	INTEGER
path_seq	INTEGER
path_id	INTEGER
node	BIGINT
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

$\mathrm{Examples} \P$

Example:

Left side driving topology with details.

SELECT * FROM pgr_withPointsKSP(

'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id', 'SELECT pid, edge_id, fraction, side from pointsOfInterest', -1, -2, 2,

driving_side := 'l', details := true);
sed | path id | path sed | pade | edge | cost | agg cost

seq	path_id	path_seq	ı	node	ı	edge	ı	cost	ı	agg_cost
+-	+		+-		+-	+			+-	
1	1	1		-1		1		0.6		0
2	1	2	-	2		4		0.7	1	0.6
3	1] 3	1	-6		4		0.3	1	1.3
4	1	4	-	5		8		1		1.6
5	1	5	1	6	1	9	١	1	1	2.6
6	1	6	1	9		15	١	1	1	3.6
7	1	7	1	12	١	15	١	0.6	1	4.6
8	1	8	Ι	-2	I	-1	ı	0	1	5.2
9	2	1	Ι	-1	I	1	ı	0.6	1	0
10	2	2	Ì	2	ĺ	4	Ĺ	0.7	Ĺ	0.6
11	2	3	Ì	-6	ĺ	4	Ĺ	0.3	Ĺ	1.3
12	2	4	Ì	5	İ	8	İ	1	İ	1.6
13	2	I 5	i	6	i	11	i	1	i	2.6
14	2	I 6	i	11	i	13	i	1	i	3.6
15 l	2	I 7	i	12	i	15	i	0.6	i	4.6
16	2	I 8	i	-2	i	-1	i	0	i	5.2
(16 rov		. 0	'	2	1	_	'	O	1	0.2

Example:

Right side driving topology with heap paths and details.

SELECT * FROM pgr_withPointsKSP(

'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id', 'SELECT pid, edge_id, fraction, side from pointsOfInterest', -1, -2, 2,

heap_paths := true, driving_side := 'r', details := true);
sed | path id | path sed | pade | cost | agg cost

-		path_seq				•				
1		+ 1		-		1		0.4		0
2	1	. 2	İ	1	İ	1		1	İ	0.4
3	1] 3	1	2		4		0.7		1.4
4	1	1 4	1	-6		4		0.3		2.1
5	1	J 5	1	5		8		1		2.4
6	1	1 6	1	6		9		1		3.4
7	1	7	1	9		15		0.4		4.4
8	1	8	1	-2		-1		0		4.8
9	1 2	1	1	-1		1		0.4		0
10	1 2	2	1	1	Ι	1	1	1		0.4

11	2	3	2	4	0.7	1.4
12	2	4	-6 l	4	0.3	2.1
13	2	5 l	5	8	1	2.4
14	2	6 I	6 l	11	1	3.4
15	2	7	11	13	1	4.4
16	2	8	12	15	1	5.4
17	2	9	9	15	0.4	6.4
18	2	10	-2	-1	0	6.8
19	3	1	-1	1	0.4	0
20	3	2	1	1	1	0.4
21	3	3	2	4	0.7	1.4
22	3	4	-6 l	4	0.3	2.1
23	3	5	5	10	1	2.4
24	3	6 I	10	12	0.6	3.4
25	3	7	-3	12	0.4	4
26	3	8	11	13	1	4.4
27	3	9	12	15	1	5.4
28	3	10	9	15	0.4	6.4
29	3	11	-2	-1	0	6.8
(29 rows)						

The queries use the Sample Data network.

History

• Proposed in version 2.2

See Also \P

• withPoints - Family of functions

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$pgr_withPointsDD - Proposed\P$ Name¶

 ${\tt pgr_withPointsDD}$ - Returns the driving distance from a starting point.

Warning

Proposed functions for next mayor release.

• They are not officially in the current release.

- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.



Boost Graph Inside

Availability: 2.2.0

Synopsis¶

Modify the graph to include points and using Dijkstra algorithm, extracts all the nodes and points that have costs less than or equal to the value distance from the starting point. The edges extracted will conform the corresponding spanning tree.

Signature Summary¶

pgr_withPointsDD(edges_sql, points_sql, start_vid, distance)
pgr_withPointsDD(edges_sql, points_sql, start_vid, distance, directed, driving_side, detai
pgr_withPointsDD(edges_sql, points_sql, start_vids, distance, directed, driving_side, deta
RETURNS SET OF (seq, node, edge, cost, agg_cost)

Signatures¶

Minimal Use¶

The minimal signature: • Is for a directed graph.

- The driving side is set as ${\bf b}$ both. So arriving/departing to/from the point(s) can be in any direction.
- No **details** are given about distance of other points of the query.

pgr_withPointsDD(edges_sql, points_sql, start_vid, distance)
 directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, node, edge, cost, agg_cost)

Example:

```
SELECT * FROM pgr_withPointsDD(
  'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
   'SELECT pid, edge_id, fraction, side from pointsOfInterest',
   -1, 3.8);
seq | node | edge | cost | agg_cost
 ---+----
            -1 l
                   0 |
  1 |
       -1 |
            1 | 0.4 |
  2 |
        1 |
                            0.4
  3 |
        2 |
              1 | 0.6 |
                             0.6
  4 I
        5 I
              4 |
                     1 |
                              1.6
  5 I
        6 |
              8 |
                     1 |
                              2.6
              7 |
  6 |
        8 |
                     1 |
                              2.6
  7 |
        10 |
              10 |
                     1 |
                              2.6
                              3.6
  8 |
       7 |
             6 |
                     1 |
  9 |
        9 |
              9 |
                     1 |
                              3.6
 10 |
       11 |
             11 |
                     1 |
                              3.6
                              3.6
 11 |
      13 | 14 |
                     1 |
(11 rows)
```

Driving distance from a single point¶

Finds the driving distance depending on the optional parameters setup.

Example:

Right side driving topology

```
SELECT * FROM pgr_withPointsDD(
  'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3.8,
   driving_side := 'r',
   details := true);
 seq | node | edge | cost | agg_cost
----+----
      -1 | -1 | 0 |
  2 |
        1 |
             1 | 0.4 |
                            0.4
  3 |
         2 |
               1 |
                    1 |
                             1.4
        -6 I
               4 | 0.7 |
  4 |
                              2.1
  5 I
         5 I
               4 | 0.3 |
                             2.4
               8 | 1 |
                             3.4
  6 |
         6 l
  7 |
         8 | 7 |
                     1 |
                              3.4
```

```
8 | 10 | 10 | 1 | 3.4
(8 rows)
```

Driving distance from many starting points¶

Finds the driving distance depending on the optional parameters setup.

```
pgr_withPointsDD(edges_sql, points_sql, start_vids, distance,
    directed:=true, driving_side:='b', details:=false, equicost:=false)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Description of the Signatures¶

Description of the edges_sql query for dijkstra like functions¶ edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
$reverse_$	cant-numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

 $SMALLINT,\,INTEGER,\,BIGINT,\,REAL,\,FLOAT$

Description of the Points SQL query \P

 $points_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	ANY-IN	TEGER(optional) Identifier of the point.
		 If column present, it can not be NULL. If column not present, a sequential identifier will be given automatically.
$edge_ic$	l ANY-IN	TEGERIdentifier of the "closest" edge to the point.
fraction	ANY-NU	MERICWalue in $<0,1>$ that indicates the relative postition from the first end point of the edge.
side	CHAR	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is:
		 In the right, left of the edge or If it doesn't matter with 'b' or NULL. If column not present 'b' is considered.

Where:

ANY-INTEGER:

smallint, int, bigint

ANY-NUMERICAL:

smallint, int, bigint, real, float

Description of the parameters of the signatures \P

Parameter	Туре
edges_sql	TEXT
points_sql	TEXT
$start_vid$	ANY-INTEGER
distance	ANY-NUMERICAL
directed	BOOLEAN

Parameter	Type
driving_side	CHAR
details	BOOLEAN
equicost	BOOLEAN

Description of the return values¶

Returns set of (seq, node, edge, cost, agg_cost)

Column	Туре
\overline{seq}	INT
node	BIGINT
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

Examples for queries marked as $\mathtt{directed}$ with \mathtt{cost} and $\mathtt{reverse_cost}$ $\mathtt{columns}\P$

The examples in this section use the following Network for queries marked as directed and cost and reverse_cost columns are used

Example:

Left side driving topology

```
SELECT * FROM pgr_withPointsDD(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
   'SELECT pid, edge_id, fraction, side from pointsOfInterest',
   -1, 3.8,
```

```
driving_side := 'l',
    details := true);
 seq | node | edge | cost | agg_cost
----+----
   1 |
         -1 |
                -1 |
                        0 |
   2 |
          2 |
                 1 | 0.6 |
                                 0.6
   3 |
         -6 I
                 4 | 0.7 |
                                 1.3
          5 I
                 4 | 0.3 |
   4 |
                                 1.6
   5 I
          1 |
                 1 |
                        1 |
                                 1.6
   6 I
          6 I
                 8 |
                        1 |
                                 2.6
   7 |
          8 |
                7 |
                        1 |
                                 2.6
   8 |
         10 |
                10 |
                                 2.6
                        1 |
   9 |
         -3 |
                12 |
                     0.6
                                 3.2
  10 I
         -4 |
                6 |
                     0.7 |
                                 3.3
  11 |
          7 |
                 6 |
                      0.3 |
                                 3.6
  12 |
          9 |
                 9 |
                        1 |
                                 3.6
  13 |
                11 |
                                 3.6
         11 |
                        1 |
         13 |
  14 |
                14 |
                                 3.6
                        1 |
(14 rows)
```

Example:

Does not matter driving side.

```
SELECT * FROM pgr_withPointsDD(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3.8,
    driving_side := 'b',
    details := true);
 seq | node | edge | cost | agg_cost
----+----
                -1 |
   1 |
         -1 |
                        0 |
                                   0
   2 |
                 1 | 0.4 |
                                 0.4
          1 |
   3 |
          2 |
                 1 | 0.6 |
                                 0.6
   4 |
         -6 |
                 4 |
                     0.7
                                 1.3
   5 I
          5 I
                 4 | 0.3 |
                                 1.6
   6 |
          6 |
                 8 |
                        1 |
                                 2.6
   7 |
          8 |
                        1 |
                 7 |
                                 2.6
   8 |
         10 |
                10 |
                        1 |
                                 2.6
   9 |
         -3 I
                12 |
                     0.6 |
                                 3.2
  10 |
         -4 |
                6 I
                      0.7 |
                                 3.3
          7 |
  11 |
                 6 I
                      0.3 |
                                 3.6
  12 |
          9 |
                 9 I
                                 3.6
                        1 |
         11 |
  13 l
                11 |
                        1 |
                                 3.6
  14 |
         13 |
                14 |
                        1 |
                                 3.6
```

(14 rows)

The queries use the Sample Data network.

History

• Proposed in version 2.2

See Also¶

- pgr_drivingDistance Driving distance using dijkstra.
- pgr_alphaShape Alpha shape computation.
- $pgr_pointsAsPolygon$ Polygon around set of points.

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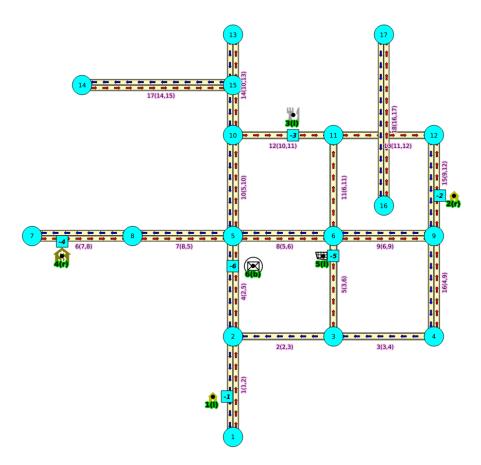
Warning

Proposed functions for next mayor release.

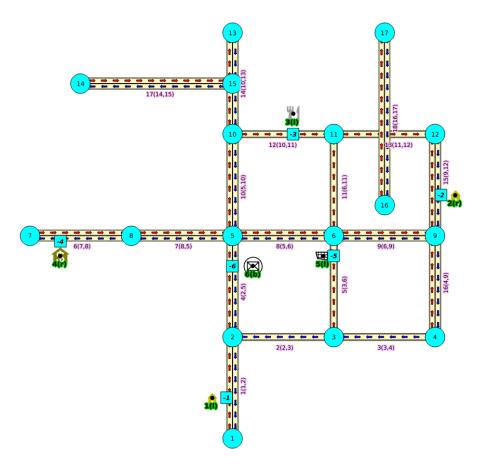
- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.

Images¶ The squared vertices are the temporary vertices, The temporary vertices are added according to the driving side, The following images visually show the differences on how depending on the driving side the data is interpreted.

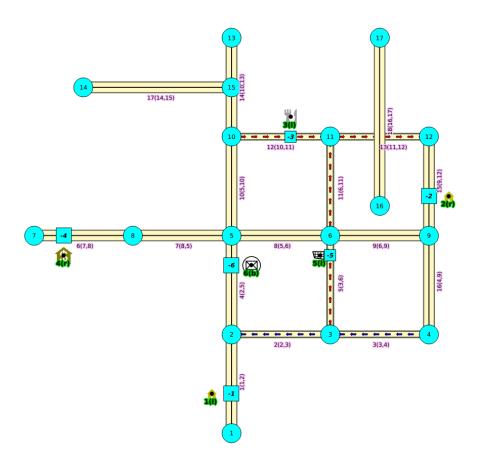
Right driving side



Left driving side



doesn't matter the driving side



Introduction¶ This family of functions was thought for routing vehicles, but might as well work for some other application that we can not think of.

The with points family of function give you the ability to route between arbitrary points located outside the original graph.

When given a point identified with a pid that its being mapped to and edge with an identifier edge_id, with a fraction along that edge (from the source to the target of the edge) and some additional information about which side of the edge the point is on, then routing from arbitrary points more accurately reflect routing vehicles in road networks,

I talk about a family of functions because it includes different functionalities.

pgr_withPoints is pgr_dijkstra based

- \bullet pgr_withPointsCost is pgr_dijkstraCost based
- pgr_withPointsKSP is pgr_ksp based
- pgr_withPointsDD is pgr_drivingDistance based

In all this functions we have to take care of as many aspects as possible:

- Must work for routing:
 - Cars (directed graph)
 - Pedestrians (undirected graph)
- Arriving at the point:
 - In either side of the street.
 - Compulsory arrival on the side of the street where the point is located.
- Countries with:
 - Right side driving
 - Left side driving
- Some points are:
 - Permanent, for example the set of points of clients stored in a table in the data base
 - Temporal, for example points given through a web application
- The numbering of the points are handled with negative sign.
 - Original point identifiers are to be positive.
 - Transformation to negative is done internally.
 - For results for involving vertices identifiers
 - * positive sign is a vertex of the original graph
 - * negative sign is a point of the temporary points

The reason for doing this is to avoid confusion when there is a vertex with the same number as identifier as the points identifier.

Graph & edges¶

- Let $\(G_d(V,E)\)$ where $\(V\)$ is the set of vertices and $\(E\)$ is the set of edges be the original directed graph.
 - An edge of the original edges_sql is \((id, source, target, cost, reverse_cost)\) will generate internally
 - * \((id, source, target, cost)\)
 - * \((id, target, source, reverse_cost)\)

Point Definition¶

- A point is defined by the quadruplet: \((pid, eid, fraction, side)\)
 - **pid** is the point identifier
 - **eid** is an edge id of the edges_sql
 - fraction represents where the edge eid will be cut.
 - **side** Indicates the side of the edge where the point is located.

Creating Temporary Vertices in the Graph¶ For edge (15, 9,12 10, 20), & lets insert point (2, 12, 0.3, r)

On a right hand side driving network

From first image above:

- We can arrive to the point only via vertex 9.
- It only affects the edge (15, 9,12, 10) so that edge is removed.
- Edge (15, 12,9, 20) is kept.
- Create new edges:
 - -(15, 9, -1, 3) edge from vertex 9 to point 1 has cost 3
 - -(15, -1, 12, 7) edge from point 1 to vertex 12 has cost 7

On a left hand side driving network

From second image above:

- We can arrive to the point only via vertex 12.
- It only affects the edge (15, 12,9 20) so that edge is removed.
- Edge (15, 9,12, 10) is kept.
- Create new edges:
 - (15, 12,-1, 14) edge from vertex 12 to point 1 has cost 14
 - (15, -1,9, 6) edge from point 1 to vertex 9 has cost 6

Remember:

that fraction is from vertex 9 to vertex 12

When driving side does not matter

From third image above:

- We can arrive to the point either via vertex 12 or via vertex 9
- Edge (15, 12,9 20) is removed.
- Edge (15, 9,12, 10) is removed.

- Create new edges:
 - (15, 12,-1, 14) edge from vertex 12 to point 1 has cost 14
 - -(15, -1.9, 6) edge from point 1 to vertex 9 has cost 6
 - -(15, 9, -1, 3) edge from vertex 9 to point 1 has cost 3
 - -(15, -1, 12, 7) edge from point 1 to vertex 12 has cost 7

See Also¶ Indices and tables

- Index
- Search Page

Cost - Category¶

- $pgr_aStarCost proposed$
- pgr_bdAstarCost Proposed
- pgr_bdDijkstraCost Proposed
- pgr_dijkstraCost
- pgr_withPointsCost Proposed

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.

General Information Characteristics

The main Characteristics are:

- Each function works as part of the family it belongs to.
- It does not return a path.
- Returns the sum of the costs of the resulting path(s) for pair combination of nodes in the graph.
- Process is done only on edges with positive costs.

- Values are returned when there is a path.
 - The returned values are in the form of a set of (start_vid, end_vid, agg_cost).
 - When the starting vertex and ending vertex are the same, there is no path.
 - * The agg_cost int the non included values (v, v) is 0.
 - When the starting vertex and ending vertex are the different and there is no path.
 - * The agg_cost in the non included values (u, v) is $\setminus (\setminus infty \setminus)$.
- Let be the case the values returned are stored in a table, so the unique index would be the pair: (start vid, end vid).
- Depending on the function and its parameters, the results can be symmetric.
 - The agg_cost of (u, v) is the same as for (v, u).
- Any duplicated value in the start_vids or in end_vids are ignored.
- The returned values are ordered:
 - start_vid ascending
 - end_vid ascending

See Also¶

Indices and tables

- Index
- Search Page

Cost Matrix - Category \P

- pgr_aStarCostMatrix proposed
- pgr_bdAstarCostMatrix proposed
- pgr bdDijkstraCostMatrix proposed
- pgr_dijkstraCostMatrix proposed
- pgr_withPointsCostMatrix proposed

Warning

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- Signature might not change. (But still can)
- Functionality might not change. (But still can)
- pgTap tests have being done. But might need more.
- Documentation might need refinement.

pgr_aStarCostMatrix - proposed¶ Name¶

pgr_aStarCostMatrix - Calculates the a cost matrix using pgr_aStar.

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.



Boost Graph Inside

Availability: 2.4.0

Synopsis

Using aStar algorithm, calculate and return a cost matrix.

Signature Summary \P

pgr_aStarCostMatrix(edges_sql, vids)
pgr_aStarCostMatrix(edges_sql, vids, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost)

Signatures¶

Minimal Signature¶

The minimal signature: • Is for a directed graph.

```
pgr_aStarCostMatrix(edges_sql, vids)
RETURNS SET OF (start_vid, end_vid, agg_cost)
Example:
Cost matrix for vertices 1, 2, 3, and 4.
SELECT * FROM pgr_aStarCostMatrix(
   'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
             1 |
         2 |
         3 |
                  1 |
                             2
        4 |
                  1 |
                             3
         1 |
                  2 |
                  2 |
         3 |
                  2 |
         4 |
                             2
         1 |
                  3 |
                              6
         2 |
                  3 l
         4 |
                  3 l
                             1
                  4 |
                             5
         1 |
         2 |
                 4 |
                   4 |
         3 |
(12 rows)
Complete Signature¶
pgr_aStarCostMatrix(edges_sql, vids, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost)
Example:
Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.
This example returns a symmetric cost matrix.
SELECT * FROM pgr_aStarCostMatrix(
   'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    directed := false, heuristic := 2
);
start_vid | end_vid | agg_cost
```

2 | 1 | 1

3	I	1	1	2
4	l	1	1	3
1	l	2	1	1
3	l	2	1	1
4	l	2	1	2
1	l	3	1	2
2	l	3	1	1
4	l	3	1	1
1	l	4	1	3
2	l	4	1	2
3		4	1	1

(12 rows)

Description of the Signatures \P

Description of the edges_sql query for a star like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
reverse_	_c any -numerical	 When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.
x1	ANY-NUMERICAL	X coordinate of source vertex.
y1	ANY-NUMERICAL	Y coordinate of source vertex.
x2	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter	Type	Description
$\overline{\mathrm{edges_sql}}$	TEXT	Edges SQL query as described above.
\mathbf{vids}	ARRAY[ANY-IN]	EXERLY of vertices_identifiers.
directed	BOOLEAN	• Optional.
		 When false the graph is considered as Undirected. Default is true which considers the graph as Directed.
heuristic	INTEGER	(optional). Heuristic number. Current valid values $0{\sim}5$. Default 5
		 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) 1: h(v) abs(max(dx, dy)) 2: h(v) abs(min(dx, dy)) 3: h(v) = dx * dx + dy * dy 4: h(v) = sqrt(dx * dx + dy * dy) 5: h(v) = abs(dx) + abs(dy)
factor	FLOAT	(optional). For units manipulation. \((factor > 0 \)). Default 1.
epsilon	FLOAT	(optional). For less restricted results. \(epsilon >= 1\). Default 1.

Description of the return values for a Cost function ¶

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vid BIGINT	Identifier of the starting ve
	starting vetrices are in the

Column Type	Description
end_vid BIGINT	Identifier of the ending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

```
{\bf Examples}\P
```

Example:

Use with tsp

```
SELECT * FROM pgr_TSP(
    SELECT * FROM pgr_aStarCostMatrix(
     'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
     (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        directed:= false, heuristic := 2
    )
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
----+----
   1 |
         1 |
                1 |
   2 |
         2 |
                1 |
                          1
                1 |
   3 |
         3 |
                          2
         4 |
   4 |
                3 |
                           3
   5 |
         1 |
                0 |
(5 rows)
```

See Also \P

- ullet a Star Family of functions
- Cost Matrix Category
- Traveling Sales Person Family of functions
- The queries use the Sample Data network.

Indices and tables

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pgr_bdAstarCostMatrix - proposed¶ Name¶

pgr_bdAstarCostMatrix - Calculates the a cost matrix using pgr_bdAstar.



Boost Graph Inside

Availability: 2.5.0

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary¶

```
pgr_bdAstarCostMatrix(edges_sql, start_vids)
pgr_bdAstarCostMatrix(edges_sql, start_vids, [, directed , heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

Signatures

Minimal Signature¶

```
pgr_bdAstarCostMatrix(edges_sql, start_vids)
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

This usage calculates the cost from the each start_vid in start_vids to each start_vid in st on a directed graph

- with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example:

Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_bdAstarCostMatrix(
   'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);</pre>
```

start_vid		end_vid	I	agg_cost
	+-		+-	
1		2	I	1
1		3		12
1	1	4		12
2	1	1		1
2		3		10
2	1	4		9
3	1	1		2
3	1	2	1	1
3	1	4		5
4	1	1		5
4	1	2		2
4	1	3	1	1
(12 rows)				

Complete Signature¶

```
pgr_bdAstarCostMatrix(edges_sql, start_vids, [, directed , heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

This usage calculates the cost from the each start_vid in start_vids to each start_vid in st if the graph is directed or undirected

- heuristic,
- and/or factor

• and/or epsilon.

Example:

Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

4 |

1 |

2 |

4 |

1 |

2 |

3 |

```
SELECT * FROM pgr_bdAstarCostMatrix(
   'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
);
 start_vid | end_vid | agg_cost
         1 |
                   2 |
                               1
                   3 |
                               2
         1 |
                               5
         1 |
                   4 |
         2 |
                   1 |
         2 |
                   3 |
                               1
```

(12 rows)

Description of the Signatures¶

2 |

3 |

3 |

3 |

4 | 4 |

4 |

Description of the edges_sql query for a star like functions¶

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

2

1

1

5

2

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point verter edge.
target	ANY-INTEGER	Identifier of the second end point veredge.
cost	ANY-NUMERICAL	Weight of the edge (source, target)

• When negative: edge (source, to not exist, therefore it's not part graph.

Column	Type Default	Description
reverse_	cant-numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.
x1	ANY-NUMERICAL	X coordinate of source vertex.
y1	ANY-NUMERICAL	Y coordinate of source vertex.
x2	ANY-NUMERICAL	X coordinate of target vertex.
y2	ANY-NUMERICAL	Y coordinate of target vertex.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter Type	Description
edges_sqlTEXT	Edges SQL query as described above.
${f start_vid}$ Any-integer	Starting vertex identifier.
${ m start_vids}$ ARRAY[ANY-IN	TEGERIng vertices identifierers.
${f end_vid}$ ANY-INTEGER	Ending vertex identifier.
${f end_vids}$ ARRAY[ANY-IN	TEGERIE vertices identifiers.
directed BOOLEAN	• Optional.

- When ${\tt false}$ the graph is considered as Undirected.
- Default is true which considers the graph as Directed.

Parameter	Type	Description
heuristic INTEGER		(optional). Heuristic number. Current valid values 0~5. Default 5
		 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) 1: h(v) abs(max(dx, dy)) 2: h(v) abs(min(dx, dy)) 3: h(v) = dx * dx + dy * dy 4: h(v) = sqrt(dx * dx + dy * dy) 5: h(v) = abs(dx) + abs(dy)
factor	FLOAT	(optional). For units manipulation. \setminus (factor $> 0 \setminus$). Default 1. see <i>Factor</i>
epsilon	FLOAT	(optional). For less restricted results. \(epsilon >= 1 \). Default 1.

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vidBIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vert ending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

```
{\bf Examples}\P
```

Example:

Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_bdAstarCostMatrix(
        'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    )
    $$,</pre>
```

randomize := false); seq | node | cost | agg_cost 1 | 1 | 2 | 2 | 3 I 1 | 2 3 | 4 | 2 | 3 2 | 4 | 5 1 | 5 I 1 | 0 | (5 rows)

See Also¶

- Bidirectional A^* Family of functions
- Cost Matrix Category
- Traveling Sales Person Family of functions
- The queries use the Sample Data network.

Indices and tables

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pgr_bdDijkstraCostMatrix - proposed¶ Name¶

 $\verb|pgr_bdDijkstraCostMatrix-Calculates| the a cost matrix using \textit|pgr_bdDijkstra|.$



Boost Graph Inside

Availability: 2.5.0

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.

- Signature might change.
- Functionality might change.
- pgTap tests might be missing.
- Might need c/c++ coding.
- May lack documentation.
- Documentation if any might need to be rewritten.
- Documentation examples might need to be automatically generated.
- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

Synopsis¶

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary¶

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vids)
pgr_bdDijkstraCostMatrix(edges_sql, start_vids, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

$Signatures \P$

Minimal Signature¶

The minimal signature: • Is for a directed graph.

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example:

Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_bdDijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);</pre>
```

agg_cost	 	end_vid	 	start_vid 	
1	 	2	 	1	
6	1	3	1	1	
5	1	4		1	
1	1	1	1	2	
5		3	1	2	

```
2 |
          4 |
3 |
          1 |
          2 |
3 |
                      1
          4 |
3 |
                      3
4 |
          1 |
                      3
4 |
          2 |
                      2
4 |
          3 |
```

(12 rows)

Complete Signature¶

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vids, directed:=true)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example:

Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_bdDijkstraCostMatrix(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
);
```

start_vid	end_vid	agg_cost
1	2	1
1	3	1 2
1	4	1 3
2	1	1
2	3	1
2	4	1 2
3	1	1 2
3	2	1
3	4	1
4	1] 3
4	2	1 2
4	3	1

(12 rows)

Description of the Signatures¶

Description of the edges_sql query for dijkstra like functions \P edges sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGI	ER	Identifier of the edge.
source	ANY-INTEGI	ER	Identifier of the first end point vertex edge.
target	ANY-INTEGI	ER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMER:	ICAL	Weight of the edge (source, target)
			• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	cant-numer:	ICAL	Weight of the edge (target, source),
			• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

ParametefType	Description	
edges_sqEXT	Edges SQL query as described above.	
start_vidBRAY[ANY-INTEGER] identifiers of the vertices.		
directed BOOLEAN	(optional). When false the graph is considered as Undirected. Default is true which considers the graph as Directed.	

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vid BIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vert ending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

$\operatorname{Examples} \P$

Example:

Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_bdDijkstraCostMatrix(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
     (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
                 1 |
   1 |
          1 |
                            0
   2 |
          2 |
                 1 |
                            1
   3 I
          3 I
                 1 |
                            2
                            3
   4 |
          4 |
                 3 |
                            6
   5 |
        1 |
                 0 |
(5 rows)
```

See Also¶

- Bidirectional Dijkstra Family of functions
- Cost Matrix Category
- Traveling Sales Person Family of functions
- The queries use the $Sample\ Data$ network.

Indices and tables

- Index
- Search Page

$pgr_dijkstraCostMatrix - proposed\P$ Name¶

pgr_dijkstraCostMatrix - Calculates the a cost matrix using pgr_dijktras.

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.



Boost Graph Inside

Availability: 2.3.0

Synopsis

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary¶

pgr_dijkstraCostMatrix(edges_sql, start_vids)
pgr_dijkstraCostMatrix(edges_sql, start_vids, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost)

Signatures¶

Minimal Signature¶

The minimal signature: • Is for a directed graph.

pgr_dijkstraCostMatrix(edges_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)

Example:

Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_dijkstraCostMatrix(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
            2 | 1
        1 |
        1 |
                3 |
                          6
        1 |
                4 |
        2 |
                1 |
                           1
        2 I
                3 |
        2 |
                 4 |
        3 |
                 1 |
                2 |
                          1
        3 |
        3 |
                4 |
                          3
        4 |
                1 |
                          3
        4 |
                2 |
                3 |
        4 |
(12 rows)
Complete\ Signature \P
pgr_dijkstraCostMatrix(edges_sql, start_vids, directed:=true)
RETURNS SET OF (start_vid, end_vid, agg_cost)
Example:
Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.
This example returns a symmetric cost matrix.
SELECT * FROM pgr_dijkstraCostMatrix(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
   false
);
 start_vid | end_vid | agg_cost
-----
            2 | 1
        1 |
        1 |
                3 I
        1 |
                4 |
        2 |
                 1 |
                           1
        2 |
                 3 |
        2 |
                 4 |
                           2
        3 |
                 1 |
                           2
```

1

3 |

2 |

(12 rows)

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	_c 4NY -NUMERICAL	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\, INTEGER,\, BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

ParameterType	Description
edges_sqlEXT	Edges SQL query as described above.

ParametefType	Description			
start_videRAY[ANY-IATEGER] identifiers of the vertices.				
directed BOOLEAN	(optional). When false the graph is considered as Undirected. Default is true which considers the graph as Directed.			

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vidBIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vertees are in the q
agg_cost FLOAT	Aggregate cost from start

```
Example:
Use with tsp

SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_dijkstraCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    )
    $$,
    randomize := false
);
    seq | node | cost | agg_cost</pre>
```

```
1 |
         1 |
                1 |
         2 |
  2 |
                1 |
                           1
  3 |
         3 |
                1 |
                           2
  4 |
         4 |
                3 |
                          3
  5 I
          1 |
                0 |
(5 rows)
```

 ${\bf Examples}\P$

See Also¶

- Dijkstra Family of functions
- Cost Matrix Category
- Traveling Sales Person Family of functions
- The queries use the *Sample Data* network.

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pgr_withPointsCostMatrix - proposed¶ Name¶

pgr_withPointsCostMatrix - Calculates the shortest path and returns only the aggregate cost of the shortest path(s) found, for the combination of points given.

Warning

Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
 - The functions make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might not change. (But still can)
 - Signature might not change. (But still can)
 - Functionality might not change. (But still can)
 - pgTap tests have being done. But might need more.
 - Documentation might need refinement.



Boost Graph Inside

Availability: 2.2.0

Signature Summary¶

pgr_withPointsCostMatrix(edges_sql, points_sql, start_vids)
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vids, directed, driving_side)
RETURNS SET OF (start_vid, end_vid, agg_cost)

Note

There is no **details** flag, unlike the other members of the withPoints family of functions.

Signatures¶

Minimal Signature¶

The minimal signature: • Is for a directed graph.

• The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.

```
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example:

```
SELECT * FROM pgr_withPointsCostMatrix(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
   'SELECT pid, edge_id, fraction from pointsOfInterest',
   array[-1, 3, 6, -6]);
start_vid | end_vid | agg_cost
```

	+		+	
-6	1	-1	1	1.3
-6	1	3	1	4.3
-6	1	6	1	1.3
-1	1	-6	1	1.3
-1	1	3	1	5.6
-1	1	6	1	2.6
3	1	-6	1	1.7
3	1	-1	1	1.6
3	1	6	1	1
6	1	-6	1	1.3
6	1	-1	1	2.6
6	1	3	1	3
(12 rows)				

(12 rows)

Complete Signature¶

Example:

returning a symmetrical cost matrix

- Using the default ${\bf side}$ value on the ${\bf points_sql}$ query
- Using an undirected graph
- Using the default ${\bf driving_side}$ value

```
SELECT * FROM pgr_withPointsCostMatrix(
```

'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
'SELECT pid, edge_id, fraction from pointsOfInterest',
array[-1, 3, 6, -6], directed := false);

start_vid	1	end_vid	I	agg_cost
	+-	+		
-6	1	-1	I	1.3
-6		3	I	1.7
-6		6		1.3
-1	1	-6		1.3
-1	1	3		1.6
-1	1	6		2.6
3	1	-6	I	1.7
3	1	-1	I	1.6
3	1	6	I	1
6	1	-6	I	1.3
6	1	-1	I	2.6
6	1	3	I	1
(10				

(12 rows)

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point verteredge.
target	ANY-INTEGER	Identifier of the second end point veredge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)

• When negative: edge (source, to not exist, therefore it's not part graph.

Column	Type	Default	Description
reverse_	_c ant -numer	ICIAL	Weight of the edge (target, source),
			• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query¶

points_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Description	
pid	ANY-INTEGER (optional) Identifier of the point.		
		. If column present it can not be NIII I	

- If column present, it can not be NULL.
- If column not present, a sequential identifier will be given automatically.

edge_id ANY-INTEGERIdentifier of the "closest" edge to the point.

 $\begin{array}{c} \textbf{fraction ANY-NUMERICWA} lue \ in < 0.1 > \ that \ indicates \ the \ relative \ postition \\ from \ the \ first \ end \ point \ of \ the \ edge. \end{array}$

side CHAR

(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is:

- In the right, left of the edge or
- If it doesn't matter with 'b' or NULL.
- If column not present 'b' is considered.

Where:

ANY-INTEGER:

smallint, int, bigint

ANY-NUMERICAL:

smallint, int, bigint, real, float

Description of the parameters of the signatures \P

Parameteffype	Description
edges_sqEXT	Edges SQL query as described above.
points_staxt	Points SQL query as described above.
start_vidRRAY[ANY-	-IATEGER identifiers of starting vertices. When negative: is a point's pid.
directed BOOLEAN	(optional). When false the graph is considered as Undirected. Default is true which considers the graph as Directed.
driving_ ShAR	 (optional) Value in ['b', 'r', 'l', NULL] indicating if the driving side is: In the right or left or If it doesn't matter with 'b' or NULL. If column not present 'b' is considered.

Description of the return values for a Cost function \P

Returns set of (start_vid, end_vid, agg_cost)

Column Type	Description
start_vidBIGINT	Identifier of the starting ve starting vetrices are in the
end_vid BIGINT	Identifier of the ending vertending vertices are in the q
agg_cost FLOAT	Aggregate cost from start

```
\operatorname{Examples} \P
```

Example:

Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_withPointsCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
        'SELECT pid, edge_id, fraction from pointsOfInterest',
        array[-1, 3, 6, -6], directed := false);
    $$,
```

```
randomize := false
);
 seq | node | cost | agg_cost
         -6 I
              1.3 |
         -1 l
              1.6
                          1.3
   3 |
         3 |
              1 |
                          2.9
   4 |
         6 | 1.3 |
                          3.9
   5 I
         -6 I
                0 |
(5 rows)
```

See Also¶

- withPoints Family of functions
- Cost Matrix Category
- Traveling Sales Person Family of functions
- sampledata network.

Indices and tables

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General Information¶ Synopsis¶

Traveling Sales Person - Family of functions needs as input a symmetric cost matrix and no edge (u, v) must value $\setminus (\inf y)$.

This collection of functions will return a cost matrix in form of a table.

Characteristics¶

The main Characteristics are:

- Can be used as input to pgr_TSP.
 - directly: when the resulting matrix is symmetric and there is no $\(\inf y \)$ value.
 - It will be the users responsibility to make the matrix symmetric.
 - * By using geometric or harmonic average of the non symmetric values.
 - * By using max or min the non symmetric values.
 - * By setting the upper triangle to be the mirror image of the lower triangle.

- * By setting the lower triangle to be the mirror image of the upper triangle.
- It is also the users responsibility to fix an $\langle (\inf y) \rangle$ value.
- Each function works as part of the family it belongs to.
- It does not return a path.
- Returns the sum of the costs of the shortest path for pair combination of nodes in the graph.
- Process is done only on edges with positive costs.
- Values are returned when there is a path.
 - The returned values are in the form of a set of (start_vid, end_vid, agg_cost).
 - When the starting vertex and ending vertex are the same, there is no path.
 - * The agg_cost int the non included values (v, v) is 0.
 - When the starting vertex and ending vertex are the different and there is no path.
 - * The agg_cost in the non included values (u, v) is $\setminus (\inf y)$.
- Let be the case the values returned are stored in a table, so the unique index would be the pair: (start vid, end vid).
- Depending on the function and its parameters, the results can be symmetric.
 - The agg_cost of (u, v) is the same as for (v, u).
- Any duplicated value in the start_vids are ignored.
- The returned values are ordered:
 - start_vid ascending
 - end_vid ascending
- Running time: approximately $\langle O(| \text{start}_\text{vids} | * (V \setminus \text{log } V + E)) \rangle$

See Also¶

• *pgr_TSP*

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KSP Category¶

- pgr_KSP Driving Distance based on pgr_dijkstra
- pgr_withPointsKSP Proposed Driving Distance based on pgr_dijkstra

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Experimental Functions¶

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

 ${\it Contraction}$ - ${\it Family~of~functions}$ - Reduce network size using contraction techniques

Graph Analysis

- $pgr_labelGraph$ - Experimental - Analyze / label subgraphs within a network

Components - Family of functions - Analyze components within a graph

- pgr_connectedComponents Experimental Return the connected components of an undirected graph
- pgr_strongComponents Experimental Return the strongly connected components of a directed graph
- pgr_biconnectedComponents Experimental Return the biconnected components of an undirected graph
- pgr_articulationPoints Experimental Return the articulation points of an undirected graph
- $\bullet \;\; pgr_bridges$ Experimental Return the bridges of an undirected graph

VRP

- $\bullet \ pgr_gsoc_vrppdtw Experimental$
- pgr_vrpOneDepot Experimental

Contraction - Family of functions¶ Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

 $pgr_contractGraph$ - Experimental

pgr_contractGraph - Experimental¶ pgr_contractGraph — Performs graph contraction and returns the contracted vertices and edges.



Boost Graph Inside

Availability: 2.3.0

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis¶

Contraction reduces the size of the graph by removing some of the vertices and edges and, for example, might add edges that represent a sequence of original edges decreasing the total time and space used in graph algorithms.

Characteristics

The main Characteristics are: • Process is done only on edges with positive costs.

- There are two types of contraction methods used namely,
 - Dead End Contraction
 - Linear Contraction
- The values returned include the added edges and contracted vertices.

- The returned values are ordered as follows:
 - column id ascending when type = v
 - column id descending when type = e

Signature Summary:¶

The pgr contractGraph function has the following signatures:

```
pgr_contractGraph(edges_sql, contraction_order)
pgr_contractGraph(edges_sql, contraction_order, max_cycles, forbidden_vertices, directed)
```

RETURNS SETOF (seq, type, id, contracted_vertices, source, target, cost)

Signatures¶

Minimal signature¶

pgr_contractGraph(edges_sql, contraction_order)

Example:

Making a dead end contraction and a linear contraction.

```
SELECT * FROM pgr_contractGraph(
  'SELECT id, source, target, cost, reverse_cost FROM edge_table',
  ARRAY[1, 2]);
seq | type | id | contracted_vertices | source | target | cost
-1 |
                                   -1 | -1
                            -1 |
                                  -1 |
                            -1 |
                                  -1 |
                                        -1
                            3 |
9 |
                                   5 |
                                   3 |
                                   11 |
                                   9 |
(7 rows)
```

Complete signature

pgr_contractGraph(edges_sql, contraction_order, max_cycles, forbidden_vertices, directed)

Example:

Making a dead end contraction and a linear contraction and vertex 2 is forbidden from contraction

SELECT * FROM pgr_contractGraph(

'SELECT id, source, target, cost, reverse_cost FROM edge_table', ARRAY[1, 2], forbidden_vertices:=ARRAY[2]);

	e id contracted_vert		•
1 v 2 v 3 v 4 v 5 e 6 e	2 {1} 5 {7,8} 15 {14} 17 {16} -1 {4} -2 {10,13} -3 {12}	-1 -1 -1 -1 -1 9 5	
(7 rows)	1 0 1 (12)	1 11	0 1 2

Description of the edges_sql query for dijkstra like functions \P

edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vertedge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
$reverse_$	c ant -numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

 $SMALLINT,\,INTEGER,\,BIGINT,\,REAL,\,FLOAT$

Description of the parameters of the signatures \P

Column	Type	Description
$\overline{\mathrm{edges_sql}}$	TEXT	SQL query as described above.
contraction ABRAYEFANY-IORREPED contraction operations. • 1 = Dead end contraction		
forbidden_	-Yn æ∌vatrffa v_	• 2 = Linear contraction • 107HGHRII). Identifiers of vertices forbidden from contraction. Default is an empty array.
max_cycle	s INTEGER	(optional). Number of times the contraction operations on contraction_order will be performed. Default is 1.
directed	BOOLEAN	 When true the graph is considered as Directed. When false the graph is considered as Undirected.

Description of the return values \P

RETURNS SETOF (seq, type, id, contracted_vertices, source, target, cost)

The function returns a single row. The columns of the row are:

Column	Type	Description
seq	INTEGER	Sequential value starting from 1.
type	TEXT	Type of the id. • 'v' when id is an identifier of a vertex.• 'e' when id is an identifier of an edge.
id	BIGINT	 Identifier of: • the vertex when type = 'v'. The vertex belongs to the edge_table passed as a parameter.
		 the edge when type = 'e'. The id is a decreasing sequence starting from -1. Representing a pseudo id as is not incorporated into the edge_table.

 ${\bf contracted_ver {\tt ARRAY}} \verb|[BIGIMT]| {\tt ray} \ of \ contracted \ vertex \ identifiers.$

Column	Type	Description
source	BIGINT	Identifier of the source vertex of the current edge id. Valid values when type = 'e'.
target	BIGINT	Identifier of the target vertex of the current edge id. Valid values when type = 'e'.
cost	FLOAT	Weight of the edge (source, target). Valid values when type = 'e'.

Examples¶ Example:

Only dead end contraction

SELECT * FROM pgr_contractGraph(

'SELECT id, source, target, cost, reverse_cost FROM edge_table', ARRAY[1]);

			contracted_vertice			_	
1 v	-		+ {1}	-	•	 -1	-
•	•		{7,8}	i		-1	•
3 v	1	10	{13}	1	-1	-1	-1
4 v	- 1	15	{14}		-1	-1	-1
5 v	- 1	17	{16}		-1	-1	-1
(5 rows)							

Example:

Only linear contraction

SELECT * FROM pgr_contractGraph(

'SELECT id, source, target, cost, reverse_cost FROM edge_table', ARRAY[2]);

-	0 -		contracted_vertices			J		
1 2 3	e e e e	-1 -2 -3	{4} {8} {8}	 	9 5 7	3 7		2 2 2

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Introduction¶ In big graphs, like the road graphs, or electric networks, graph contraction can be used to speed up some graph algorithms. Contraction reduces the size of the graph by removing some of the vertices and edges and, for example, might add edges that represent a sequence of original edges decreasing the total time and space used in graph algorithms.

This implementation gives a flexible framework for adding contraction algorithms in the future, currently, it supports two algorithms:

- 1. Dead end contraction
- 2. Linear contraction

Allowing the user to:

- Forbid contraction on a set of nodes.
- Decide the order of the contraction algorithms and set the maximum number of times they are to be executed.

Note

UNDER DISCUSSION: Forbid contraction on a set of edges

Dead end contraction¶ In the algorithm, dead end contraction is represented by 1.

Dead end nodes¶

The definition of a dead end node is different for a directed and an undirected graph.

In case of a undirected graph, a node is considered a dead end node if

• The number of adjacent vertices is 1.

In case of an directed graph, a node is considered a dead end node if

- There are no outgoing edges and has at least one incoming edge.
- There is one incoming and one outgoing edge with the same identifier.

Examples

- The green node B represents a dead end node
- The node A is the only node connecting to B.

- Node A is part of the rest of the graph and has an unlimited number of incoming and outgoing edges.
- Directed graph

Operation: Dead End Contraction¶

The dead end contraction will stop until there are no more dead end nodes. For example from the following graph:

- Node A is connected to the rest of the graph by an unlimited number of edges.
- Node B is connected to the rest of the graph with one incoming edge.
- Node B is the only node connecting to C.
- The green node C represents a Dead End node

After contracting C, node B is now a Dead End node and is contracted:

Node B gets contracted

Nodes B and C belong to node A.

Not Dead End nodes¶

In this graph B is not a dead end node.

Linear contraction¶ In the algorithm, linear contraction is represented by 2.

Linear nodes¶

A node is considered a linear node if satisfies the following:

- The number of adjacent vertices are 2.
- Should have at least one incoming edge and one outgoing edge.

Examples

- The green node B represents a linear node
- $\bullet\,$ The nodes A and C are the only nodes connecting to B.
- Node A is part of the rest of the graph and has an unlimited number of incoming and outgoing edges.
- Node C is part of the rest of the graph and has an unlimited number of incoming and outgoing edges.
- · Directed graph

Operation: Linear Contraction¶

The linear contraction will stop until there are no more linear nodes. For example from the following graph:

- Node A is connected to the rest of the graph by an unlimited number of edges.
- Node B is connected to the rest of the graph with one incoming edge and one outgoing edge.
- Node C is connected to the rest of the graph with one incoming edge and one outgoing edge.
- Node D is connected to the rest of the graph by an unlimited number of edges.
- $\bullet\,$ The green nodes B and C represents Linear nodes.

After contracting B, a new edge gets inserted between A and C which is represented by red color.

Node C is linear node and gets contracted.

Nodes B and C belong to edge connecting \boldsymbol{A} and \boldsymbol{D} which is represented by red color.

Not Linear nodes¶

In this graph B is not a linear node.

The cycle¶ Contracting a graph, can be done with more than one operation. The order of the operations affect the resulting contracted graph, after applying one operation, the set of vertices that can be contracted by another operation changes.

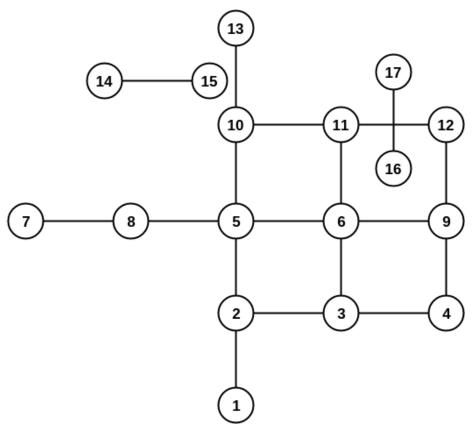
This implementation, cycles max_cycles times through operations_order .

```
<input>
do max_cycles times {
    for (operation in operations_order)
        { do operation }
}
<output>
```

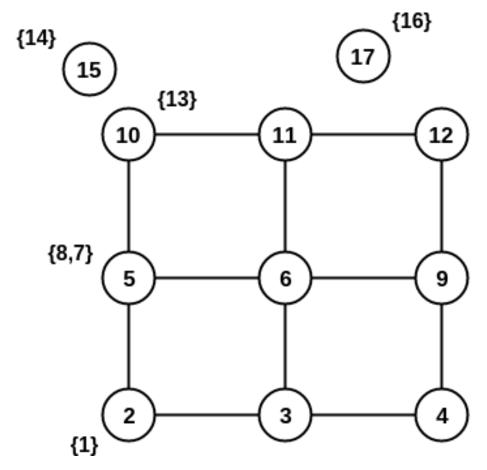
Contracting Sample Data¶ In this section, building and using a contracted graph will be shown by example.

- The Sample Data for an undirected graph is used
- a dead end operation first followed by a linear operation.

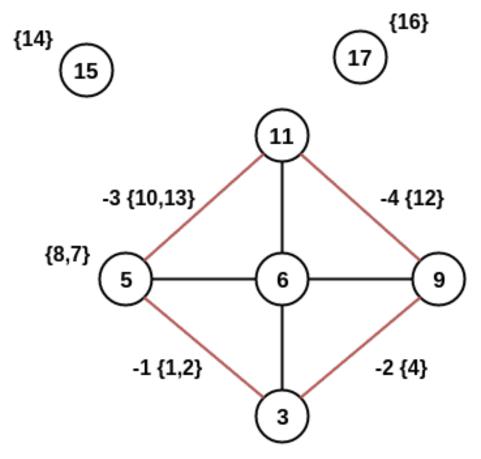
The original graph:



After doing a dead end contraction operation:



Doing a linear contraction operation to the graph above



There are five cases, in this documentation, which arise when calculating the shortest path between a given source and target. In this examples, pgr_dijkstra is used.

- Case 1: Both source and target belong to the contracted graph.
- Case 2: Source belongs to a contracted graph, while target belongs to a edge subgraph.
- Case 3: Source belongs to a vertex subgraph, while target belongs to an edge subgraph.
- Case 4: Source belongs to a contracted graph, while target belongs to an vertex subgraph.
- Case 5: The path contains a new edge added by the contraction algorithm.

Construction of the graph in the database \P

Original Data

The following query shows the original data involved in the contraction operation.

Contraction Results

```
SELECT * FROM pgr_contractGraph(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   array[1,2], directed:=true);
 seq | type | id | contracted_vertices | source | target | cost
1 | v
          | 5 | {7,8}
                                - 1
                                     -1 |
                                            -1 l
                                                  -1
  2 | v
          | 15 | {14}
                               - 1
                                    -1 |
                                            -1 |
                                                  -1
  3 | v
         | 17 | {16}
                               -1 |
                                            -1 |
                                                  -1
  4 | e
         | -1 | \{1,2\}
                               3 |
                                             5 I
                                             3 |
  5 | e
         | -2 | {4}
                               9 |
                                                   2
          | -3 | {10,13}
                               6 | e
                                                   2
                                     5 I
                                            11 |
          | -4 | {12}
  7 | e
                                11 |
                                             9 |
                                                   2
(7 rows)
```

The above results do not represent the contracted graph. They represent the changes done to the graph after applying the contraction algorithm. We can see that vertices like 6 and 11 do not appear in the contraction results because they were not affected by the contraction algorithm.

step 1

Adding extra columns to the edge_table and edge_table_vertices_pgr tables:

Column	Description			
contracted_vefhicesrtices set belonging to the vertex/edge				
is_contracted On a vertex table: when true the vertex is contracted, so is				
not part of the contracted graph.				

is_contracted On an edge table: when **true** the edge was generated by the contraction algorithm.

Using the following queries:

```
ALTER TABLE edge_table ADD contracted_vertices BIGINT[];
ALTER TABLE
ALTER TABLE edge_table_vertices_pgr ADD contracted_vertices BIGINT[];
```

```
ALTER TABLE
ALTER TABLE edge_table ADD is_contracted BOOLEAN DEFAULT false;
ALTER TABLE
ALTER TABLE edge_table_vertices_pgr ADD is_contracted BOOLEAN DEFAULT false;
ALTER TABLE
SET client_min_messages TO NOTICE;
SET
step 2
For simplicity, in this documentation, store the results of the call to
pgr contractGraph in a temporary table
SELECT * INTO contraction_results
FROM pgr_contractGraph(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    array[1,2], directed:=true);
SELECT 7
step 3
Update the vertex and edge tables using the results of the call to
pgr_contraction
  • In edge_table_vertices_pgr.is_contracted indicate the vertices that are
    contracted.
UPDATE edge_table_vertices_pgr
SET is_contracted = true
WHERE id IN (SELECT unnest(contracted_vertices) FROM contraction_results);
UPDATE 10
```

• Add to edge_table_vertices_pgr.contracted_vertices the contracted vertices belonging to the vertices.

```
UPDATE edge_table_vertices_pgr
SET contracted_vertices = contraction_results.contracted_vertices
FROM contraction_results
WHERE type = 'v' AND edge_table_vertices_pgr.id = contraction_results.id;
UPDATE 3
```

• Insert the new edges generated by pgr_contractGraph.

INSERT INTO edge_table(source, target, cost, reverse_cost, contracted_vertices, is_contract
SELECT source, target, cost, -1, contracted_vertices, true
FROM contraction_results
WHERE type = 'e';
INSERT 0 4

step 3.1

Verify visually the updates.

 \bullet On the edge_table_vertices_pgr

SELECT id, contracted_vertices, is_contracted
FROM edge_table_vertices_pgr
ORDER BY id;

id cont	racted_vertices	is_contracted
1		t
2		t
3		f
4		t
5 {7,8	}	f
6		f
7		t
8		t
9		f
10		t
11		f
12		t
13		t
14		t
15 {14}		f
16		t
17 {16}		f
(17 rows)		

 $\bullet\,$ On the edge_table

SELECT id, source, target, cost, reverse_cost, contracted_vertices, is_contracted FROM edge_table ORDER BY id;

1	1	2	1	1	f
2	2	3	-1	1	f

```
3 |
                              -1 |
                                                1 |
                                                                            | f
  4 |
            2 |
                       5 I
                               1 |
                                                1 |
                                                                            | f
                                               -1 |
  5 I
            3 |
                       6 |
                                                                            | f
                               1 |
            7 |
                                                1 |
  6 |
                      8 |
                               1 |
                                                                            | f
  7 |
            8
                      5 I
                               1 |
                                                1 |
                                                                            | f
  8 |
            5 I
                       6 I
                                                1 |
                                                                            | f
                               1 I
  9 |
                      9 |
            6 I
                                                                            | f
                               1 |
                                                1 |
 10 |
            5 I
                     10 |
                                                                            | f
                               1 |
                                                1 |
 11 |
            6 I
                     11 |
                                               -1 l
                                                                            | f
                               1 |
 12 |
           10 |
                     11 |
                               1 |
                                               -1 l
                                                                            | f
 13 |
                     12 |
                                               -1 |
                                                                            | f
           11 |
                               1 |
           10 |
                     13 |
                                                                            | f
 14 |
                               1 |
                                                1 |
 15 |
            9 |
                     12 |
                               1 |
                                                1 |
                                                                            | f
 16 |
            4 |
                      9 |
                               1 |
                                                1 |
                                                                            | f
 17 |
           14 |
                     15 |
                               1 |
                                                1 |
                                                                            | f
 18 |
           16 |
                      17 |
                               1 |
                                                1 |
                                                                            | f
 19 |
            3 |
                      5 I
                               2 |
                                               -1 \mid \{1,2\}
                                                                            | t
20 |
            9 |
                      3 |
                               2 |
                                               -1 | {4}
                                                                            | t
21 |
            5 I
                                               -1 \mid \{10,13\}
                     11 |
                               2 |
                                                                            | t
22 |
           11 |
                      9 |
                               2 |
                                               -1 | {12}
                                                                            | t
(22 rows)
```

• vertices that belong to the contracted graph are the non contracted vertices

```
SELECT id FROM edge_table_vertices_pgr
WHERE is_contracted = false
ORDER BY id;
id
----
3
5
6
9
11
15
17
(7 rows)
```

case 1: Both source and target belong to the contracted graph.

Inspecting the contracted graph above, vertex 3 and vertex 11 are part of the contracted graph. In the following query:

• vertices_in_graph hold the vertices that belong to the contracted graph.

 when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 11: 3 -> 6 -> 11, and in the contracted graph, it is also 3 -> 6 -> 11. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra(
   $$
   WITH
   vertices_in_graph AS (
    SELECT id FROM edge_table_vertices_pgr WHERE is_contracted = false)
   SELECT id, source, target, cost, reverse cost
   FROM edge table
   WHERE source IN (SELECT * FROM vertices_in_graph)
   AND target IN (SELECT * FROM vertices_in_graph)
   $$,
   3, 11, false);
seq | path_seq | node | edge | cost | agg_cost
 ---+----+----
            1 |
                  3 | 5 |
                              1 |
  2 |
            2 | 6 | 11 | 1 |
                                        1
            3 |
                 11 | -1 |
                               0 |
  3 I
                                         2
(3 rows)
```

case 2: Source belongs to the contracted graph, while target belongs to a edge subgraph.

Inspecting the contracted graph above, vertex 3 is part of the contracted graph and vertex expand1 holds the contracted vertices of the edge where vertex 1 belongs. (belongs to edge 19).

- vertices_in_graph hold the vertices that belong to the contracted graph and also the contracted vertices of edge 19.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 1: 3 -> 2 -> 1, and in the contracted graph, it is also 3 -> 2 -> 1. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra(
    $$
```

```
WITH
  expand_edges AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_table),
   expand1 AS (SELECT contracted_vertices FROM edge_table
      WHERE id IN (SELECT id FROM expand_edges WHERE vertex = 1)),
   vertices_in_graph AS (
    SELECT id FROM edge_table_vertices_pgr WHERE is_contracted = false
       UNION
       SELECT unnest(contracted_vertices) FROM expand1)
   SELECT id, source, target, cost, reverse_cost
   FROM edge_table
   WHERE source IN (SELECT * FROM vertices_in_graph)
   AND target IN (SELECT * FROM vertices_in_graph)
   $$,
   3, 1, false);
seq | path_seq | node | edge | cost | agg_cost
---+----
            1 |
                   3 |
                          2 |
  1 |
                                 1 |
             2 |
                   2 |
                         1 |
  2 |
                                 1 |
                                           1
             3 I
                   1 l
                         -1 l
                                 0 I
                                            2
  3 |
(3 rows)
```

case 3: Source belongs to a vertex subgraph, while target belongs to an edge subgraph.

Inspecting the contracted graph above, vertex 7 belongs to the contracted subgraph of vertex 5 and vertex 13 belongs to the contracted subgraph of edge 21. In the following query:

- expand7 holds the contracted vertices of vertex where vertex 7 belongs. (belongs to vertex 5)
- $\bullet\,$ expand 13 holds the contracted vertices of edge where vertex 13 belongs. (belongs to edge 21)
- vertices_in_graph hold the vertices that belong to the contracted graph, contracted vertices of vertex 5 and contracted vertices of edge 21.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 7 to 13: 7 -> 8 -> 5 -> 10 -> 13, and in the contracted graph, it is also 7 -> 8 -> 5 -> 10 -> 13. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra(
    $$
```

```
expand_vertices AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_table_ver
  expand7 AS (SELECT contracted_vertices FROM edge_table_vertices_pgr
     WHERE id IN (SELECT id FROM expand_vertices WHERE vertex = 7)),
  expand_edges AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_table),
    expand13 AS (SELECT contracted_vertices FROM edge_table
       WHERE id IN (SELECT id FROM expand_edges WHERE vertex = 13)),
    vertices_in_graph AS (
     SELECT id FROM edge_table_vertices_pgr WHERE is_contracted = false
        UNION
        SELECT unnest(contracted_vertices) FROM expand13
        UNION
        SELECT unnest(contracted_vertices) FROM expand7)
    SELECT id, source, target, cost, reverse_cost
   FROM edge table
    WHERE source IN (SELECT * FROM vertices in graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    $$,
    7, 13, false);
 seq | path_seq | node | edge | cost | agg_cost
                     7 I
  1 |
              1 |
                            6 |
                                    1 |
                                               0
  2 |
              2 |
                     8 |
                            7 |
                                    1 |
                                               1
  3 |
              3 |
                     5 l
                           10 |
                                    1 |
  4 |
              4 |
                    10 |
                           14 l
                                               3
                                    1 I
              5 |
                    13 l
                                               4
  5 I
                           -1 l
                                    0 |
(5 rows)
```

case 4: Source belongs to the contracted graph, while target belongs to an vertex subgraph.

Inspecting the contracted graph above, vertex 3 is part of the contracted graph and vertex 7 belongs to the contracted subgraph of vertex 5. In the following query:

- expand7 holds the contracted vertices of vertex where vertex 7 belongs. (belongs to vertex 5)
- vertices_in_graph hold the vertices that belong to the contracted graph and the contracted vertices of vertex 5.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 7: 3 -> 2 -> 5 -> 8 -> 7, but in the contracted graph, it is 3 -> 5 -> 8 -> 7. The results, on the contracted graph do not match the results as if it was done on the original graph. This is because the path contains edge 19 which is added by the contraction algorithm.

```
SELECT * FROM pgr_dijkstra(
    $$
    WITH
  expand_vertices AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_table_ver
  expand7 AS (SELECT contracted_vertices FROM edge_table_vertices_pgr
      WHERE id IN (SELECT id FROM expand_vertices WHERE vertex = 7)),
    vertices_in_graph AS (
     SELECT id FROM edge_table_vertices_pgr WHERE is_contracted = false
        SELECT unnest(contracted_vertices) FROM expand7)
    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    $$,
    3, 7, false);
 seq | path_seq | node | edge | cost | agg_cost
              1 |
                     3 |
                            19 |
                                    2 |
                                               0
   1 |
   2 |
              2 |
                     5 I
                            7 |
                                    1 |
                                               2
   3 |
              3 |
                     8 |
                            6 |
                                    1 |
                                               3
              4 I
                     7 |
                           -1 l
   4 |
                                    0 1
(4 rows)
```

case 5: The path contains an edge added by the contraction algorithm.

In the previous example we can see that the path from vertex 3 to vertex 7 contains an edge which is added by the contraction algorithm.

Inspecting the contracted graph above, edge 19 should be expanded. In the following query:

- first_dijkstra holds the results of the dijkstra query.
- edges_to_expand holds the edges added by the contraction algorithm and included in the path.
- vertices_in_graph hold the vertices that belong to the contracted graph, vertices of the contracted solution and the contracted vertices of the edges added by the contraction algorithm and included in the contracted solution.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 7: 3 -> 2 -> 5 -> 8 -> 7, and in the contracted graph, it is also 3 -> 2 -> 5 -> 8 -> 7. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT unnest(contracted_vertices) FROM expand7)
           SELECT id, source, target, cost, reverse_cost
           FROM edge_table
           WHERE source IN (SELECT * FROM vertices_in_graph)
           AND target IN (SELECT * FROM vertices_in_graph)
           3, 7, false)),
   -- edges that need expansion and the vertices to be expanded.
   edges_to_expand AS (
       SELECT edge, contracted_vertices
       FROM first_dijkstra JOIN edge_table
       ON (edge = id)
       WHERE is_contracted = true),
   vertices_in_graph AS (
       -- the nodes of the contracted solution
       SELECT node FROM first_dijkstra
       UNION
       -- the nodes of the expanding sections
       SELECT unnest(contracted_vertices) FROM edges_to_expand)
   SELECT id, source, target, cost, reverse_cost
   FROM edge_table
   WHERE source IN (SELECT * FROM vertices_in_graph)
   AND target IN (SELECT * FROM vertices_in_graph)
   -- not including the expanded edges
   AND id NOT IN (SELECT edge FROM edges_to_expand)
   $$,
   3, 7, false);
seq | path_seq | node | edge | cost | agg_cost
---+----+----
  1 |
            1 | 3 | 2 | 1 |
          1 | 3 | 2 | 1 |
2 | 2 | 4 | 1 |
3 | 5 | 7 | 1 |
4 | 8 | 6 | 1 |
  2 |
  3 |
  4 |
            5 | 7 | -1 | 0 |
  5 |
(5 rows)
```

See Also \P

- $\bullet \ \ http://www.cs.cmu.edu/afs/cs/academic/class/15210-f12/www/lectures/lecture16.pdf$
- http://algo2.iti.kit.edu/documents/routeplanning/geisberger_dipl.pdf

• The queries use $pgr_contractGraph$ - Experimental function and the $Sam-ple\ Data$ network.

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Flow - Family of functions¶

- pgr_maxFlow Proposed Only the Max flow calculation using Push and Relabel algorithm.
- pgr_boykovKolmogorov Proposed Boykov and Kolmogorov with details of flow on edges.
- pgr_edmondsKarp Proposed Edmonds and Karp algorithm with details of flow on edges.
- pgr_pushRelabel Proposed Push and relabel algorithm with details of flow on edges.
- Applications
 - $-\ pgr_edgeDisjointPaths$ Proposed Calculates edge disjoint paths between two groups of vertices.
 - pgr_maxCardinalityMatch Proposed Calculates a maximum cardinality matching in a graph.

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

pgr_maxFlow - Proposed¶ Synopsis¶

pgr_maxFlow — Calculates the maximum flow in a directed graph from the source(s) to the targets(s) using the Push Relabel algorithm.



Boost Graph Inside

Availability: 2.4.0

Warning

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 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - $-\,$ Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Characteristics

- The graph is **directed**.
- When the maximum flow is 0 then there is no flow and **0** is returned.
 - There is no flow when a ${\bf source}$ is the same as a ${\bf target}.$
- Any duplicated value in the source(s) or target(s) are ignored.
- Uses the $pgr_pushRelabel$ algorithm.
- Running time: $\langle (O(V^3) \rangle)$

Signature Summary¶

```
pgr_maxFlow(edges_sql, source, target)
pgr_maxFlow(edges_sql, sources, target)
pgr_maxFlow(edges_sql, source, targets)
pgr_maxFlow(edges_sql, sources, targets)
RETURNS BIGINT
One to One \P
Calculates the maximum flow from the source to the target.
pgr_maxFlow(edges_sql, source, target)
RETURNS BIGINT
Example:
SELECT * FROM pgr_maxFlow(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, 11
);
pgr_maxflow
         230
(1 row)
One to Many¶
Calculates the maximum flow from the source to all of the targets.
pgr_maxFlow(edges_sql, source, targets)
RETURNS BIGINT
Example:
SELECT * FROM pgr_maxFlow(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
```

```
, 6, ARRAY[11, 1, 13]
);
pgr_maxflow
         340
(1 row)
Many to One\P
Calculates the maximum flow from all the sources to the target.
pgr_maxFlow(edges_sql, sources, target)
RETURNS BIGINT
Example:
SELECT * FROM pgr_maxFlow(
    'SELECT id,
             source,
            target,
             capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], 11
);
pgr_maxflow
         230
(1 row)
Many to Many¶
Calculates the maximum flow from all of the sources to all of the targets.
pgr_maxFlow(edges_sql, sources, targets)
RETURNS BIGINT
Example:
SELECT * FROM pgr_maxFlow(
    'SELECT id,
             source,
             target,
             capacity,
             reverse_capacity
```

```
FROM edge_table'
, ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
pgr_maxflow
-----
360
(1 row)
```

Description of the Signatures \P

Description of the edges_sql query for Max-flow like functions \P

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Description
Identifier of the edge.
Identifier of the first end point vertex edge.
Identifier of the second end point ver edge.
Weight of the edge (source, target)
 When negative: edge (source, t does not exist, therefore it's not the graph. Weight of the edge (target, source), When negative: edge (target, so does not exist, therefore it's not the graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

Description of the Parameters of the Flow Signatures ¶

Column	Type	Default	Description
edges_sq	l TEXT		The edges SQL que above.

Column	Type Default	Description
source	BIGINT	Identifier of the star flow.
sources	ARRAY[BIGINT]	Array of identifiers vertices of the flow.
target	BIGINT	Identifier of the end flow.
targets	ARRAY[BIGINT]	Array of identifiers of the flow.

Description of the return value \P

Type	Description
BIGINT	Maximum flow possible from the source(s) to the $target(s)$

See Also \P

- Flow Family of functions
- $\bullet \ \ http://www.boost.org/libs/graph/doc/push_relabel_max_flow.html$
- $\bullet \ \ https://en.wikipedia.org/wiki/Push\%E2\%80\%93relabel_maximum_flow_algorithm$

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$\mathbf{pgr_pushRelabel - Proposed} \P \quad \mathrm{Synopsis} \P$

pgr_pushRelabel — Calculates the flow on the graph edges that maximizes the flow from the sources to the targets using Push Relabel Algorithm.



Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr maxFlowPushRelabel
- New in 2.3.0

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Characteristics

- The graph is **directed**.
- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
 - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
 - Edges with zero flow are omitted.
- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by $pgr_maxFlow$ when executed with the same parameters and can be calculated:
 - By aggregation of the outgoing flow from the sources
 - By aggregation of the incoming flow to the targets

• Running time: $\langle (O(V^3) \rangle)$

Signature Summary¶

```
pgr_pushRelabel(edges_sql, source, target) - Proposed
pgr_pushRelabel(edges_sql, sources, target) - Proposed
pgr_pushRelabel(edges_sql, source, targets) - Proposed
pgr_pushRelabel(edges_sql, sources, targets) - Proposed
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

One to One¶

Calculates the flow on the graph edges that maximizes the flow from the source to the target.

```
pgr_pushRelabel(edges_sql, source, target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example:

```
SELECT * FROM pgr_pushRelabel(
   'SELECT id,
         source,
         target,
         capacity,
         reverse_capacity
   FROM edge_table'
   , 6, 11
);
seq | edge | start_vid | end_vid | flow | residual_capacity
10 |
              5 |
                       10 | 100 |
  1 |
      8 |
              6 |
6 |
  2 I
                       5 | 100 |
```

10 |

One to Many¶

3 |

4 |

(4 rows)

11 |

12 |

Calculates the flow on the graph edges that maximizes the flow from the source to all of the targets.

11 | 130 |

11 | 100 |

30

0

0

```
pgr_pushRelabel(edges_sql, source, targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
Example:
SELECT * FROM pgr_pushRelabel(
   'SELECT id,
           source,
           target,
           capacity,
           reverse capacity
   FROM edge table'
   , 6, ARRAY[11, 1, 13]
);
seq | edge | start_vid | end_vid | flow | residual_capacity
                              1 | 130 |
  1 |
         1 |
                    2 |
                                                        0
                              2 |
  2 |
         2 |
                    3 |
                                    80 |
                                                       20
  3 |
         3 |
                    4 |
                              3 |
                                    80 |
                                                       50
  4 |
                   5 l
                              2 |
                                    50 l
                                                       0
         4 |
  5 I
         7 |
                   5 I
                             8 |
                                    50 I
                                                       80
  6 |
        10 |
                   5 l
                            10 | 80 |
                                                       50
  7 |
        8 I
                   6 l
                             5 | 130 |
                                                       0
                   6 I
  8 I
         9 |
                             9 |
                                   80 I
                                                       50
                    6 |
  9 |
        11 |
                            11 | 130 |
                                                        0
                    7 |
                              8 I
  10 |
         6 l
                                   50 l
                                                        0
  11 |
         6 I
                    8 I
                              7 |
                                    50 I
                                                       50
         7 |
                    8 I
                              5 |
                                    50 I
                                                        0
  12 |
                                                        0
  13 |
        16 |
                    9 |
                              4 |
                                    80 |
 14 |
        12 |
                   10 |
                             11 |
                                    80 |
                                                       20
(14 rows)
```

Many to One \P

Calculates the flow on the graph edges that maximizes the flow from all of the sources to the target.

```
pgr_pushRelabel(edges_sql, sources, target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example:

SELECT * FROM pgr_pushRelabel(

Many to Many¶

Calculates the flow on the graph edges that maximizes the flow from all of the sources to all of the targets.

```
pgr_pushRelabel(edges_sql, sources, targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

```
SELECT * FROM pgr_pushRelabel(
   'SELECT id,
          source,
          target,
          capacity,
          reverse_capacity
   FROM edge_table'
   , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
----+----+-----
                     1 | 50 |
3 | 80 |
             2 |
  1 |
       1 |
                                                80
                4 |
  2 |
       3 |
                                               50
                5 |
                         2 | 50 |
  3 |
       4 |
                                                0
               5 |
6 |
                       10 | 100 |
  4 | 10 |
                                               30
                         5 | 130 |
  5 |
       8 |
                                                0
       9 | 6 | 9 | 30 |
11 | 6 | 11 | 130 |
                         9 | 30 |
  6 |
                                               100
  7 |
                                                 0
```

8		7	I	8	1	5		20	1	30
9		16		9	-	4		80	1	0
10		12		10	-	11		100	1	0
11		15		12	-	9		50	1	0
(11 rows)										

Description of the Signatures \P

Description of the edges_sql query for Max-flow like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
$\overline{\mathbf{id}}$	ANY-INTEG	ER	Identifier of the edge.
source	ANY-INTEG	ER	Identifier of the first end point vertex edge.
target	ANY-INTEG	ER	Identifier of the second end point ver edge.
capacity	ANY-INTEG	ER	Weight of the edge (source, target)
reverse_c	сафисіі утес	ERI	 When negative: edge (source, t does not exist, therefore it's not the graph. Weight of the edge (target, source), When negative: edge (target, so does not exist, therefore it's not the graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

Description of the Parameters of the Flow Signatures \P

Column	Type	Default	Description
edges_sq	l TEXT		The edges SQL que above.
source	BIGINT		Identifier of the star flow.

Column	Type	Default	Description
sources	ARRAY[BIGIN	r]	Array of identifiers vertices of the flow.
target	BIGINT		Identifier of the end flow.
targets	ARRAY[BIGIN	Γ]	Array of identifiers of the flow.

Description of the Return Values¶

Column	Type	Description
\overline{seq}	INT	Sequential value starting from 1.
$edge_id$	BIGINT	Identifier of the edge in the original query(edges_sql).
source	BIGINT	Identifier of the first end point vertex of the edge.
target	BIGINT	Identifier of the second end point vertex of the edge.
flow	BIGINT	Flow through the edge in the direction (source, target).
residual_ca	pad it ¢INT	Residual capacity of the edge in the direction (source, target).

See Also \P

- Flow Family of functions, pgr_boykovKolmogorov, pgr_edmondsKarp
- $\bullet \ \ http://www.boost.org/libs/graph/doc/push_relabel_max_flow.html$
- $\bullet \ \ https://en.wikipedia.org/wiki/Push\%E2\%80\%93relabel_maximum_flow_algorithm$

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$pgr_edmondsKarp - Proposed\P$ Synopsis¶

pgr_edmondsKarp — Calculates the flow on the graph edges that maximizes the flow from the sources to the targets using Push Relabel Algorithm.



Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maxFlowEdmondsKarp
- New in 2.3.0

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Characteristics

- The graph is directed.
- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
 - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
 - Edges with zero flow are omitted.

- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by $pgr_maxFlow$ when executed with the same parameters and can be calculated:
 - By aggregation of the outgoing flow from the sources
 - By aggregation of the incoming flow to the targets
- Running time: $\langle (O(V * E^2) \rangle)$

Signature Summary¶

```
pgr_edmondsKarp(edges_sql, source, target) - Proposed
pgr_edmondsKarp(edges_sql, sources, target) - Proposed
pgr_edmondsKarp(edges_sql, source, targets) - Proposed
pgr_edmondsKarp(edges_sql, sources, targets) - Proposed
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

One to One¶

Calculates the flow on the graph edges that maximizes the flow from the source to the target.

```
pgr_edmondsKarp(edges_sql, source, target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

-		0		_		-			residual_capacity
	٠.	10	٠.		٠.	10	٠.		
2		8	-	6	1	5	1	100	30
3		11	-	6	1	11	1	130	0
4		12	-	10	1	11	1	100	0
(4 ro	ws	3)							

One to Many¶

Calculates the flow on the graph edges that maximizes the flow from the source to all of the targets.

```
pgr_edmondsKarp(edges_sql, source, targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
Example:
SELECT * FROM pgr edmondsKarp(
    'SELECT id,
           source,
           target,
           capacity,
           reverse_capacity
   FROM edge_table'
   , 6, ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
----+----+-----
       1 | 2 | 1 | 50 |
3 | 4 | 3 | 80 |
4 | 5 | 2 | 50 |
   1 |
                                                     80
   2 |
                                                    50
   3 l
                                                      0
                        10 | 80 |
5 | 130 |
9 | 80 |
   4 | 10 |
                  5 |
                                                     50
                 6 |
        8 |
   5 I
                                                      0
        9 |
                  6 I
   6 I
                            9 | 80 |
                                                     50
   7 | 11 | 6 | 11 | 130 | 8 | 16 | 9 | 4 | 80 |
                                                     0
                   9 |
   8 |
                            4 | 80 |
                                                      0
      16 |
                 10 | 11 | 80 |
   9 |
        12 |
                                                      20
(9 rows)
```

Many to One¶

Calculates the flow on the graph edges that maximizes the flow from all of the sources to the target.

```
pgr_edmondsKarp(edges_sql, sources, target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

```
SELECT * FROM pgr_edmondsKarp(
   'SELECT id,
         source,
         target,
         capacity,
         reverse_capacity
   FROM edge_table'
  , ARRAY[6, 8, 12], 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
____+
     10 | 5 | 10 | 100 |
  1 |
               6 |
6 |
       8 |
  2 |
                        5 | 100 |
                                            30
                                             0
  3 |
     11 |
                       11 | 130 |
               10 |
  4 | 12 |
                       11 | 100 |
                                              0
(4 rows)
```

Many to Many¶

Calculates the flow on the graph edges that maximizes the flow from all of the sources to all of the targets.

```
pgr_edmondsKarp(edges_sql, sources, targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

```
SELECT * FROM pgr_edmondsKarp(
   'SELECT id,
         source,
         target,
         capacity,
         reverse_capacity
   FROM edge_table'
  , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
----+----+-----
              2 |
                      1 | 50 |
  1 |
       1 |
                                             80
               4 |
                        3 | 80 |
  2 |
       3 |
                                             50
  3 I
               5 I
                        2 l 50 l
       4 I
                                             0
  4 I
      10 |
               5 l
                       10 | 100 |
                                             30
                    5 | 130 |
9 | 80 |
                6 |
  5 |
       8 |
                                              0
       9 | 6 |
  6 I
                                              50
```

7	1	11		6	-	11	130	1	0	
8		7		8	-	5 l	20	1	30	
9		16		9	-	4	80	1	0	
10	1	12		10	-	11	100	1	0	
(10 r	(10 rows)									

Description of the Signatures \P

Description of the edges_sql query for Max-flow like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point ver edge.
capacity	ANY-INTEGER	Weight of the edge (source, target)
reverse	ca que cily Tegerl	 When negative: edge (source, t does not exist, therefore it's not the graph. Weight of the edge (target, source), When negative: edge (target, source) does not exist, therefore it's not exist.
		the graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

Description of the Parameters of the Flow Signatures \P

Column	Type	Default	Description
edges_sq	l TEXT		The edges SQL queabove.
source	BIGINT		Identifier of the star flow.

Column	Type	Default	Description
sources	ARRAY[BIGIN	r]	Array of identifiers vertices of the flow.
target	BIGINT		Identifier of the end flow.
targets	ARRAY[BIGIN	Γ]	Array of identifiers of the flow.

Description of the Return Values¶

Column	Type	Description
\overline{seq}	INT	Sequential value starting from 1.
$edge_id$	BIGINT	Identifier of the edge in the original query(edges_sql).
source	BIGINT	Identifier of the first end point vertex of the edge.
target	BIGINT	Identifier of the second end point vertex of the edge.
flow	BIGINT	Flow through the edge in the direction (source, target).
residual_ca	npa dsty INT	Residual capacity of the edge in the direction (source, target).

See Also¶

- $\bullet \ \ Flow \ \ Family \ of \ functions, \ pgr_boykovKolmogorov, \ pgr_PushRelabel$
- http://www.boost.org/libs/graph/doc/edmonds_karp_max_flow.html
- $\bullet \ \, https://en.wikipedia.org/wiki/Edmonds\%E2\%80\%93Karp_algorithm$

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$pgr_boykovKolmogorov - Proposed\P \quad \mathrm{Synopsis}\P$

 ${\tt pgr_boykovKolmogorov}$ — Calculates the flow on the graph edges that maximizes the flow from the sources to the targets using Boykov Kolmogorov algorithm.



Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maxFlowBoykovKolmogorov
- New in 2.3.0

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Characteristics

- The graph is directed.
- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
 - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
 - Edges with zero flow are omitted.

- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by $pgr_maxFlow$ when executed with the same parameters and can be calculated:
 - By aggregation of the outgoing flow from the sources
 - By aggregation of the incoming flow to the targets
- Running time: Polynomial

Signature Summary¶

```
pgr_boykovKolmogorov(edges_sql, source, target) - Proposed
pgr_boykovKolmogorov(edges_sql, sources, target) - Proposed
pgr_boykovKolmogorov(edges_sql, source, targets) - Proposed
pgr_boykovKolmogorov(edges_sql, sources, targets) - Proposed
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

One to One¶

Calculates the flow on the graph edges that maximizes the flow from the source to the target.

```
pgr_boykovKolmogorov(edges_sql, source, target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

SE	P.	ı	eage	ı	start_vid	ı	ena_via	ı	IIOW	ı	residual_capacity
		+-		+-		+-		+-		+-	
	1	1	10	-	5		10		100		30
	2	1	8	-	6		5		100	I	30
	3		11	-	6		11		130	I	0
	4		12	-	10		11		100	I	0
(4	ro	ws	3)								

One to Many¶

Calculates the flow on the graph edges that maximizes the flow from the source to all of the targets.

```
pgr_boykovKolmogorov(edges_sql, source, targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
Example:
SELECT * FROM pgr boykovKolmogorov(
      'SELECT id.
                source,
                target,
                capacity,
                reverse_capacity
     FROM edge_table'
      , 6, ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
----+----+-----

    1 | 1 | 2 | 1 | 50 |

    2 | 3 | 4 | 3 | 80 |

    3 | 4 | 5 | 2 | 50 |

    4 | 10 | 5 | 10 | 80 |

    5 | 8 | 6 | 5 | 130 |

    6 | 9 | 6 | 9 | 80 |

    7 | 11 | 6 | 11 | 130 |

    8 | 16 | 9 | 4 | 80 |

    10 | 12 | 10 | 11 | 80 |

                                                                               80
                                                                              50
                                                                                0
                                                                               50
                                                                                0
                                                                              50
                                                                                0
                                                                                0
            12 | 10 | 11 | 80 |
    9 |
                                                                                20
(9 rows)
```

Many to One¶

Calculates the flow on the graph edges that maximizes the flow from all of the sources to the target.

```
pgr_boykovKolmogorov(edges_sql, sources, target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

```
SELECT * FROM pgr_boykovKolmogorov(
   'SELECT id,
         source,
         target,
         capacity,
         reverse_capacity
   FROM edge_table'
   , ARRAY[6, 8, 12], 11
);
seq | edge | start_vid | end_vid | flow | residual_capacity
____+
                                  30
  1 | 10 | 5 | 10 | 100 |
              6 |
6 |
      8 |
  2 |
                       5 | 100 |
                                           30
                      11 | 130 |
     11 |
                                           0
  3 |
            10 |
  4 | 12 |
                      11 | 100 |
                                            0
(4 rows)
```

Many to Many¶

Calculates the flow on the graph edges that maximizes the flow from all of the sources to all of the targets.

```
pgr_boykovKolmogorov(edges_sql, sources, targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

```
SELECT * FROM pgr_boykovKolmogorov(
   'SELECT id,
         source,
         target,
         capacity,
         reverse_capacity
   FROM edge_table'
   , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
----+----+-----
              2 |
                      1 | 50 |
  1 |
       1 |
                                             80
               4 |
                        3 | 80 |
  2 |
       3 |
                                            50
  3 I
       4 l
               5 l
                        2 l 50 l
                                             0
  4 I
     10 |
               5 |
                      10 | 100 |
                                             30
                    5 | 130 |
9 | 80 |
               6 |
  5 I
       8 |
                                             0
       9 | 6 |
  6 I
                                             50
```

7		11	1	6	-	11	130	1	0
8		7	1	8	-	5	20	1	30
9	1	16	1	9	-	4	80	1	0
10		12	1	10	-	11	100	1	0
(10 r	ows)							

Description of the Signatures \P

Description of the edges_sql query for Max-flow like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point ver edge.
capacity	ANY-INTEGER	Weight of the edge (source, target)
reverse_0	ca qua ci ly Tegeri	• When negative: edge (source, t does not exist, therefore it's no the graph. Weight of the edge (target, source),
		• When negative: edge (target, so does not exist, therefore it's no the graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

Description of the Parameters of the Flow Signatures \P

Column	Type	Default	Description
edges_sq	l TEXT		The edges SQL que above.
source	BIGINT		Identifier of the star flow.

Column	Type	Default	Description
sources	ARRAY[BIGIN]	г]	Array of identifiers vertices of the flow.
target	BIGINT		Identifier of the end flow.
targets	ARRAY[BIGIN]	r]	Array of identifiers of the flow.

Description of the Return Values¶

Column	Type	Description
$\overline{\text{seq}}$	INT	Sequential value starting from 1.
$edge_id$	BIGINT	Identifier of the edge in the original query(edges_sql).
source	BIGINT	Identifier of the first end point vertex of the edge.
target	BIGINT	Identifier of the second end point vertex of the edge.
flow	BIGINT	Flow through the edge in the direction (source, target).
residual_cap	oa df GINT	Residual capacity of the edge in the direction (source, target).

See Also \P

- $\bullet \ \ Flow \ \ Family \ of \ functions, \ pgr_pushRelabel, \ pgr_EdmondsKarp$
- $\bullet \ \, http://www.boost.org/libs/graph/doc/boykov_kolmogorov_max_flow.html$

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$\mathbf{pgr_maxCardinalityMatch-Proposed}\P \quad \mathrm{Synopsis}\P$

 ${\tt pgr_maxCardinalityMatch}$ — Calculates a maximum cardinality matching in a graph.

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting



Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maximumCardinalityMatching
- New in 2.3.0

Characteristics

- A matching or independent edge set in a graph is a set of edges without common vertices.
- A maximum matching is a matching that contains the largest possible number of edges.
 - There may be many maximum matchings.
 - Calculates **one** possible maximum cardinality matching in a graph.
- The graph can be directed or undirected.
- Running time: $(O(E*V*\lambda(E,V)))$

 $- (\langle Alpha(E,V) \rangle)$ is the inverse of the Ackermann function.

Signature Summary¶

```
pgr_MaximumCardinalityMatching(edges_sql) - Proposed
pgr_MaximumCardinalityMatching(edges_sql, directed) - Proposed
```

RETURNS SET OF (seq, edge_id, source, target)
OR EMPTY SET

Minimal Use¶

```
pgr_MaximumCardinalityMatching(edges_sql)
RETURNS SET OF (seq, edge_id, source, target) OR EMPTY SET
```

The minimal use calculates one possible maximum cardinality matching on a directed graph.

Example:

```
SELECT * FROM pgr_maxCardinalityMatch(
   'SELECT id, source, target, cost AS going, reverse_cost AS coming FROM edge_table'
);
```

seq	١	edge	١	source	1	target
	+-		+-		+-	
1		1		1	-	2
2	1	3	1	4	-	3
3	I	9	1	6	1	9
4	I	6	1	7	1	8
5	1	14	I	10	1	13
6	I	13	1	11	1	12
7	I	17	1	14	1	15
8	1	18	Ι	16	1	17
(8 ro	ws	3)				

Complete signature¶

```
pgr_MaximumCardinalityMatching(edges_sql, directed)
RETURNS SET OF (seq, edge_id, source, target) OR EMPTY SET
```

The complete signature calculates one possible maximum cardinality matching. Example:

SELECT * FROM pgr_maxCardinalityMatch(
 'SELECT id, source, target, cost AS going, reverse_cost AS coming FROM edge_table',
 directed := false
);

seq	edge	I	source	1	target
+-	+	_		+-	
1	1		1		2
2	3		3	-	4
3	9	l	6	-	9
4	6	١	7	1	8
5	14	l	10	-	13
6	13	l	11	1	12
7	17	١	14	1	15
8	18	l	16	1	17
(8 rows	3)				

Description of the Signatures \P

Description of the SQL query¶

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex of the edge.
target	ANY-INTEGER	Identifier of the second end point vertex of the edge.
going	ANY-NUMERIC	A positive value represents the existence of the edge (source, target).
coming	ANY-NUMERIC	A positive value represents the existence of the edge (target, source).

Where:

- ANY-INTEGER: SMALLINT, INTEGER, BIGINT
- ANY-NUMERIC: SMALLINT, INTEGER, BIGINT, REAL, DOUBLE PRECISION

Description of the parameters of the signatures \P

Column	Type	Description
$\overline{\mathrm{edges_sql}}$	TEXT	SQL query as described above.
directed	BOOLEAN	(optional) Determines the type of the graph. Default TRUE.

Description of the Result \P

Column	Type	Description
seq	INT	Sequential value starting from 1.
edge	BIGINT	Identifier of the edge in the original query(edges_sql).
source	BIGINT	Identifier of the first end point of the edge.
target	BIGINT	Identifier of the second end point of the edge.

See Also \P

- Flow Family of functions
- http://www.boost.org/libs/graph/doc/maximum_matching.html
- $\bullet\ https://en.wikipedia.org/wiki/Matching_\%28graph_theory\%29$
- $\bullet \ \ https://en.wikipedia.org/wiki/Ackermann_function$

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$\mathbf{pgr_edgeDisjointPaths} - \mathbf{Proposed}\P \quad \mathrm{Name}\P$

 ${\tt pgr_edgeDisjointPaths} -- {\tt Calculates} \ {\tt edge} \ {\tt disjoint} \ {\tt paths} \ {\tt between} \ {\tt two} \ {\tt groups} \ {\tt of} \ {\tt vertices}.$



Boost Graph Inside

Availability: 2.3.0

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis¶

Calculates the edge disjoint paths between two groups of vertices. Utilizes underlying maximum flow algorithms to calculate the paths.

Characteristics:¶

The main characterics are: • Calculates the edge disjoint paths between any two groups of vertices.

- Returns EMPTY SET when source and destination are the same, or cannot be reached.
- The graph can be directed or undirected.
- One to many, many to one, many to many versions are also supported.
- Uses pgr_boykovKolmogorov Proposed to calculate the paths.

Signature Summary¶

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid)
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid, directed)
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vids, directed)
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vid, directed)
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vids, directed)
```

```
RETURNS SET OF (seq, path_id, path_seq, [start_vid,] [end_vid,] node, edge, cost, agg_cost) OR EMPTY SET
```

Signatures

Minimal use¶

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
OR EMPTY SET
```

The minimal use is for a **directed** graph from one **start_vid** to one **end_vid**. Example:

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    3, 5
);
```

seq	path_id p	=			_			
1	1	 1	_	٠.	2		+- 	0
2	1	2	2	I	4	1	1	1
3	1	3 l	5	1	-1	0	1	2
4	2	1	3	1	5	1	1	0
5	2	2	6	1	8	1	1	1
6	2	3	5	1	-1	0	1	2
(6 rows)							

One to One¶

This signature finds the set of dijoint paths from one start_vid to one end_vid:

on a directed graph when directed flag is missing or is set to
true.

• on an undirected graph when directed flag is set to false.

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
OR EMPTY SET
```

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
```

```
3, 5,
   directed := false
);
seq | path_id | path_seq | node | edge | cost | agg_cost
----+----+-----
                              2 |
  1 |
           1 |
                    1 |
                          3 l
                                    1 |
                                               0
                   2 |
                              4 |
  2 |
           1 |
                        2 |
                                    1 |
                                               1
                        5 |
                    3 |
                               -1 l
                                      0 |
                                               2
  3 |
           1 |
  4 |
           2 |
                    1 |
                          3 I
                               3 l
                                     -1 |
                    2 |
  5 I
           2 |
                          4 |
                               16 |
                                      1 |
                                               -1
  6 |
           2 |
                    3 |
                          9 |
                               9 |
                                      1 |
                                               0
           2 |
  7 |
                    4 |
                          6 |
                                8 |
                                      1 |
                                               1
  8 |
           2 |
                    5 I
                          5 |
                               -1 |
                                      0 |
  9 I
           3 |
                    1 |
                          3 |
                               5 |
                                      1 l
                                               0
 10 |
           3 |
                   2 |
                         6 l
                               11 |
                                      1 |
                                               1
 11 |
           3 |
                   3 |
                         11 |
                               12 |
                                     -1 |
           3 |
                   4 |
                         10 |
 12 |
                               10 |
                                     1 |
                                               1
 13 |
           3 |
                   5 | 5 | -1 |
                                      0 |
                                                2
(13 rows)
```

One to Many \P

This signature finds the sset of disjoint paths from the start_vid to each one of the end_vid on a directed graph when directed flag is missing or is set to true.

- on an undirected graph when directed flag is set to false.
- The result is equivalent to the union of the results of the one to one pgr_edgeDisjointPaths.
- The extra end_vid in the result is used to distinguish to which path it belongs.

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vids, directed)
RETURNS SET OF (seq, path_id, path_seq, end_vid, node, edge, cost, agg_cost)
OR EMPTY SET
```

2	1	2	4	6 I	9	1	1
3	1	3	4	9	16	1	2
4	1	4	4	4	-1	0	3
5	2	1	5	3	2	1	0
6 l	2	2	5	2	4	1	1
7	2	3	5	5 I	-1	0	2
8	3	1	5	3	5	1	0
9	3	2	5	6	8	1	1
10	3	3	5	5	-1	0	2
11	4	1	10	3	2	1	0
12	4	2	10	2	4	1	1
13	4	3	10	5 l	10	1	2
14	4	4	10	10	-1	0	3
(14 rows)							

Many to One¶

This signature finds the set of disjoint paths from each one of the start_vid in start_vids to on a directed graph when directed flag is missing or is set to true.

- on an undirected graph when directed flag is set to false.
- The result is equivalent to the union of the results of the one to one pgr_edgeDisjointPaths.
- The extra start_vid in the result is used to distinguish to which path it belongs.

```
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vid, directed)
RETURNS SET OF (seq, path_id, path_seq, start_vid, node, edge, cost, agg_cost)
OR EMPTY SET
```

```
SELECT * FROM pgr_edgeDisjointPaths(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[3, 6], 5
);
seq | path_id | path_seq | start_vid | node | edge | cost | agg_cost
  1 |
                                                   2 |
                                                                      0
   2 |
             1 |
                        2 |
                                    0 |
                                            2 |
                                                   4 |
                                                          1 |
                                                                      1
  3 |
                        3 |
                                            5 |
                                                  -1 |
                                                          0 |
                                                                      2
             1 |
                                    0 |
   4 |
             2 |
                        1 |
                                    1 |
                                            3 |
                                                   5 |
                                                                      0
                                                          1 |
   5 |
             2 |
                        2 |
                                    1 |
                                            6 |
                                                   8 |
                                                          1 |
                                                                      1
             2 |
                        3 |
   6 I
                                    1 |
                                            5 |
                                                  -1 |
                                                                      2
```

7	3	1	2	6 I	8	1	0
8	3	2	2	5	-1	0	1
9	4	1	3	6 I	9	1	0
10	4	2	3	9	16	1	1
11	4	3	3	4	3	1	2
12	4	4	3	3	2	1	3
13	4	5	3	2	4	1	4
14	4	6 l	3	5	-1	0	5
(14 rows)							

Many to Many¶

This signature finds the set of disjoint paths from each one of the start_vid in start_vids to on a directed graph when directed flag is missing or is set to

- on an undirected graph when directed flag is set to false.
- The result is equivalent to the union of the results of the one to one pgr_edgeDisjointPaths.
- The extra start_vid and end_vid in the result is used to distinguish to which path it belongs.

```
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vids, directed)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost)
OR EMPTY SET
```

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[3, 6], ARRAY[4, 5, 10]
);
seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
  1 |
                     1 |
                                0 |
           1 |
                                         4 |
                                                3 |
                                                      5 |
                                                             1 |
                                                                       0
  2 |
           1 |
                     2 |
                                0 |
                                         4 |
                                                6 |
                                                      9 |
                                                             1 |
                                                                       1
  3 |
           1 |
                     3 |
                                0 |
                                         4 |
                                               9 |
                                                     16 l
                                                             1 |
                                                                       2
  4 |
                     4 |
                                0 |
                                         4 |
                                               4 |
                                                     -1 |
                                                             0 |
                                                                       3
           1 |
                                                     2 |
  5 I
           2 |
                     1 |
                                         5 I
                                                3 |
                                                                       0
                                1 |
                                                             1 |
  6 I
           2 |
                     2 |
                                1 |
                                         5 I
                                                2 |
                                                      4 |
                                                                       1
                                                             1 |
  7 |
           2 |
                     3 I
                                1 |
                                         5 l
                                               5 I
                                                     -1 |
                                                             0 |
                                                                       2
                                2 |
           3 |
                                         5 |
                                                3 |
                                                                       0
  8 |
                     1 |
                                                     5 I
                                                             1 |
  9 |
           3 |
                     2 |
                                2 |
                                         5 |
                                                6 |
                                                      8 |
                                                             1 |
                                                                       1
 10 |
           3 |
                     3 |
                                2 |
                                         5 |
                                                5 |
                                                     -1 |
                                                             0 |
                                                                       2
                                3 |
 11 |
           4 |
                     1 |
                                        10 |
                                                3 |
                                                      2 |
                                                             1 |
```

Description of the Signatures \P

Type

Column

Description of the edges_sql query for dijkstra like functions \P edges_sql:

Default

an SQL query, which should return a set of rows with the following columns: $\,$

id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	c ent -numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Description

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Column	Type	Default	Description
$\overline{\mathrm{sql}}$	TEXT		SQL query as desc
start_vid	BIGINT		Identifier of the stapath.
$start_vid$	sARRAY[BIGINT]]	Array of identifiers
end_vid	BIGINT		Identifier of the enepath.
end_vids	ARRAY[BIGINT]]	Array of identifiers
directed	BOOLEAN	true	When true (DirectedWhen false considered as

Description of the return values for a path \P

Returns set of (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)

Column	Туре
\overline{seq}	INT
$\operatorname{path_id}$	INT
path _seq	INT
start_vid	BIGINT
end_vid	BIGINT
node	BIGINT

Column	Type
edge	BIGINT
cost	FLOAT
agg_cost	FLOAT

See Also \P

• Flow - Family of functions

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Flow Functions General Information¶ Characteristics

- The graph is **directed**.
- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and $\bf EMPTY~\bf SET$ is returned.
 - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
 - Edges with zero flow are omitted.
- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by $pgr_maxFlow$ when executed with the same parameters and can be calculated:
 - By aggregation of the outgoing flow from the sources
 - By aggregation of the incoming flow to the targets

pgr_maxFlow is the maximum Flow and that maximum is guaranteed
to be the same on the functions pgr_pushRelabel, pgr_edmondsKarp,
pgr_boykovKolmogorov, but the actual flow through each edge may vary.

Problem definition¶ A flow network is a directed graph where each edge has a capacity and a flow. The flow through an edge must not exceed the capacity of the edge. Additionally, the incoming and outgoing flow of a node must be equal except the for source which only has outgoing flow, and the destination(sink) which only has incoming flow.

Maximum flow algorithms calculate the maximum flow through the graph and the flow of each edge.

The maximum flow through the graph is guaranteed to be the same with all implementations, but the actual flow through each edge may vary. Given the following query:

```
\label{lem:condition} $$  \operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{\colored}_{\operatorname{
```

The weighted directed graph, $\backslash (G(V,E)\backslash)$, is defined as:

- the set of vertices $\setminus (V \setminus)$
- the set of edges $\backslash (E \backslash)$

Maximum flow problem

Given:

- $\backslash (G(V,E)\backslash)$
- \(source_vertex \in V\) the source vertex
- \(\sink\\ vertex \in V\\) the sink vertex

Then:

Where:

\(\boldsymbol{\Phi}\) is a subset of the original edges with their residual capacity and flow. The maximum flow through the graph can be obtained by aggregating on the source or sink and summing the flow from/to it. In particular:

- $\setminus (id_i = i \setminus)$
- $\ensuremath{\mbox{(edge_id=id_i\mbox{)} in edges_sql}}$

See Also¶

 $\bullet \ \ https://en.wikipedia.org/wiki/Maximum_flow_problem$

Indices and tables

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$pgr_labelGraph - Experimental\P$

Name¶ pgr_labelGraph — Locates and labels sub-networks within a network which are not topologically connected.

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis¶ Must be run after pgr_createTopology(). No use of geometry column. Only id, source and target columns are required.

The function returns:

- OK when a column with provided name has been generated and populated successfully. All connected edges will have unique similar integer values. In case of rows_where condition, non participating rows will have -1 integer values.
- FAIL when the processing cannot be finished due to some error. Notice will be thrown accordingly.
- rows_where condition generated 0 rows when passed SQL condition has not been fulfilled by any row.

varchar pgr_labelGraph(text, text, text, text, text, text)

Description¶ A network behind any routing query may consist of subnetworks completely isolated from each other. Possible reasons could be:

- An island with no bridge connecting to the mainland.
- An edge or mesh of edges failed to connect to other networks because of human negligence during data generation.
- The data is not properly noded.
- Topology creation failed to succeed.

pgr_labelGraph() will create an integer column (with the name provided by the user) and will assign same integer values to all those edges in the network which are connected topologically. Thus better analysis regarding network structure is possible. In case of rows_where condition, non participating rows will have -1 integer values.

Prerequisites: Must run pgr_createTopology() in order to generate source and target columns. Primary key column id should also be there in the network table.

Function accepts the following parameters:

edge table:

text Network table name, with optional schema name.

id:

text Primary key column name of the network table. Default is id.

source

text Source column name generated after pgr_createTopology(). Default is source.

```
target:
```

text Target column name generated after pgr_createTopology(). Default is target.

subgraph:

text Column name which will hold the integer labels for each sub-graph. Default is subgraph.

 $rows_where:$

 ${\tt text}$ The SQL where condition. Default is ${\tt true},$ means the processing will be done on the whole table.

Example Usage¶ The sample data, has 3 subgraphs.

```
SET client_min_messages TO WARNING;
SET

SELECT pgr_labelGraph('edge_table', 'id', 'source', 'target', 'subgraph');
pgr_labelgraph

OK
(1 row)

SELECT DISTINCT subgraph FROM edge_table ORDER BY subgraph;
subgraph

1
2
3
(3 rows)
```

See Also¶

• pgr_createTopology to create the topology of a table based on its geometry and tolerance value.

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Components - Family of functions¶ Warning

Experimental functions

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 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting
- pgr_connectedComponents Experimental Return the connected components of an undirected graph.
- pgr_strongComponents Experimental Return the strongly connected components of a directed graph.
- pgr_biconnectedComponents Experimental Return the biconnected components of an undirected graph.
- pgr_articulationPoints Experimental Return the articulation points of an undirected graph.
- pgr_bridges Experimental Return the bridges of an undirected graph.

pgr_connectedComponents - Experimental¶ pgr_connectedComponents — Return the connected components of an undirected graph using a DFS-based approach. In particular, the algorithm implemented by Boost.Graph.



Boost Graph Inside

Warning

Experimental functions

- They are not officially of the current release.
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 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis

A connected component of an undirected graph is a set of vertices that are all reachable from each other. This implementation can only be used with an undirected graph.

Characteristics

The main Characteristics are:

- Components are described by vertices
- The returned values are ordered:
 - component ascending
 - node ascending
- Running time: $\langle (O(V + E) \rangle)$

$Signatures \P$

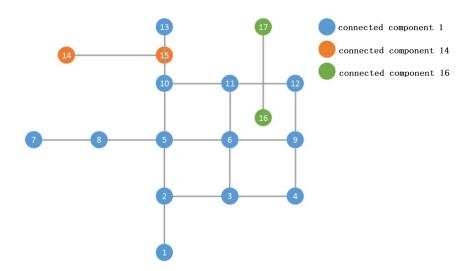
```
pgr_connectedComponents(edges_sql)
```

```
RETURNS SET OF (seq, component, n_seq, node)
OR EMPTY SET
```

The signature is for a **undirected** graph.

SELECT * FROM pgr_connectedComponents(
 'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);

seq	component	١	n_seq	١	node
+		+-		+-	
1	1		1		1
2	1	1	2		2
3	1		3		3
4	1	1	4	1	4
5 l	1	1	5	1	5
6 I	1	1	6	1	6
7	1	1	7	1	7
8	1	1	8	1	8
9	1	1	9	1	9
10	1	1	10	1	10
11	1	1	11	1	11
12	1	1	12	1	12
13	1	1	13	1	13
14	14	1	1	1	14
15	14	1	2	1	15
16	16	1	1	1	16
17	16	1	2	1	17
(17 row	rs)				



Description of the Signatures \P

Description of the edges_sql query for components functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns: $\,$

Column	Type	Default	Description
id	ANY-INTEGE	ER	Identifier of the edge.
source	ANY-INTEGE	ER	Identifier of the first end point vertex edge.
target	ANY-INTEGE	ER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMER	ICAL	Weight of the edge (source, target)
			• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	cant-numer:	ICAL	Weight of the edge (target, source),
			• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

 $SMALLINT,\,INTEGER,\,BIGINT,\,REAL,\,FLOAT$

Description of the parameters of the signatures \P

Parameter	Type	Default	Description
edges_sql	TEXT		SQL query as described above.

Description of the return values for connected components and strongly connected components \P

Returns set of (seq, component, n_seq, node)

Column	Type	Description
\overline{seq}	INT	Sequential value starting f
compone	ntBIGINT	Component identifier. It is identifier in the componen
n_seq	INT	It is a sequential value sta

Column	Туре	Description
node	BIGINT	Identifier of the vertex.

See Also \P

- $\bullet \ \, http://en.wikipedia.org/wiki/Connected_component_\%28graph_theory\%29$
- The queries use the *Sample Data* network.

Indices and tables

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pgr_strongComponents - Experimental¶ pgr_strongComponents — Return the strongly connected components of a directed graph using Tarjan's algorithm based on DFS. In particular, the algorithm implemented by Boost.Graph.



Boost Graph Inside

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.

- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

Synopsis¶

A strongly connected component of a directed graph is a set of vertices that are all reachable from each other. This implementation can only be used with a directed graph.

$Characteristics \P$

The main Characteristics are:

- Components are described by vertices
- The returned values are ordered:
 - component ascending
 - node ascending
- Running time: $\langle O(V + E) \rangle$

$Signatures \P$

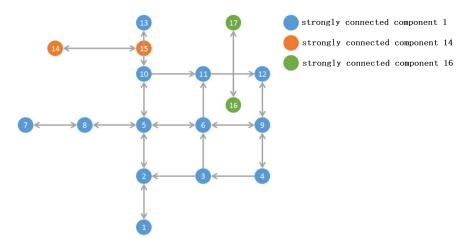
The signature is for a directed graph.

Example:

```
SELECT * FROM pgr_strongComponents(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);
```

seq	component		n_seq		node
 1	 1	+- I	1	r- I	1
Τ Ι	1	ı	1	ı	1
2	1		2		2
3	1		3		3
4	1		4		4
5 I	1		5		5
6	1		6		6
7	1		7		7
8	1		8	١	8
9	1		9		9

10	1	1	10	1	10
11	1	1	11	1	11
12	1	1	12	1	12
13	1	1	13	1	13
14	14	1	1	1	14
15	14	1	2	1	15
16	16	1	1	1	16
17	16	1	2	1	17
(17 rows)					



Description of the Signatures \P

Description of the edges_sql query for components functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)

• When negative: edge (source, to not exist, therefore it's not part graph.

Column Type Default	Description
reverse_cant-numerical	Weight of the edge (target, source),
	• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter	Type	Default	Description
$\overline{\mathrm{edges_sql}}$	TEXT		SQL query as described above.

Description of the return values for connected components and strongly connected components \P

Returns set of (seq, component, n_seq, node)

Column	Туре	Description
$\overline{\text{seq}}$	INT	Sequential value starting f
componer	ntBIGINT	Component identifier. It is identifier in the componen
n_seq	INT	It is a sequential value sta
\mathbf{node}	BIGINT	Identifier of the vertex.

See Also \P

- $\bullet \ \, \text{http://en.wikipedia.org/wiki/Strongly_connected_component}\\$
- ullet The queries use the $Sample\ Data$ network.

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pgr_biconnectedComponents - Experimental¶ pgr_biconnectedComponents
Return the biconnected components of an undirected graph. In particular, the algorithm implemented by Boost.Graph.



Boost Graph Inside

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis¶

The biconnected components of an undirected graph are the maximal subsets of vertices such that the removal of a vertex from particular component will not disconnect the component. Unlike connected components, vertices may belong to multiple biconnected components. Vertices can be present in multiple biconnected components, but each edge can only be contained in a single biconnected component. So, the output only has edge version.

This implementation can only be used with an undirected graph.

$Characteristics \P$

The main Characteristics are:

- Components are described by edges
- The returned values are ordered:
 - component ascending
 - edge ascending
- Running time: $\langle (O(V + E) \rangle)$

$Signatures \P$

OR EMPTY SET

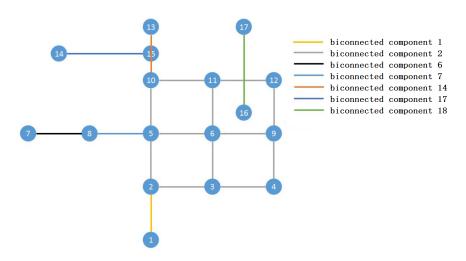
```
pgr_biconnectedComponents(edges_sql)
RETURNS SET OF (seq, component, n_seq, edge)
```

The signature is for a **undirected** graph.

Example:

```
SELECT * FROM pgr_biconnectedComponents(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);
```

seq	component	1	n_seq		edge
1	1	l	1		1
2	2	l	1	1	2
3	2	١	2	1	3
4	2	١	3	1	4
5	2	١	4	1	5
6 l	2	١	5	1	8
7	2		6		9
8	2	l	7		10
9	2		8		11
10	2	l	9		12
11	2	l	10		13
12	2	١	11		15
13	2	l	12		16
14	6	١	1		6
15	7	l	1		7
16	14		1		14
17	17		1		17
18	18		1		18
(18 row	rs)				



Description of the Signatures \P

Type

Description of the edges_sql query for components functions \P edges_sql:

Default

an SQL query, which should return a set of rows with the following columns:

ANY-INTEGER	Identifier of the edge.
ANY-INTEGER	Identifier of the first end point vertex edge.
ANY-INTEGER	Identifier of the second end point vert edge.
ANY-NUMERICAL	Weight of the edge (source, target)
c any -numerical	• When negative: edge (source, ta not exist, therefore it's not part graph. Weight of the edge (target, source),
	• When negative: edge (target, so not exist, therefore it's not part graph.
	ANY-INTEGER ANY-INTEGER

Description

Where:

Column

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter	Type	Default	Description
edges_sql	TEXT		SQL query as described above.

Description of the return values for biconnected components, connected components (edge version) and strongly connected components \P

Returns set of (seq, component, n_seq, edge)

Column	Туре	Description
\overline{seq}	INT	Sequential value starting f
compone	ntBIGINT	Component identifier. It is identifier in the componen
n_seq	INT	It is a sequential value sta
\mathbf{edge}	BIGINT	Identifier of the edge.

See Also \P

- $\bullet \ \, http://en.wikipedia.org/wiki/Biconnected_component$
- The queries use the $Sample\ Data$ network.

Indices and tables

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pgr_articulationPoints - Experimental¶ pgr_articulationPoints - Return the articulation points of an undirected graph. In particular, the algorithm implemented by Boost.Graph.



Boost Graph Inside

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis¶

Those vertices that belong to more than one biconnected component are called articulation points or, equivalently, cut vertices. Articulation points are vertices whose removal would increase the number of connected components in the graph. This implementation can only be used with an undirected graph.

Characteristics¶

The main Characteristics are:

- The returned values are ordered:
 - node ascending
- Running time: $\langle O(V + E) \rangle$

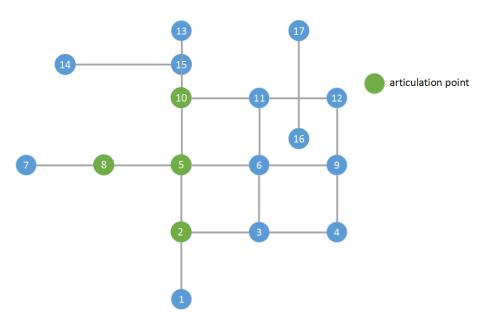
Signatures

```
pgr_articulationPoints(edges_sql)
```

RETURNS SET OF (seq, node)
OR EMPTY SET

The signature is for a **undirected** graph.

Example:



Description of the Signatures \P

Description of the edges_sql query for components functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns: $\,$

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.

Column	Type	Default	Description	
$\overline{\mathrm{cost}}$	ANY-NUMER	ICAL	Weight of the e	dge (source, target)
morrowco	CANW NIMED	T.C.A.I.	not exist, graph.	gative: edge (source, ta therefore it's not part
reverse_	_c ant -numer	TOME		dge (target, source), gative: edge (target, so
				therefore it's not part

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter	Type	Default	Description
edges_sql	TEXT		SQL query as described above.

Description of the return values for articulation points \P

Returns set of (seq, node)

Column	Type	Description	
\overline{seq}	INT	Sequential value starting from 1.	
node	BIGINT	Identifier of the vertex.	

See Also \P

- $\bullet \ \, http://en.wikipedia.org/wiki/Biconnected_component$
- $\bullet\,$ The queries use the $Sample\ Data$ network.

Indices and tables

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- Search Page

pgr_bridges - Experimental¶ pgr_bridges - Return the bridges of an undirected graph.



Boost Graph Inside

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

$Synopsis\P$

A bridge is an edge of an undirected graph whose deletion increases its number of connected components. This implementation can only be used with an undirected graph.

Characteristics

The main Characteristics are:

- The returned values are ordered:
 - edge ascending

• Running time: $\langle (O(E * (V + E)) \rangle)$

${\bf Signatures} \P$

```
pgr_bridges(edges_sql)
```

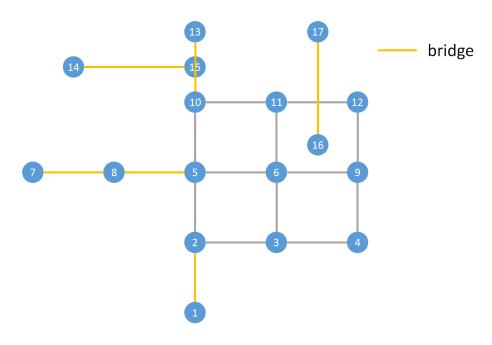
RETURNS SET OF (seq, node)
OR EMPTY SET

The signature is for a ${\bf undirected}$ graph.

 ${\bf Example:}$

```
SELECT * FROM pgr_bridges(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);
seq | edge
```

	. 1	•	0
		+	
	1	1	1
	2	1	6
	3		7
	4	1	14
	5		17
	6	1	18
(6	ro	ows)	



Description of the Signatures \P

Description of the edges_sql query for components functions \P

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	c4N Y-NUMERI€IAL	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\, INTEGER,\, BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter	Type	Default	Description
edges_sql	TEXT		SQL query as described above.

Description of the return values for bridges \P

Returns set of (seq, node)

Column	Type	Description
\overline{seq}	INT	Sequential value starting from 1.
edge	BIGINT	Identifier of the edge.

See Also¶

- http://en.wikipedia.org/wiki/Bridge_%28graph_theory%29
- The queries use the Sample Data network.

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The problem definition¶ Connected components

A connected component of an undirected graph is a set of vertices that are all reachable from each other.

Notice: This problem defines on an undirected graph.

Given the following query:

```
\label{eq:connectedComponentsV} $$ pgr\_connectedComponentsV(\(sql\)) $$ where \(sql = \(id_i, source_i, target_i, cost_i, reverse\_cost_i)\) $$ and
```

- \(source = \bigcup source_i\),\(target = \bigcup target_i\),
- The graphs are defined as follows:

The weighted undirected graph, $\langle (G(V,E) \rangle)$, is definied by:

- the set of vertices $\(V\)$ $\(V = source \setminus cup target\)$
- the set of edges $\backslash (E \backslash)$

Given:

• $\backslash (G(V,E)\backslash)$

Then:

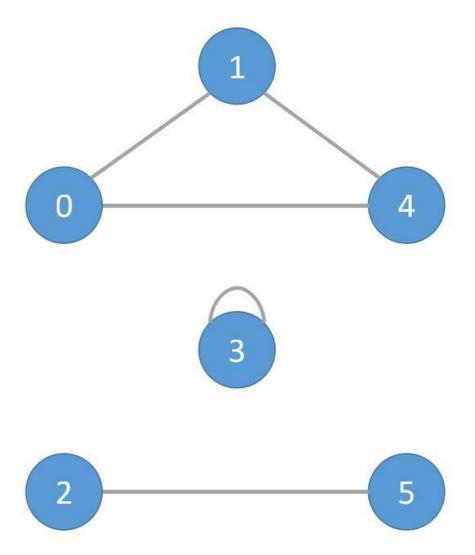
```
\(\boldsymbol{\pi} = \{(component_i, n\_seq_i, node_i)\}\)
```

where: • $\langle \text{component}_i = \min \{ \text{node}_j \mid \text{node}_j \in \text{component}_i \} \rangle$

- (n_seq_i) is a sequential value starting from 1 in a component.
- \(node_i \in component_i\)
- The returned values are ordered:
 - component ascending
 - node ascending

Example: • The first component is composed of nodes 0, 1 and 4.

- The second component is composed of node 3.
- The third component is composed of nodes 2 and 5.



Strongly connected components

A strongly connected component of a directed graph is a set of vertices that are all reachable from each other.

 $\bf Notice:$ This problem defines on a directed graph.

Given the following query:

```
pgr\_strongComponentsV(\backslash(sql\backslash)) where \backslash(sql=\backslash\{(id\_i,\,source\_i,\,target\_i,\,cost\_i,\,reverse\backslash\_cost\_i)\backslash\}\backslash) and
```

```
• (source = \bigcup source_i),
```

•
$$\langle (target = \langle bigcup \ target_i \rangle),$$

The graphs are defined as follows:

The weighted directed graph, $\backslash (G_d(V,E) \backslash)$, is definied by:

- the set of vertices $\langle (V \rangle)$
 - $(V = source \cup target \cup {start_{vid}} \cup {end_{vid}}))$
- the set of edges $\backslash (E \backslash)$

Given:

• $\backslash (G(V,E)\backslash)$

Then:

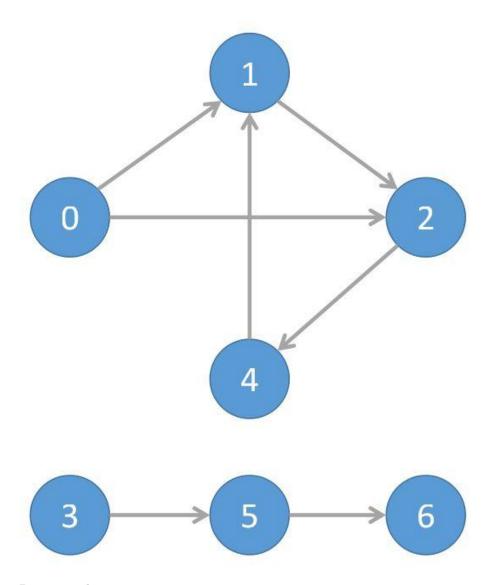
```
\( \boldsymbol{\pi} = \ (\component_i, n\eq_i, node_i) \)
```

where: • \((component_i = \min {node_j | node_j \in component_i}\))

- $\langle (n \subseteq seq_i) \rangle$ is a sequential value starting from 1 in a component.
- \(node_i \in component_i\)
- The returned values are ordered:
 - component ascending
 - node ascending

Example: • The first component is composed of nodes 1, 2 and 4.

- The second component is composed of node 0.
- The third component is composed of node 3.
- The fourth component is composed of node 5.
- The fifth component is composed of node 6.



Biconnected components

The biconnected components of an undirected graph are the maximal subsets of vertices such that the removal of a vertex from particular component will not disconnect the component. Unlike connected components, vertices may belong to multiple biconnected components. Vertices can be present in multiple biconnected components, but each edge can only be contained in a single biconnected component. So, the output only has edge version.

Notice: This problem defines on an undirected graph.

Given the following query:

```
\label{eq:content} $\operatorname{pgr\_biconnectedComponents}(\(\operatorname{sql}))$ where $$\(\operatorname{id_i}, \operatorname{source_i}, \operatorname{target_i}, \operatorname{cost_i}, \operatorname{reverse}_\operatorname{cost_i})\)$ and
```

- $\langle \text{source} = \text{bigcup source}_i \rangle$,
- \(target = \bigcup target_i\),

The graphs are defined as follows:

The weighted undirected graph, $\backslash (G(V,E) \backslash)$, is definied by:

- the set of vertices $\langle (V \rangle)$
 - $(V = source \setminus cup target)$
- the set of edges $\backslash (E \backslash)$

Given:

• $\backslash (G(V,E) \backslash)$

Then:

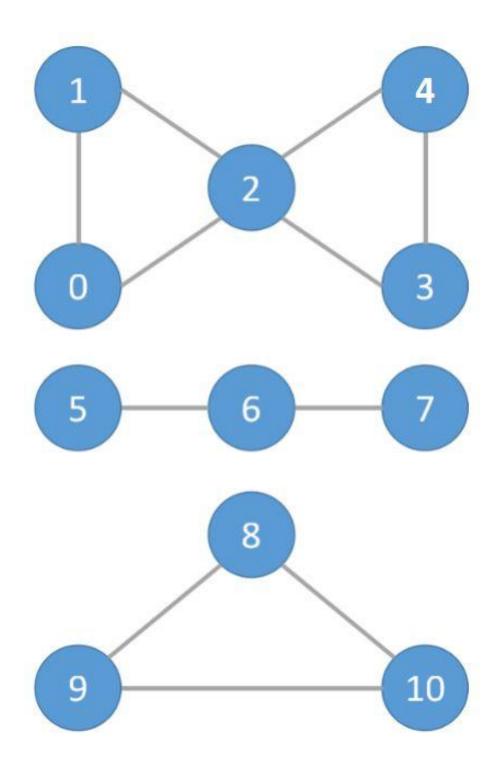
```
\( \boldsymbol{ pi} = \ (\component_i, n\eq_i, node_i) \)
```

where: • $\langle \text{component}_i = \min \{ \text{node}_j \mid \text{node}_j \mid \text{node}_i \} \rangle$

- $\langle (n \subseteq i \rangle)$ is a sequential value starting from 1 in a component.
- \(edge_i \in component_i\)
- The returned values are ordered:
 - component ascending
 - edge ascending

Example: • The first component is composed of edges 1-2, 0-1 and 0-2.

- The second component is composed of edges 2-4, 2-3 and 3-4.
- $\bullet\,$ The third component is composed of edge $5\,$ $\,6.$
- The fourth component is composed of edge 6 7.
- The fifth component is composed of edges 8 9, 9 10 and 8 10.



Articulation Points

Those vertices that belong to more than one biconnected component are called articulation points or, equivalently, cut vertices. Articulation points are vertices whose removal would increase the number of connected components in the graph.

Notice: This problem defines on an undirected graph.

Given the following query:

- $\setminus (\text{source} = \setminus \text{bigcup source}_i \setminus),$
- $\t (target = \bigcup target_i\),$

The graphs are defined as follows:

The weighted undirected graph, $\langle (G(V,E) \rangle)$, is definied by:

- the set of vertices $\setminus (V \setminus)$
 - $(V = source \setminus cup target)$
- the set of edges $\backslash (E \backslash)$

Given:

• $\backslash (G(V,E)\backslash)$

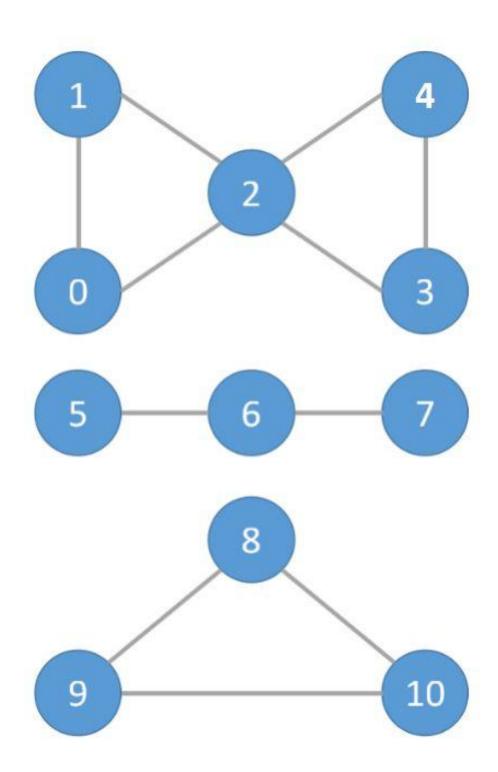
Then:

 $\(\big) = \left(\big) = \left(\big)$

where: • \(node_i\) is an articulation point.

- The returned values are ordered:
 - $-\ {\rm node}\ {\rm ascending}$

Example: • Articulation points are nodes 2 and 6.



Bridges

A bridge is an edge of an undirected graph whose deletion increases its number of connected components.

Notice: This problem defines on an undirected graph.

Given the following query:

```
\label{eq:cost_invariant} $\operatorname{pgr\_bridges}(\sql\sl)$ where $$ \sline = \sline i, source_i, target_i, cost_i, reverse\sline scot_i)\sline scot_i, source_i, target_i, cost_i, reverse\sline scot_i)\sline scot_i, source_i, target_i, cost_i, reverse\sline scot_i)\sline scot_i, source_i, target_i, cost_i, reverse\sline scot_i, source_i, target_i, cost_i, reverse\sline scot_i, source_i, source_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot_i, scot
```

- $\bullet \ \ \backslash (source = \backslash bigcup \ source_i \backslash),$
- \((target = \bigcup target i\),

The graphs are defined as follows:

The weighted undirected graph, $\backslash (G(V,E)\backslash)$, is definied by:

- the set of vertices \((V\)\)
 \((V = source \cup target\)\)
- the set of edges $\backslash (E \backslash)$

Given:

• $\backslash (G(V,E)\backslash)$

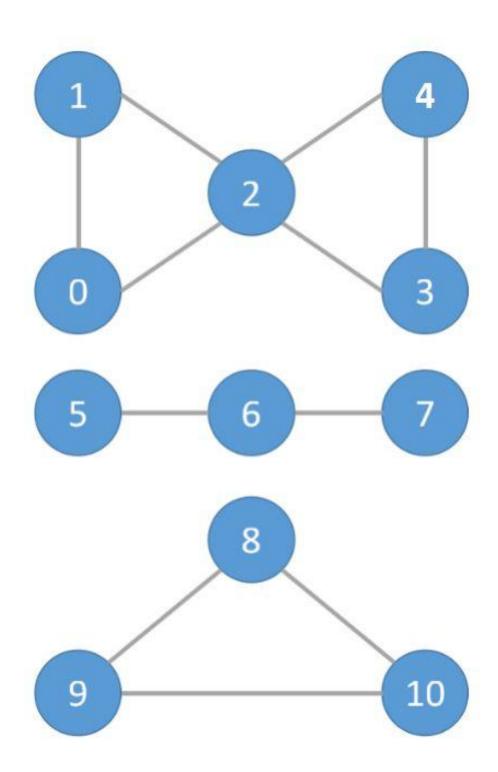
Then:

```
\( \big) = \left( \left( \frac{i}{j} \right) = \left( \frac{i}{j} \right) \right)
```

where: $\cdot (edge_i)$ is an edge.

- The returned values are ordered:
 - edge ascending

Example: • Bridges are edges $5 \leftarrow -> 6$ and $6 \leftarrow -> 7$.



Description of the edges_sql query for components functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns: $\,$

Column	Type Default	Description
$\overline{\mathrm{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
$reverse_$	_c ant -numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Parameter	Type	Default	Description
edges_sql	TEXT		SQL query as described above.

Description of the return values for connected components and strongly connected components \P

Returns set of (seq, component, n_seq, node)

Column	Туре	Description
$\overline{\text{seq}}$	INT	Sequential value starting f
componentBIGINT		Component identifier. It is identifier in the componen
n_seq	INT	It is a sequential value sta
\mathbf{node}	BIGINT	Identifier of the vertex.

Description of the return values for biconnected components, connected components (edge version) and strongly connected components \P

Returns set of (seq, component, n_seq, edge)

componentBIGINT Component identifier. It is identifier in the component			
componentBIGINT Component identifier. It is identifier in the component identifier in the component identifier. It is a sequential value state of the component identifier. It is a sequential value state of the component identifier. It is a sequential value state of the component identifier. It is a sequential value state of the component identifier. It is a sequential value state of the component identifier. It is a sequential value state of the component identifier in the component identifier in the component identifier.	Column	Type	Description
n_seq INT identifier in the component It is a sequential value state.	\overline{seq}	INT	Sequential value starting f
- •	componentBIGINT		Component identifier. It is identifier in the componen
edge BIGINT Identifier of the edge.	n_{seq}	INT	It is a sequential value sta
	edge	BIGINT	Identifier of the edge.

Description of the return values for articulation points \P

Returns set of (seq, node)

Column	Type	Description
$\overline{\text{seq}}$	INT	Sequential value starting from 1.
node	BIGINT	Identifier of the vertex.

Description of the return values for bridges \P

Returns set of (seq, node)

Column	Type	Description
\overline{seq}	INT	Sequential value starting from 1.
edge	BIGINT	Identifier of the edge.

See Also¶ Indices and tables

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pgr_gsoc_vrppdtw - Experimental¶

Name¶ pgr_gsoc_vrppdtw — Returns a solution for Pick and Delivery with time windows Vehicle Routing Problem

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Signature Summary¶

```
pgr_gsoc_vrppdtw(sql, vehicle_num, capacity)
RETURNS SET OF pgr_costResult[]:
```

Signatures Complete signature

```
pgr_gsoc_vrppdtw(sql, vehicle_num, capacity)
Returns set of pgr_costResult[]:
```

Example: Show the id1

```
SELECT DISTINCT(id1) FROM pgr_gsoc_vrppdtw(
    'SELECT * FROM customer ORDER BY id', 25, 200)

ORDER BY id1;
id1
----

1
2
3
4
5
6
7
8
9
10
(10 rows)
```

$\textbf{Description of the Signatures} \P \quad \text{Description of the sql query} \P$

Column	Type	Description	
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the customer.	
		• A value of 0 identifies the starting location	
x	ANY-NUMERICA	AK coordinate of the location.	
y	ANY-NUMERICAN coordinate of the location.		
demand	${\tt ANY-NUMERICA}{\tt H}{\tt ow}$ much is added / removed from the vehicle.		
	Negative value is a delivery,Positive value is a pickup,		
${\bf openTime}$	ANY-NUMERICAIThe time relative to 0, when the customer opens.		
${\bf close Time}$	ANY-NUMERICAIThe time relative to 0, when the customer closes.		
serviceTimeANY-NUMERICAThe duration of the loading / unloading.			
pIndex	ANY-INTEGER	Value used when the current customer is a Delivery to find the corresponding Pickup	
dIndex	ANY-INTEGER	Value used when the current customer is a Pickup to find the corresponding Delivery	

Description of the parameters of the signatures \P

Column	Type	Description
$\overline{\mathrm{sql}}$	TEXT	SQL query as described above.
vehicle_num	INTEGER	Maximum number of vehicles in the result. (currently is ignored)
capacity	INTEGER	Capacity of the vehicle.

Description of the result \P

RETURNS SET OF pgr_costResult[]:

Column	Туре	Description
\overline{seq}	INTEGER	Sequential value starting from 1.
id1	INTEGER	Current vehicle identifier.
id2	INTEGER	Customer identifier.
cost	FLOAT	Previous cost plus travel tim when id2 = 0 for the same id1, then has the current id1

$\textbf{Examples} \P \quad \text{Example: Total number of rows returned}$

SELECT count(*) FROM pgr_gsoc_vrppdtw(

```
'SELECT * FROM customer ORDER BY id', 25, 200);
 count
   126
(1 row)
Example: Results for only id1 values: 1, 5, and 9
SELECT * FROM pgr_gsoc_vrppdtw(
    'SELECT * FROM customer ORDER BY id', 25, 200)
    WHERE id1 in (1, 5, 9);
 seq | id1 | id2 |
                         cost
   1 |
         1 |
              0 |
   2 |
         1 | 13 | 120.805843601499
   3 |
        1 | 17 | 214.805843601499
   4 |
         1 | 18 | 307.805843601499
```

```
19 | 402.805843601499
              15 | 497.805843601499
   7 |
              16 | 592.805843601499
   8
              14 | 684.805843601499
   9
              12 | 777.805843601499
  10 |
              50 | 920.815276724293
              52 | 1013.97755438446
  11 |
  12 |
              49 | 1106.97755438446
  13 |
              47 | 1198.97755438446
  14 I
         1 |
               0 | 1217.00531076178
 57 I
         5 |
               0 |
         5 |
 58 |
              90 | 110.615528128088
 59 |
              87 | 205.615528128088
 60 I
         5 I
              86 | 296.615528128088
 61 |
         5 |
              83 | 392.615528128088
 62 |
              82 | 485.615528128088
              84 | 581.446480022934
 63 |
         5 |
                    674.27490714768
 64 |
         5 |
              85 |
                    767.27490714768
 65 I
              88 |
  66 I
              89 | 860.103334272426
 67
              91 |
                    953.70888554789
         5 I
               0 | 976.069565322888
 68 I
               0 |
 105 |
         9 |
 106 |
              67 | 102.206555615734
              65 | 193.206555615734
 107 |
         9 |
108 |
         9 |
              63 | 285.206555615734
              62 | 380.206555615734
109 |
         9 |
110 |
         9 |
              74 | 473.206555615734
111 |
         9 |
              72 | 568.206555615734
112 I
         9 |
              61 | 661.206555615734
113 |
         9 |
             64 | 663.206555615734
         9 | 102 | 753.206555615734
115 |
         9 I
              68 | 846.206555615734
         9 |
               0 | 866.822083743822
116 |
(38 rows)
```

See Also¶

- The examples use Pick & Deliver Data
- $\bullet \ \, {\rm http://en.wikipedia.org/wiki/Vehicle_routing_problem}$

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pgr_vrpOneDepot - Experimental¶ Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

No documentation available

Example: ¶ Current Result ¶

```
BEGIN;
BEGIN
SET client_min_messages TO NOTICE;
SET
SELECT * FROM pgr_vrpOneDepot(
    'SELECT * FROM vrp orders'.
    'SELECT * FROM vrp_vehicles',
    'SELECT * FROM vrp_distance',
    1);
 oid | opos | vid | tarrival | tdepart
          1 |
                5 I
 -1 l
                           0 |
                                      0
          2 |
                           0 |
  66 l
                5 |
                                      0
  25 I
          3 |
                5 I
                           0 |
                                      0
          4 |
                5 I
                           0 |
  21 |
                                      0
  84 I
          5 | 5 |
                           0 |
                                      0
          6 |
               5 |
                           0 |
  50 l
                                      0
  49 |
          7 |
                5 |
                           0 |
                                      0
          8 I
                5 I
                           0 |
  24 I
```

22	9	5	0	0
20	10	5	0	1 0
				0
	12		11	l 21
84	13	5	30	l 45
24	14	5	71	l 81
	15		83	93
	17		114	124
50	18	5	131	141
21	19	5	144	l 154
25	20	5	158	168
	21		179	189
	22		234	234
	1		0	0
31	2	6	0	0
32	3	6	0	1 0
	4	6	0	Ι 0
	5		I 0	1 0
	6		0	0
	7		0	0
33	8	6	0	0
28	9	6	0	1 0
27	10	6	0	1 0
	11		15	25
			61	
	12			71
	13		78	88
31	14	6	97	107
35	15	6	112	122
27	16	6	134	144
	17			162
	18			206
	19		221	231
	20	6	238	238
-1	1	3	0	1 0
16	2	3	0	1 0
	3			. 0
	4		0	0
	J 5		0	0
17	l 6			0
15	7	3	0	1 0
13	8		0	Ι 0
	. 9			. 0
	1 10			1 0
	11			45
48	12	3	l 48	58

13	13	3	l 64	74
16	14	3	82	92
17	15	3	94	l 104
10	l 16	3	115	125
11	17		130	140
14			l 147	157
18				179
-1				219
-1				0
				1 0
				0
				0
				0
				0
				0
			•	1 0
		8	•	1 0
	10	8	•	l 44
40	11	8	l 49	59
39	12	8	61	l 85
41	13	8	J 90	l 100
42	14	8	111	121
44	l 15	8		l 141
				176
				208
				228
				l 0
				1 0
				l 0
				l 0
			•	l 0
			•	•
				•
			•	48
			•	65
				96
		•		145
		•		158
	12			175
				l 202
	14	1	222	222
				0
92	1 2	13	0	I 0
52	3	13	0	I 0
57	4	13	0	I 0
85	J 5		0	I 0
68				0

63	7	13	0	0
63	8	13	l 29	62
68	J 9	13	l 69	l 80
52	10	13	104	114
85	11	13	123	133
57	12	13	142	152
92	13	13	159	177
-1	14	13	189	189
-1	1	7	0	0
		7	0	0
29		1 7	0	0
		7	0	0
	5		0	1 0
			0	0
			51	61
			I 70	80
			85	95
			149	159
			162	172
		, , 7		217
		2		0
		2		1 0
		2		1 0
		2		1 0
		2		
				l 0 l 0
		•		
	•	2	•	77
	-	2	•	106
		2	•	121
	•	2	•	134
	•	2	•	165
	-	•	192	192
_	-		0	0
	-	14	•	0
		14		0
	•	14		0
96	•	•	0	0
			0	0
		14		44
			l 63	73
		•	l 83	93
	-	-	119	129
	-	•	154	164
	-	14	180	180
-1		15	0	0
67	1 2	15	0	0

73	3	15	l 0	0
95	4	15	0	1 0
82			0	0
72			I 0	. 0
73			I 27	1 40
72	8		50	75
82	9		91	101
95				
	10	•	114	124
67	11	•	144	154
-1	12	-	167	167
-1	•		0	0
78	-		0	0
26	-		0	0
87	-		0	1 0
23	5		l 0	0
87	6	11	J 32	J 97
23	7	11	118	128
78	8	11	l 149	l 160
26	9	11	l 172	l 182
-1	10	11	l 227	227
-1		4	Ι 0	I 0
60	2	4	Ι 0	Ι 0
59			Ι 0	Ι 0
100			Ι 0	Ι 0
54			I 0	0
60			l 42	52
100			I 74	87
54			103	113
59	9		153	163
-1	1 10		211	211
-1			0	0
86			l 0	I 0
90				
			0	0
65	•	•	0	0
53		•	0	0
53	l 6	10	25	62
65			82	92
		10		121
	9		140	154
	10	•	l 206	1 206
	1		0	0
	•	12	0	0
80		12	0	0
7	4	12	0	0
56	5	12	0	0
6	6	12	l 40	51

80	7	12	73	l 99
7	8	12	113	123
56	9	12	142	152
-1	10	12	166	l 166
-1	1	19	0	I 0
88	1 2	19	0	I 0
70	J 3	19	0	I 0
58	4	19	0	I 0
99	5	19	0	I 0
70	6	19	9	J 51
99	7	19		l 66
88	8	19	97	107
	9	19	125	l 135
-1	10	19	162	l 162
-1	1	17	0	I 0
75	2	17	0	0
98	3	17	0	0
76	4	17	0	I 0
76	5	17	57	l 84
	6	17	97	l 130
75	7	17	146	l 156
-1	8	17	192	l 192
-1	1	16	0	0
69	2	16	0	0
79	3	16	0	0
74	4	16	0	0
	5	16		87
79	l 6	16	94	104
		-		154
		16		164
_		9	0	0
		9	•	0
				0
			•	I 0
			•	53
				74
			•	104
				120
	1			0
	1 2			I 0
	3			1 0
	4			l 69
	5			109
	6		•	113
				0
83	1 2	20	0	0

```
20 |
                          67 I
                                    67
  -1 l
          4 |
          0 |
                0 |
                          -1 |
  -1 |
                                  3712
(241 rows)
ROLLBACK;
ROLLBACK
Data¶
drop table if exists vrp_orders cascade;
create table vrp_orders (
    id integer not null primary key,
    order_unit integer,
    open_time integer,
    close_time integer,
    service_time integer,
   x float8,
    y float8
);
copy vrp_orders (id, x, y, order_unit, open_time, close_time, service_time) from stdin;
   40.000000
                50.000000
                            0
                                0
                                    240 0
   25.000000
                85.000000
                               145 175 10
2
                            20
3
   22.000000
                75.000000
                            30
                                50 80 10
4
   22.000000
                85.000000
                            10
                                109 139 10
5
   20.000000
                80.000000
                            40
                                141 171 10
6
   20.000000
                85.000000
                            20
                                41 71 10
7
   18.000000
                75.000000
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                                    125 10
8
   15.000000
                75.000000
                            20
                                79 109 10
9
   15.000000
                80.000000
                            10
                                91 121 10
10 10.000000
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                            20
                                91 121 10
11 10.000000
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                                119 149 10
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12 8.000000
                40.000000
                            40
                                59 89
                                        10
13 8.000000
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                            20
                                64
                                    94
                                        10
14
   5.000000
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                            10
                                142 172 10
   5.000000
                45.000000
                            10
                                35
15
                                    65
                                        10
16 2.000000
                40.000000
                            20
                                58
                                   88
                                        10
17 0.000000
                40.000000
                            20
                                72 102 10
18 0.000000
                45.000000
                            20
                                149 179 10
19 44.000000
                5.000000
                            20
                                87 117 10
20 42.000000
                10.000000
                            40
                                72 102 10
21 42.000000
                                122 152 10
                15.000000
                            10
```

83 |

22 40.000000

5.000000

3 |

20 |

15 |

52

67 97 10

10

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23
    40.000000
                 15.000000
                                   92
                                        122 10
                               40
24
    38.000000
                 5.000000
                               30
                                   65
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25
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                                   148 178 10
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26
    35.000000
                 5.000000
                               20
                                   154 184 10
27
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                                       145
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28
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                                   62
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                                            10
                                   62
29
    92.000000
                 30.000000
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                                            10
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                 30.000000
                                   74
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                               10
                                        91
32
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                 35.000000
                               20
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                                            10
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33
                 30.000000
                               10
                                   131 161 10
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    85.000000
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                               10
                                   51
                                        81
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                 35.000000
                                   111 141 10
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                                   139 169 10
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38
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                                   113 143 10
48
    2.000000
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                                   116 146 10
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                                   139 169 10
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                 60.000000
                                   140 170 10
                               16
57
    50.000000
                 35.000000
                               19
                                   130 160 10
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                 25.000000
                               23
                                   96
                                        126 10
59
    15.000000
                 10.000000
                               20
                                   152 182 10
    10.000000
                 20.000000
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60
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    15.000000
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                               17
                                   155 185 10
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                  20.000000
                               6
                                   39
65
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                 30.000000
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                                        83
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40.000000
                 60.000000
                                  144 174 10
                              21
70
   31.000000
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                                  41
                                     71
                                          10
                                  180 210 10
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                                          10
73
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                                      60
                                          10
74
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                                  90
80
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                              30
                                  89
                                      119 10
    47.000000
                 47.000000
                                  192 222 10
81
                              13
82
    49.000000
                                  86
                                      116 10
                 58.000000
                             10
83
    27.000000
                 43.000000
                             9
                                  42
                                      72
                                          10
84
   37.000000
                 31.000000
                              14
                                  35
                                      65
                                          10
85
   57.000000
                 29.000000
                              18
                                  96
                                      126 10
86
   63.000000
                 23.000000
                                  87
                                      117 10
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    21.000000
                 24.000000
                              28
                                  87
                                      117 10
88
    12.000000
                 24.000000
                              13
                                  90
                                      120 10
89
    24.000000
                 58.000000
                              19
                                  67
                                      97
                                          10
    67.000000
                 5.000000
                                  144 174 10
90
                              25
    37.000000
                 47.000000
                                     116 10
91
                              6
                                  86
92
   49.000000
                 42.000000
                              13
                                  167 197 10
93
   53.000000
                 43.000000
                              14
                                  14
                                      44
                                          10
94
    61.000000
                 52.000000
                              3
                                  178 208 10
95
    57.000000
                 48.000000
                              23
                                  95
                                      125 10
96
    56.000000
                 37.000000
                                  34
                                      64
                             6
                                          10
97
    55.000000
                 54.000000
                              26
                                  132 162 10
98
   4.000000
                                  120 150 10
                 18.000000
                              35
99
   26.000000
                 52.000000
                             9
                                  46
                                     76 10
100 26.000000
                 35.000000
                                  77
                                      107 10
                              15
                                  180 210 10
101 31.000000
                 67.000000
                             3
١.
drop table if exists vrp_vehicles cascade;
create table vrp_vehicles (
    vehicle_id integer not null primary key,
    capacity integer,
    case_no integer
);
copy vrp_vehicles (vehicle_id, capacity, case_no) from stdin;
1
    200 5
2
    200 5
3
    200 5
```

```
200 5
5
    200 5
6
    200 5
7
    200 5
8
    200 5
9
    200 5
10 200 5
11 200 5
12 200 5
13 200 5
14
   200 5
   200 5
15
16
    200 5
17
   200 5
18 200 5
19 200 5
20 200 5
١.
drop table if exists vrp_distance cascade;
create table vrp_distance (
    src_id integer,
    dest_id integer,
    cost Float8,
    distance Float8,
    traveltime Float8
);
copy vrp_distance (src_id, dest_id, cost, distance, traveltime) from stdin;
        38.078866
                                38.078866
    2
                    38.078866
1
    3
        30.805844
                    30.805844
                                30.805844
1
    4
        39.357337
                    39.357337
                                39.357337
1
    5
        36.055513
                    36.055513
                                36.055513
                    40.311289
1
    6
        40.311289
                                40.311289
1
    7
        33.301652
                    33.301652
                                33.301652
1
    8
        35.355339
                    35.355339
                                35.355339
1
    9
        39.051248
                    39.051248
                                39.051248
                    33.541020
1
    10 33.541020
                                33.541020
1
    11
        31.622777
                    31.622777
                                31.622777
1
    12
        33.526109
                    33.526109
                                33.526109
    13
        32.388269
                    32.388269
                                32.388269
1
    14
        38.078866
                    38.078866
                                38.078866
1
1
    15
        35.355339
                    35.355339
                                35.355339
                                39.293765
    16
        39.293765
                    39.293765
1
                    41.231056
    17
        41.231056
                                41.231056
1
1
    18 40.311289
                    40.311289
                                40.311289
```

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45.177428
                      45.177428
                                   45.177428
1
    19
1
    20
        40.049969
                      40.049969
                                   40.049969
    21
                      35.057096
1
        35.057096
                                   35.057096
1
    22
        45.000000
                      45.000000
                                   45.000000
1
    23
        35.000000
                      35.000000
                                   35.000000
    24
        45.044423
                      45.044423
                                   45.044423
1
    25
                      35.057096
1
        35.057096
                                   35.057096
1
    26
        45.276926
                      45.276926
                                   45.276926
        58.523500
                      58.523500
                                   58.523500
1
    27
    28
        57.008771
                      57.008771
                                   57.008771
1
    29
1
        55.713553
                      55.713553
                                  55.713553
1
    30
        52.201533
                      52.201533
                                   52.201533
                      52.000000
1
    31
        52.000000
                                   52.000000
                      50.289164
    32
1
        50.289164
                                  50.289164
1
    33
        51.078371
                      51.078371
                                  51.078371
1
    34
        51.478151
                      51.478151
                                   51.478151
1
    35
        47.434165
                      47.434165
                                   47.434165
    36
        44.204072
                      44.204072
                                   44.204072
1
1
    37
        43.011626
                      43.011626
                                  43.011626
1
    38
        40.607881
                      40.607881
                                   40.607881
1
    39
        37.202150
                      37.202150
                                   37.202150
        36.055513
                      36.055513
                                   36.055513
    40
1
1
    41
        40.311289
                      40.311289
                                   40.311289
1
    42
        30.805844
                      30.805844
                                   30.805844
1
    43
        33.541020
                      33.541020
                                   33.541020
1
    44
        38.078866
                      38.078866
                                   38.078866
1
    45
        35.341194
                      35.341194
                                   35.341194
    46
        37.735925
                      37.735925
                                   37.735925
1
    47
        37.202150
                      37.202150
                                   37.202150
1
    48
        38.327536
                      38.327536
                                  38.327536
1
1
    49
        45.044423
                      45.044423
                                   45.044423
1
    50
        38.052595
                      38.052595
                                   38.052595
    51
        35.341194
                      35.341194
                                   35.341194
1
    52
        33.541020
                      33.541020
                                  33.541020
1
1
    53
        25.000000
                      25.000000
                                   25.000000
1
    54
        20.000000
                      20.000000
                                   20.000000
    55
        18.027756
                      18.027756
                                   18.027756
1
    56
1
        14.142136
                      14.142136
                                   14.142136
1
    57
        18.027756
                      18.027756
                                   18.027756
        26.925824
1
    58
                      26.925824
                                   26.925824
    59
        47.169906
                      47.169906
1
                                   47.169906
    60
        42.426407
                      42.426407
1
                                   42.426407
1
    61
        26.925824
                      26.925824
                                   26.925824
1
    62
        15.811388
                      15.811388
                                   15.811388
    63
                      29.154759
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        17.804494
                     17.804494
                                  17.804494
100 89
        23.086793
                     23.086793
                                 23.086793
100 90
        50.803543
                     50.803543
                                 50.803543
100 91
        16.278821
                     16.278821
                                 16.278821
100 92
        24.041631
                     24.041631
                                  24.041631
100 93
        28.160256
                     28.160256
                                  28.160256
100 94
        38.910153
                     38.910153
                                  38.910153
100 95
        33.615473
                     33.615473
                                  33.615473
100 96
        30.066593
                     30.066593
                                  30.066593
100 97
        34.669872
                     34.669872
                                  34.669872
100 98
                     27.802878
        27.802878
                                  27.802878
100 99
        17.000000
                     17.000000
                                  17.000000
```

```
100 101 32.388269
                     32.388269
                                 32.388269
101 1
        19.235384
                     19.235384
                                 19.235384
101 2
        18.973666
                     18.973666
                                  18.973666
101 3
        12.041595
                     12.041595
                                  12.041595
101 4
        20.124612
                     20.124612
                                 20.124612
101 5
        17.029386
                     17.029386
                                  17.029386
101 6
        21.095023
                     21.095023
                                 21.095023
101 7
        15.264338
                     15.264338
                                  15.264338
101 8
        17.888544
                     17.888544
                                  17.888544
101 9
        20.615528
                     20.615528
                                  20.615528
101 10
        38.275318
                     38.275318
                                 38.275318
101 11
        34.205263
                     34.205263
                                 34.205263
101 12
        35.468296
                     35.468296
                                 35.468296
101 13
        31.827661
                     31.827661
                                 31.827661
101 14
        41.231056
                     41.231056
                                 41.231056
101 15
        34.058773
                     34.058773
                                 34.058773
101 16
        39.623226
                     39.623226
                                 39.623226
101 17
        41.109610
                     41.109610
                                 41.109610
101 18
        38.013156
                     38.013156
                                 38.013156
101 19
        63.348244
                     63.348244
                                 63.348244
101 20
        58.051701
                     58.051701
                                 58.051701
101 21
        53.150729
                     53.150729
                                 53.150729
        62.649820
101 22
                     62.649820
                                  62.649820
101 23
        52.773099
                     52.773099
                                 52.773099
101 24
        62.393910
                     62.393910
                                  62.393910
101 25
        52.469038
                     52.469038
                                 52.469038
101 26
        62.128898
                     62.128898
                                  62.128898
101 27
                     73.925638
        73.925638
                                 73.925638
101 28
        71.554175
                     71.554175
                                 71.554175
101 29
                     71.344236
        71.344236
                                 71.344236
101 30
        67.119297
                     67.119297
                                  67.119297
        67.955868
                     67.955868
                                  67.955868
101 31
101 32
        65.368188
                     65.368188
                                  65.368188
101 33
        67.119297
                     67.119297
                                 67.119297
101 34
        68.410526
                     68.410526
                                  68.410526
101 35
        62.769419
                     62.769419
                                 62.769419
101 36
        40.249224
                     40.249224
                                 40.249224
101 37
        38.470768
                     38.470768
                                 38.470768
101 38
        37.161808
                     37.161808
                                  37.161808
101 39
        33.615473
                     33.615473
                                  33.615473
101 40
        31.780497
                     31.780497
                                  31.780497
101 41
        34.132096
                     34.132096
                                  34.132096
101 42
        28.160256
                     28.160256
                                  28.160256
101 43
        27.294688
                     27.294688
                                  27.294688
                     30.000000
101 44
                                  30.000000
        30.000000
101 45
        28.301943
                     28.301943
                                  28.301943
```

```
101 46
        18.601075
                     18.601075
                                  18.601075
101 47
        18.384776
                     18.384776
                                  18.384776
101 48
        36.400549
                     36.400549
                                 36.400549
101 49
        62.968246
                     62.968246
                                  62.968246
101 50
        56.089215
                     56.089215
                                 56.089215
101 51
        52.009614
                     52.009614
                                 52.009614
101 52
        52.773099
                     52.773099
                                 52.773099
101 53
        37.483330
                     37.483330
                                  37.483330
101 54
        20.248457
                     20.248457
                                  20.248457
101 55
        25.000000
                     25.000000
                                  25.000000
101 56
        7.071068
                     7.071068
                                 7.071068
101 57
        37.215588
                     37.215588
                                 37.215588
101 58
        42.011903
                     42.011903
                                 42.011903
101 59
                     59.203040
        59.203040
                                 59.203040
101 60
        51.478151
                     51.478151
                                 51.478151
101 61
        17.464249
                     17.464249
                                  17.464249
101 62
        14.142136
                     14.142136
                                 14.142136
101 63
        46.690470
                     46.690470
                                 46.690470
101 64
        58.008620
                     58.008620
                                 58.008620
101 65
        39.560081
                     39.560081
                                 39.560081
101 66
        27.294688
                     27.294688
                                  27.294688
101 67
        31.622777
                     31.622777
                                 31.622777
101 68
        41.400483
                     41.400483
                                 41.400483
101 69
        11.401754
                     11.401754
                                  11.401754
101 70
        15.000000
                     15.000000
                                  15.000000
101 71
        4.472136
                     4.472136
                                  4.472136
101 72
        36.055513
                     36.055513
                                  36.055513
101 73
        32.062439
                     32.062439
                                  32.062439
                                  29.832868
101 74
        29.832868
                     29.832868
101 75
        48.270074
                     48.270074
                                 48.270074
101 76
        67.230945
                     67.230945
                                  67.230945
101 77
        62.177166
                     62.177166
                                  62.177166
101 78
        64.498062
                     64.498062
                                  64.498062
101 79
        25.495098
                     25.495098
                                  25.495098
101 80
        25.019992
                     25.019992
                                  25.019992
101 81
        25.612497
                     25.612497
                                  25.612497
101 82
        20.124612
                     20.124612
                                 20.124612
101 83
        24.331050
                     24.331050
                                  24.331050
101 84
        36.496575
                     36.496575
                                  36.496575
101 85
        46.043458
                     46.043458
                                  46.043458
101 86
        54.405882
                     54.405882
                                  54.405882
101 87
        44.147480
                     44.147480
                                  44.147480
101 88
        47.010637
                     47.010637
                                  47.010637
101 89
        11.401754
                     11.401754
                                  11.401754
101 90
                     71.693793
                                 71.693793
        71.693793
101 91
        20.880613
                     20.880613
                                  20.880613
```

```
101 92 30.805844
                   30.805844
                               30.805844
101 93 32.557641
                   32.557641
                               32.557641
101 94
       33.541020
                   33.541020
                               33.541020
101 95
       32.202484
                   32.202484
                               32.202484
101 96
       39.051248
                   39.051248
                               39.051248
101 97
       27.294688
                   27.294688
                               27.294688
101 98 55.946403
                   55.946403
                               55.946403
101 99 15.811388
                   15.811388
                               15.811388
101 100 32.388269
                   32.388269
                               32.388269
```

See Also¶

• http://en.wikipedia.org/wiki/Vehicle_routing_problem

Indices and tables

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Graph Operations

Transformation - Family of functions - Maps a given graph to a new form

- pgr_lineGraph Experimental Transformation algorithm for generating a Line Graph.
- pgr_lineGraphFull Experimental Transformation algorithm for generating a Line Graph out of each vertex in the input graph.

 $\label{pgr_lineGraph} \textbf{-Experimental} \P \quad \text{pgr_lineGraph} \ -- \ \text{Transforms a given graph into its corresponding edge-based graph}.$



Boost Graph Inside

Warning

Experimental functions

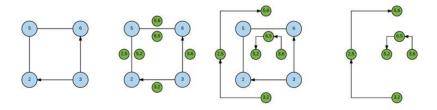
- They are not officially of the current release.
- They likely will not be officially be part of the next release:

- The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
- Name might change.
- Signature might change.
- Functionality might change.
- pgTap tests might be missing.
- Might need c/c++ coding.
- May lack documentation.
- Documentation if any might need to be rewritten.
- Documentation examples might need to be automatically generated.
- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

Synopsis¶ Given a graph G, its line graph L(G) is a graph such that:-

- each vertex of L(G) represents an edge of G
- two vertices of L(G) are adjacent if and only if their corresponding edges share a common endpoint in G.

The following figures show a graph (left, with blue vertices) and its Line Graph (right, with green vertices).



Signature Summary¶

Signatures¶ Minimal signature¶

```
pgr_lineGraph(edges_sql)
RETURNS SET OF (seq, source, target, cost, reverse_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph:

Example:

1	-18	18	1	1
2	-17	17	1	1
3	-16	-3	1	-1
4	-16	16	1	1
5 I	-15	-9 I	1	1
6 I	-15	15	1	1
7	-14	-10	1	1
8	-14	12	1	-1
9	-14	14	1	1
10	-10	-7 I	1	1
11	-10	-4 I	1	1
12	-10	8	1	1
13	-10	10	1	1
14	-9	-8	1	1
15	-9	9	1	1
16	-9	11	1	-1
17	-8	-7 I	1	1
18	-8	-4	1	1
19	-8	8	1	1
20	-7	-6 I	1	1
21	-6	6 l	1	1
22	-4	-1	1	1
23 I	-4	4	1	1
24 l	-3	-2	1	-1
25 I	-3	5	1	-1
26 I	-2	-1	1	-1
27	-2	4	1	-1
28	-1 l	1	1	1
29 l	5 l	-8	1	-1
30 l	5 l	9	1	-1
31	5 I	11	1	-1
32	7	-7	1	1
33	7	-4	1	1
34	8	11	1	-1

```
10 |
 35 |
                    12 |
 36 I
           11 |
                    13 |
                             1 |
                                           -1
                             1 |
 37 |
           12 |
                    13 |
                                           -1
                                           -1
 38 |
           13 |
                   -15 |
                             1 |
 39 |
           16 l
                    -9 |
                             1 |
                                            1
 40 l
           16 l
                    15 |
                                            1
(40 rows)
```

$Complete\ Signature \P$

```
pgr_lineGraph(edges_sql, directed);
RETURNS SET OF (seq, source, target, cost, reverse_cost) or EMPTY SET
```

This signature returns the Line Graph of the current graph: • on a directed graph when directed flag is missing or is set to true.

• on an undirected graph when directed flag is set to false.

Example:

```
SELECT * FROM pgr_lineGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    FALSE
);
```

seq	source	target		cost	reverse_cost
1	-3	-2		1	l -1
2	-3	5	1	1	-1
3	-2	4		1	-1
4	1	4		1	-1
5	4	8		1	-1
6	4	10		1	-1
7	5	9		1	-1
8	5	11		1	-1
9	6 l	7		1	-1
10	7	8		1	-1
11	7	10		1	-1
12	8	9		1	-1
13	8	11		1	-1
14	9	15		1	-1
15	10	12		1	-1
16	10	14	1	1	-1
17	11	13	1	1	-1
18	12	13	1	1	-1
19	16	15		1	-1
(19 row	s)				

Description of the Signatures ¶ Description of the edges_sql query for dijkstra like functions ¶

 $edges_sql:$

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
$\overline{\mathbf{id}}$	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
	AND MINISTERIAL	• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	cant-numerical	Weight of the edge (target, source),
		 When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Column	Type	Description	
edges_sql	TEXT	SQL query as described above.	
directed	BOOLEAN	 When true the graph is considered as Directed. When false the graph is considered as Undirected. 	

Description of the parameters of the signatures \P

Description of the return values¶ RETURNS SETOF (seq, source, target, cost, reverse_cost)

Column	Type	Description	
seq	INTEGER	Sequential value starting from 1.	
source	BIGINT	Identifier of the source vertex of the current edge id.	
		• When negative: the source is the reverse edge in the original graph.	
target	BIGINT	Identifier of the target vertex of the current edge id.	
		• When negative: the target is the reverse edge in the original graph.	
cost	FLOAT	Weight of the edge (source, target).	
		• When negative: edge (source, target) does not exist, therefore it's not part of the graph.	
${\bf reverse_cost}$	FLOAT	Weight of the edge (target, source).	
		• When negative: edge (target, source) does not exist, therefore it's not part of the graph.	

See Also \P

- $\bullet \ \ https://en.wikipedia.org/wiki/Line_graph$
- ullet The queries use the $Sample\ Data$ network.

Indices and tables

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- Search Page

 $\label{lineGraphFull-Experimental} $$ pgr_lineGraphFull-Transforms a given graph into a new graph where all of the vertices from the original graph are converted to line graphs.$

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis¶ pgr_lineGraphFull, converts original directed graph to a directed line graph by converting each vertex to a complete graph and keeping all the original edges. The new connecting edges have a cost 0 and go between the adjacent original edges, respecting the directionality.

A possible application of the resulting graph is "routing with two edge restrictions":

Setting a cost of using the vertex when routing between edges on the connecting edge

• Forbid the routing between two edges by removing the connecting edge

This is possible because each of the intersections (vertices) in the original graph are now complete graphs that have a new edge for each possible turn across that intersection.

$Characteristics\P$

The main characteristics are: • This function is for directed graphs.

- Results are undefined when a negative vertex id is used in the input graph.
- Results are undefined when a duplicated edge id is used in the input graph.
- Running time: TBD

Signature Summary \P

Signatures¶

22 |

-8 |

-5 l

${\bf Minimal\ signature} \P$

```
pgr_lineGraphFull(TEXT edges_sql)
RETURNS SET OF (seq, source, target, cost, edge) OR EMPTY SET
Example:
SELECT * FROM pgr_lineGraphFull(
    'SELECT id, source, target, cost, reverse_cost
      FROM edge table
      WHERE id IN (4,7,8,10)'
);
 seq | source | target | cost | edge
   1 |
           -1 |
                     5 I
                            1 |
   2 |
           2 |
                    -1 l
                            0 |
                                    0
   3 |
           -2 |
                     2 |
                             1 |
                                   -4
                                   -7
   4 |
           -3 |
                     8 |
                            1 |
           -4 |
   5 I
                     6 I
                            1 |
                                   8
   6 I
           -5 I
                    10 |
                            1 |
                                   10
   7 |
           5 |
                    -2 |
                             0 |
                                    0
   8 |
           5 |
                    -3 |
                             0 |
                                    0
   9 I
           5 I
                    -4 I
                             0 |
                                    0
  10 |
           5 |
                    -5 |
                             0 |
                                    0
  11 |
           -6 |
                    -2 |
                             0 |
                                    0
  12 |
           -6 |
                    -3 |
                             0 |
                                    0
                    -4 I
  13 l
           -6 |
                            0 |
                                    0
           -6 I
                    -5 I
                            0 |
                                    0
  14 |
           -7 |
                    -2 |
                             0 |
                                    0
  15 |
  16 |
           -7 I
                    -3 |
                             0 |
                                    0
  17 I
           -7 I
                    -4 I
                             0 |
                                    0
           -7 |
                    -5 I
  18 I
                             0 |
                                    0
           -8 |
                    -2 |
  19 |
                             0 |
                                    0
                    -3 |
  20 |
           -8 |
                             0 |
                                    0
                    -4 |
 21 |
           -8 |
                             0 |
                                    0
```

0

0 |

Description of the Signatures \P Description of the edges_sql query for dijkstra like functions \P

edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
$\overline{\mathbf{id}}$	ANY-INTEG	ER	Identifier of the edge.
source	ANY-INTEG	ER	Identifier of the first end point vertex edge.
target	ANY-INTEG	ER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMER	ICAL	Weight of the edge (source, target)
			• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	cant-numer	ICIAL	Weight of the edge (target, source),
			• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

 ${\bf SMALLINT,\,INTEGER,\,BIGINT}$

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

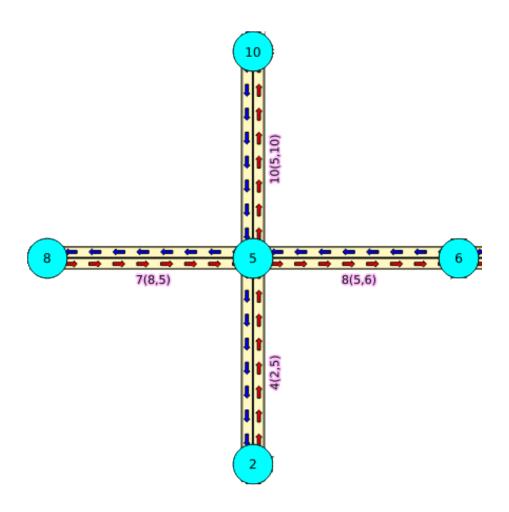
Column	Type	Default	Description
$\overline{\mathrm{sql}}$	TEXT		SQL quer

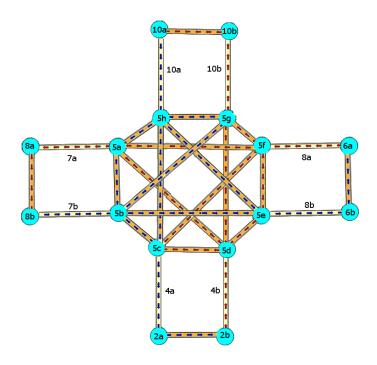
Additional Examples \P The examples of this section are based on the *Sample Data* network.

The examples include the subgraph including edges 4, 7, 8, and 10 with reverse_cost.

Example for generating the LineGraphFull¶ This example displays how this graph transformation works to create additional edges for each possible turn in a graph.

```
SELECT id, source, target, cost, reverse_cost
FROM edge_table
WHERE id IN (4,7,8,10);
```





In the transformed graph, all of the edges from the original graph are still present (yellow), but we now have additional edges for every turn that could be made across vertex 6 (orange).

Example for creating table that identifies transformed vertices¶ The vertices in the transformed graph are each created by splitting up the vertices in the original graph. Unless a vertex in the original graph is a leaf vertex, it will generate more than one vertex in the transformed graph. One of the newly created vertices in the transformed graph will be given the same vertex-id as the vertex that it was created from in the original graph, but the rest of the newly created vertices will have negative vertex ids. Following is an example of how to generate a table that maps the ids of the newly created vertices with the original vertex that they were created from

The first step is to store your results graph into a table and then create the vertex mapping table with one row for each distinct vertex id in the results

```
graph.
```

```
CREATE TABLE lineGraph_edges AS SELECT * FROM pgr_lineGraphFull(
    $$SELECT id, source, target, cost, reverse_cost
    FROM edge_table WHERE id IN (4,7,8,10)$$
);

SELECT 28

CREATE TABLE lineGraph_vertices AS

SELECT *, NULL::BIGINT AS original_id

FROM (SELECT source AS id FROM lineGraph_edges
    UNION
    SELECT target FROM lineGraph_edges) as foo

ORDER BY id;

SELECT 16
```

Next, we set the original_id of all of the vertices in the results graph that were given the same vertex id as the vertex that it was created from in the original graph.

```
UPDATE lineGraph_vertices AS r
   SET original_id = v.id
   FROM edge_table_vertices_pgr AS v
   WHERE v.id = r.id;
UPDATE 5
```

Then, we cross reference all of the other newly created vertices that do not have the same original_id and set their original_id values.

```
WITH
unassignedVertices
  AS (SELECT e.id, e.original_id
       FROM lineGraph_vertices AS e
       WHERE original_id IS NOT NULL),
edgesWithUnassignedSource
  AS (SELECT *
        FROM lineGraph_edges
     WHERE cost = 0 and source IN (SELECT id FROM unassignedVertices)),
{\tt edgesWithUnassignedSourcePlusVertices}
  AS (SELECT *
        FROM edgesWithUnassignedSource
        JOIN lineGraph_vertices
        ON(source = id)),
verticesFromEdgesWithUnassignedSource
 AS (SELECT DISTINCT edgesWithUnassignedSourcePlusVertices.target,
```

```
edgesWithUnassignedSourcePlusVertices.original id
        FROM edgesWithUnassignedSourcePlusVertices
        JOIN lineGraph_vertices AS r
        ON(target = r.id AND r.original_id IS NULL))
UPDATE lineGraph_vertices
 SET original_id = verticesFromEdgesWithUnassignedSource.original_id
 FROM verticesFromEdgesWithUnassignedSource
 WHERE verticesFromEdgesWithUnassignedSource.target = id;
UPDATE 8
WITH
unassignedVertices
  AS (SELECT e.id, e.original_id
        FROM lineGraph_vertices AS e
        WHERE original_id IS NOT NULL),
edgesWithUnassignedTarget
  AS (SELECT *
        FROM lineGraph_edges
     WHERE cost = 0 and target IN (SELECT id FROM unassignedVertices)),
{\tt edgesWithUnassignedTargetPlusVertices}
  AS (SELECT *
        FROM edgesWithUnassignedTarget
        JOIN lineGraph_vertices
        ON(target = id)),
verticesFromEdgesWithUnassignedTarget
 AS (SELECT DISTINCT edgesWithUnassignedTargetPlusVertices.source,
                 edgesWithUnassignedTargetPlusVertices.original_id
        {\tt FROM}\ edges {\tt WithUnassignedTargetPlusVertices}
        JOIN lineGraph_vertices AS r
        ON(source = r.id AND r.original_id IS NULL))
UPDATE lineGraph_vertices
 SET original_id = verticesFromEdgesWithUnassignedTarget.original_id
 FROM verticesFromEdgesWithUnassignedTarget
  WHERE verticesFromEdgesWithUnassignedTarget.source = id;
UPDATE 3
```

The only vertices left that have not been mapped are a few of the leaf vertices from the original graph. The following sql completes the mapping for these leaf vertices (in the case of this example graph there are no leaf vertices but this is necessary for larger graphs).

```
WITH
unassignedVertexIds
AS (SELECT id
FROM lineGraph_vertices
WHERE original_id IS NULL),
```

```
edgesWithUnassignedSource
  AS (SELECT source, edge
        FROM lineGraph_edges
        WHERE source IN (SELECT id FROM unassignedVertexIds)),
\verb|originalEdgesWithUnassignedSource| \\
  AS (SELECT id, source
        FROM edge_table
        WHERE id IN (SELECT edge FROM edgesWithUnassignedSource))
UPDATE lineGraph_vertices AS d
  SET original_id = (SELECT source
                        FROM originalEdgesWithUnassignedSource
                       WHERE originalEdgesWithUnassignedSource.id =
                          (SELECT edge
                             FROM edgesWithUnassignedSource
                     WHERE edgesWithUnassignedSource.source = d.id))
  WHERE id IN (SELECT id FROM unassignedVertexIds);
UPDATE 0
WITH
unassignedVertexIds
  AS (SELECT id
        FROM lineGraph_vertices
        WHERE original_id IS NULL),
{\tt edgesWithUnassignedTarget}
  AS (SELECT target, edge
        FROM lineGraph_edges
        WHERE target IN (SELECT id FROM unassignedVertexIds)),
\verb|originalEdgesWithUnassignedTarget|\\
  AS (SELECT id, target
        FROM edge_table
        WHERE id IN (SELECT edge FROM edgesWithUnassignedTarget))
UPDATE lineGraph_vertices AS d
  SET original_id = (SELECT target
                        {\tt FROM\ original Edges With Unassigned Target}
                       WHERE originalEdgesWithUnassignedTarget.id =
                          (SELECT edge
                             FROM edgesWithUnassignedTarget
                     WHERE edgesWithUnassignedTarget.target = d.id))
  WHERE id IN (SELECT id FROM unassignedVertexIds);
UPDATE 0
Now our vertex mapping table is complete:
SELECT * FROM lineGraph_vertices;
 id | original id
----+-----
```

2	2
5	5
6	6
8	8
10	10
-11	10
-10	6
-9	8
-5	5
-4	5
-3	5
-2	5
-1	2
-8	5
-7	5
-6	5
(16 rows)	

Example for running a dijkstra's shortest path with turn penalties¶ One use case for this graph transformation is to be able to run a shortest path

search that takes into account the cost or limitation of turning. Below is an example of running a dijkstra's shortest path from vertex 2 to vertex 8 in the original graph, while adding a turn penalty cost of 100 to the turn from edge 4 to edge -7.

First we must increase set the cost of making the turn to 100:

UPDATE 1

Then we must run a dijkstra's shortest path search using all of the vertices in the new graph that were created from vertex 2 as the starting point, and all of the vertices in the new graph that were created from vertex 8 as the ending point.

```
SELECT * FROM
  (SELECT * FROM pgr_dijkstra($$SELECT seq AS id, * FROM lineGraph_edges$$,
    (SELECT array_agg(id) FROM lineGraph_vertices where original_id = 2),
```

(SELECT array_agg(id) FROM lineGraph_vertices where original_id = 8)
)) as shortestPaths

WHERE start_vid = 2 AND end_vid = 8 AND (cost != 0 OR edge = -1)) as b; seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost

	+		+	+	+-	
29	2	2		1 1 1		0
31	4	2	8 -4	5	1	1
33	6 l	2	8 -10	25	1	2
35	8	2	8 -3	4	1	3
36	9	2	8 8	-1	0	4
(5 rows)						

Normally the shortest path from vertex 2 to vertex 8 would have an aggregate cost of 2, but since there is a large penalty for making the turn needed to get this cost, the route goes through vertex 6 to avoid this turn.

If you cross reference the node column in the dijkstra results with the vertex id mapping table, this will show you that the path goes from v2 -> v5 -> v6 -> v5 -> v8 in the original graph.

See Also¶

- http://en.wikipedia.org/wiki/Line_graph
- http://en.wikipedia.org/wiki/Complete_graph

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Transformation - Family of functions¶

- pgr_lineGraph Experimental Transformation algorithm for generating a Line Graph.
- pgr_lineGraphFull Experimental Transformation algorithm for generating a Line Graph out of each vertex in the input graph.

pgr_lineGraph - Experimental¶ pgr_lineGraph — Transforms a given graph into its corresponding edge-based graph.



Boost Graph Inside

Warning

Experimental functions

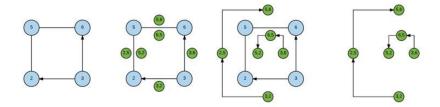
- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis

Given a graph G, its line graph L(G) is a graph such that:-

- each vertex of L(G) represents an edge of G
- two vertices of L(G) are adjacent if and only if their corresponding edges share a common endpoint in G.

The following figures show a graph (left, with blue vertices) and its Line Graph (right, with green vertices).



Signature Summary \P

 $Signatures \P$

 ${\bf Minimal\ signature}\P$

pgr_lineGraph(edges_sql)
RETURNS SET OF (seq, source, target, cost, reverse_cost) or EMPTY SET

The minimal signature is for a **directed** graph:

Example:

SELECT * FROM pgr_lineGraph(
 'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);

<i>,</i>				
seq	source	target	cost	reverse_cost
1	-18	18	1	1
2	-17		1	1
3	-16	-3	1 I	-1
4	-16	16	1 I	1
5	-15		1	1
6 I	-15	15	1	1
7	-14	-10	1	1
8	-14		1	-1
9	-14	14	1	1
10 l	-10	-7	1	1
11	-10	-4	1	1
12 l	-10	8	1	1
13	-10	10	1	1
14 l	-9	-8	1	1
15	-9	9	1	1
16 l	-9	11	1	-1
17 l	-8	-7	1	1
18	-8	-4	1	1
19	-8	8	1	1
20	-7	-6	1	1
21	-6	6	1	1
22	-4	-1	1	1
23	-4	4	1	1
24	-3	-2	1	-1
25 I	-3	J 5 J	1	-1
26 I	-2	-1	1	-1

```
27 |
         -2 |
                4 |
                                   -1
 28 I
         -1 |
                 1 |
                       1 |
                                   1
                                   -1
 29 |
         5 |
                 -8 |
                     1 |
                 9 |
                                   -1
 30 |
         5 |
                      1 |
 31 |
         5 |
                11 |
                       1 |
                                   -1
                                   1
 32 |
         7 |
                -7 I
                      1 |
         7 |
                -4 I
 33 |
                     1 |
                                   1
 34 l
                11 |
                                   -1
         8 I
                     1 |
 35 l
         10 |
                12 |
                     1 |
                                   -1
 36 I
         11 |
               13 |
                      1 |
                                   -1
                                   -1
 37 |
         12 |
               13 |
                      1 |
 38 |
         13 |
                -15 |
                      1 |
                                   -1
                                   1
 39 |
         16 |
                -9 |
                       1 |
 40 I
         16 |
                15 |
                     1 |
(40 rows)
```

Complete Signature¶

```
pgr_lineGraph(edges_sql, directed);
RETURNS SET OF (seq, source, target, cost, reverse_cost) or EMPTY SET
```

This signature returns the Line Graph of the current graph: • on a directed graph when directed flag is missing or is set to true.

- on an ${\bf undirected}$ graph when ${\bf directed}$ flag is set to false.

Example:

4 |

5 I

1 |

4 |

```
SELECT * FROM pgr_lineGraph(
   'SELECT id, source, target, cost, reverse_cost FROM edge_table',
   FALSE
);
seq | source | target | cost | reverse_cost
----+-----
         -3 |
                -2 | 1 |
  2 |
         -3 |
               5 | 1 |
                                 -1
  3 |
         -2 |
                4 | 1 |
                                 -1
```

-1

-1

1 | 1 |

4 |

8 |

12	8	9	1	-1
13	8	11	1	-1
14	9	15	1	-1
15	10	12	1	-1
16	10	14	1	-1
17	11	13	1	-1
18	12	13	1	-1
19	16	15	1	-1
(19 rows)				

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex edge.
target	ANY-INTEGER	Identifier of the second end point vert edge.
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)
		• When negative: edge (source, ta not exist, therefore it's not part graph.
reverse_	cont-numerical	Weight of the edge (target, source),
		• When negative: edge (target, so not exist, therefore it's not part graph.

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Column	Type	Description
edges_sql	TEXT	SQL query as described above.
directed	BOOLEAN	 When true the graph is considered as Directed. When false the graph is considered as Undirected.

Description of the return values \P

RETURNS SETOF (seq, source, target, cost, reverse_cost)

Column	Type	Description
$\overline{\text{seq}}$	INTEGER	Sequential value starting from 1.
source	BIGINT	Identifier of the source vertex of the current edge id.
		• When negative: the source is the reverse edge in the original graph.
target	BIGINT	Identifier of the target vertex of the current edge id.
		• When negative: the target is the reverse edge in the original graph.
\mathbf{cost}	FLOAT	Weight of the edge (source, target).
		• When negative: edge (source, target) does not exist, therefore it's not part of the graph.
${\bf reverse_cost}$	FLOAT	Weight of the edge (target, source).
		• When negative: edge (target, source) does not exist, therefore it's not part of the graph.

See Also \P

- $\bullet \ \ https://en.wikipedia.org/wiki/Line_graph$
- $\bullet\,$ The queries use the $Sample\ Data$ network.

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pgr_lineGraphFull - Experimental¶ pgr_lineGraphFull — Transforms a given graph into a new graph where all of the vertices from the original graph are converted to line graphs.

Warning

Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
 - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
 - Name might change.
 - Signature might change.
 - Functionality might change.
 - pgTap tests might be missing.
 - Might need c/c++ coding.
 - May lack documentation.
 - Documentation if any might need to be rewritten.
 - Documentation examples might need to be automatically generated.
 - Might need a lot of feedback from the comunity.
 - Might depend on a proposed function of pgRouting
 - Might depend on a deprecated function of pgRouting

Synopsis

pgr_lineGraphFull, converts original directed graph to a directed line graph by converting each vertex to a complete graph and keeping all the original edges. The new connecting edges have a cost 0 and go between the adjacent original edges, respecting the directionality.

A possible application of the resulting graph is "routing with two edge restrictions":

- Setting a cost of using the vertex when routing between edges on the connecting edge
- Forbid the routing between two edges by removing the connecting edge

This is possible because each of the intersections (vertices) in the original graph are now complete graphs that have a new edge for each possible turn across that intersection.

Characteristics¶

The main characteristics are: • This function is for directed graphs.

- Results are undefined when a negative vertex id is used in the input graph.
- Results are undefined when a duplicated edge id is used in the input graph.
- Running time: TBD

14 |

15 |

-6 I

-7 I

-5 I

-2 |

```
Signature Summary \P
pgr_lineGraphFull(edges_sql)
RETURNS SET OF (seq, source, target, cost, edge)
    OR EMPTY SET
Signatures¶
Minimal signature
pgr_lineGraphFull(TEXT edges_sql)
RETURNS SET OF (seq, source, target, cost, edge) OR EMPTY SET
Example:
SELECT * FROM pgr_lineGraphFull(
    'SELECT id, source, target, cost, reverse_cost
      FROM edge_table
      WHERE id IN (4,7,8,10)'
);
 seq | source | target | cost | edge
----+----
   1 |
           -1 |
                    5 |
                            1 |
   2 |
           2 |
                    -1 l
                            0 |
                                  0
   3 |
           -2 |
                    2 |
                            1 l
                                 -4
           -3 |
                                 -7
   4 |
                    8 |
                            1 |
   5 |
           -4 |
                    6 |
                            1 |
   6 I
           -5 |
                   10 |
                            1 |
                                 10
   7 |
           5 |
                    -2 |
                            0 |
                                  0
   8 |
           5 |
                    -3 |
                            0 |
                                  0
   9 |
           5 I
                    -4 |
                            0 |
                                   0
  10 I
           5 I
                    -5 I
                            0 |
                                   0
           -6 I
                    -2 I
                            0 |
                                   0
  11 |
           -6 I
                   -3 |
                            0 |
                                   0
  12 |
  13 |
           -6 I
                    -4 |
                            0 |
                                   0
```

0

0

0 |

0 |

```
16 |
                      -3 |
                                        0
  17 |
                      -4 |
                               0 |
                                        0
            -7 |
  18 |
                      -5 |
                               0 |
                                        0
                      -2 |
  19 |
            -8 |
                               0 |
                                        0
  20 |
            -8 |
                      -3 |
                               0 |
                                        0
  21 |
            -8 |
                      -4 |
                               0 |
                                        0
            -8 |
                      -5 I
                               0 |
                                        0
  22 |
                                        7
  23 I
            -9 I
                      -6 I
                               1 |
  24 |
             8 |
                      -9 I
                               0 |
                                        0
 25 I
           -10 l
                      -7 I
                                       -8
                               1 |
 26 |
             6 I
                     -10 |
                               0 |
                                        0
 27 |
           -11 |
                      -8 |
                               1 |
                                     -10
  28 |
            10 |
                     -11 |
                               0 |
(28 rows)
```

Description of the Signatures \P

Description of the edges_sql query for dijkstra like functions \P edges_sql:

an SQL query, which should return a set of rows with the following columns:

Column	Type Default	Description				
id	ANY-INTEGER	Identifier of the edge.				
source	ANY-INTEGER	Identifier of the first end point vertex edge.				
target	ANY-INTEGER	Identifier of the second end point vertedge.				
\mathbf{cost}	ANY-NUMERICAL	Weight of the edge (source, target)				
		• When negative: edge (source, ta not exist, therefore it's not part graph.				
$reverse_$	cant-numerical	Weight of the edge (target, source),				
		• When negative: edge (target, so not exist, therefore it's not part graph.				

Where:

ANY-INTEGER:

SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL:

SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures \P

Column	Type	Default	Description
$\overline{\mathrm{sql}}$	TEXT		SQL quer

Additional Examples \P

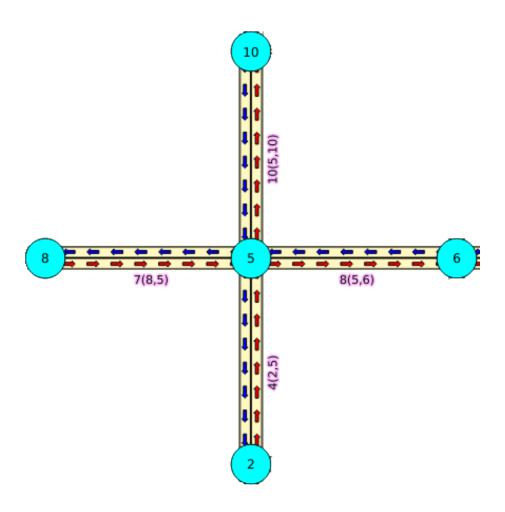
The examples of this section are based on the $Sample\ Data$ network.

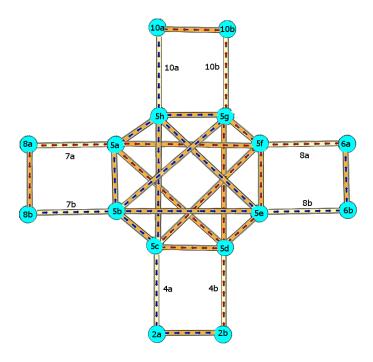
The examples include the subgraph including edges $4,\ 7,\ 8,\ {\rm and}\ 10$ with reverse_cost.

Example for generating the LineGraphFull \P

This example displays how this graph transformation works to create additional edges for each possible turn in a graph.

```
SELECT id, source, target, cost, reverse_cost
FROM edge_table
WHERE id IN (4,7,8,10);
```





In the transformed graph, all of the edges from the original graph are still present (yellow), but we now have additional edges for every turn that could be made across vertex 6 (orange).

Example for creating table that identifies transformed vertices \P

The vertices in the transformed graph are each created by splitting up the vertices in the original graph. Unless a vertex in the original graph is a leaf vertex, it will generate more than one vertex in the transformed graph. One of the newly created vertices in the transformed graph will be given the same vertex-id as the vertex that it was created from in the original graph, but the rest of the newly created vertices will have negative vertex ids. Following is an example of how to generate a table that maps the ids of the newly created vertices with the original vertex that they were created from

The first step is to store your results graph into a table and then create the vertex mapping table with one row for each distinct vertex id in the results graph.

```
CREATE TABLE lineGraph_edges AS SELECT * FROM pgr_lineGraphFull(
    $$SELECT id, source, target, cost, reverse_cost
    FROM edge_table WHERE id IN (4,7,8,10)$$
);
SELECT 28
CREATE TABLE lineGraph_vertices AS
SELECT *, NULL::BIGINT AS original_id
FROM (SELECT source AS id FROM lineGraph_edges
    UNION
    SELECT target FROM lineGraph_edges) as foo
ORDER BY id;
SELECT 16
```

Next, we set the original_id of all of the vertices in the results graph that were given the same vertex id as the vertex that it was created from in the original graph.

```
UPDATE lineGraph_vertices AS r
   SET original_id = v.id
   FROM edge_table_vertices_pgr AS v
   WHERE v.id = r.id;
UPDATE 5
```

Then, we cross reference all of the other newly created vertices that do not have the same original_id and set their original_id values.

```
WITH
unassignedVertices
  AS (SELECT e.id, e.original_id
       FROM lineGraph_vertices AS e
       WHERE original_id IS NOT NULL),
edgesWithUnassignedSource
  AS (SELECT *
        FROM lineGraph_edges
     WHERE cost = 0 and source IN (SELECT id FROM unassignedVertices)),
{\tt edgesWithUnassignedSourcePlusVertices}
  AS (SELECT *
        FROM edgesWithUnassignedSource
        JOIN lineGraph vertices
        ON(source = id)),
{\tt verticesFromEdgesWithUnassignedSource}
 AS (SELECT DISTINCT edgesWithUnassignedSourcePlusVertices.target,
                  edgesWithUnassignedSourcePlusVertices.original_id
        {\tt FROM}\ edges {\tt WithUnassignedSourcePlusVertices}
```

JOIN lineGraph_vertices AS r

```
ON(target = r.id AND r.original_id IS NULL))
UPDATE lineGraph_vertices
 SET original_id = verticesFromEdgesWithUnassignedSource.original_id
 {\tt FROM}\ {\tt verticesFromEdgesWithUnassignedSource}
  WHERE verticesFromEdgesWithUnassignedSource.target = id;
UPDATE 8
WITH
unassignedVertices
  AS (SELECT e.id, e.original_id
        FROM lineGraph_vertices AS e
        WHERE original_id IS NOT NULL),
edgesWithUnassignedTarget
  AS (SELECT *
        FROM lineGraph_edges
     WHERE cost = 0 and target IN (SELECT id FROM unassignedVertices)),
{\tt edgesWithUnassignedTargetPlusVertices}
  AS (SELECT *
        FROM edgesWithUnassignedTarget
        JOIN lineGraph_vertices
        ON(target = id)),
verticesFromEdgesWithUnassignedTarget
 AS (SELECT DISTINCT edgesWithUnassignedTargetPlusVertices.source,
                 edgesWithUnassignedTargetPlusVertices.original_id
        FROM edgesWithUnassignedTargetPlusVertices
        JOIN lineGraph_vertices AS r
        ON(source = r.id AND r.original_id IS NULL))
UPDATE lineGraph_vertices
 SET original_id = verticesFromEdgesWithUnassignedTarget.original_id
 FROM verticesFromEdgesWithUnassignedTarget
  WHERE verticesFromEdgesWithUnassignedTarget.source = id;
UPDATE 3
```

The only vertices left that have not been mapped are a few of the leaf vertices from the original graph. The following sql completes the mapping for these leaf vertices (in the case of this example graph there are no leaf vertices but this is necessary for larger graphs).

```
WITH
unassignedVertexIds
AS (SELECT id
FROM lineGraph_vertices
WHERE original_id IS NULL),
edgesWithUnassignedSource
AS (SELECT source,edge
FROM lineGraph_edges
```

```
WHERE source IN (SELECT id FROM unassignedVertexIds)),
\verb|originalEdgesWithUnassignedSource| \\
  AS (SELECT id, source
        FROM edge_table
        WHERE id IN (SELECT edge FROM edgesWithUnassignedSource))
UPDATE lineGraph_vertices AS d
  SET original_id = (SELECT source
                       FROM originalEdgesWithUnassignedSource
                      WHERE originalEdgesWithUnassignedSource.id =
                          (SELECT edge
                             FROM edgesWithUnassignedSource
                    WHERE edgesWithUnassignedSource.source = d.id))
  WHERE id IN (SELECT id FROM unassignedVertexIds);
UPDATE 0
WITH
unassignedVertexIds
  AS (SELECT id
        FROM lineGraph_vertices
        WHERE original_id IS NULL),
edgesWithUnassignedTarget
  AS (SELECT target, edge
        FROM lineGraph_edges
        WHERE target IN (SELECT id FROM unassignedVertexIds)),
\verb|originalEdgesWithUnassignedTarget|\\
  AS (SELECT id, target
        FROM edge_table
        WHERE id IN (SELECT edge FROM edgesWithUnassignedTarget))
UPDATE lineGraph_vertices AS d
  SET original_id = (SELECT target
                       FROM originalEdgesWithUnassignedTarget
                      WHERE originalEdgesWithUnassignedTarget.id =
                          (SELECT edge
                             FROM edgesWithUnassignedTarget
                    WHERE edgesWithUnassignedTarget.target = d.id))
  WHERE id IN (SELECT id FROM unassignedVertexIds);
UPDATE 0
Now our vertex mapping table is complete:
SELECT * FROM lineGraph_vertices;
 id | original_id
----+----
   2 |
                 2
   5 |
```

6 I

```
8 |
  10 |
                  10
 -11 |
                  10
 -10 |
                   6
  -9 I
                   8
  -5 I
                   5
  -4 |
                   5
  -3 |
                   5
  -2 |
                   5
  -1 l
                   2
  -8 I
                   5
  -7 |
                   5
  -6 |
                   5
(16 rows)
```

UPDATE 1

Example for running a dijkstra's shortest path with turn penalties¶

One use case for this graph transformation is to be able to run a shortest path search that takes into account the cost or limitation of turning. Below is an example of running a dijkstra's shortest path from vertex 2 to vertex 8 in the original graph, while adding a turn penalty cost of 100 to the turn from edge 4 to edge -7.

First we must increase set the cost of making the turn to 100:

Then we must run a dijkstra's shortest path search using all of the vertices in the new graph that were created from vertex 2 as the starting point, and all of the vertices in the new graph that were created from vertex 8 as the ending point.

```
SELECT * FROM
  (SELECT * FROM
   (SELECT * FROM pgr_dijkstra($$SELECT seq AS id, * FROM lineGraph_edges$$,
      (SELECT array_agg(id) FROM lineGraph_vertices where original_id = 2),
      (SELECT array_agg(id) FROM lineGraph_vertices where original_id = 8)
      )) as shortestPaths
WHERE start_vid = 2 AND end_vid = 8 AND (cost != 0 OR edge = -1)) as b;
```

	- -	ct_vid end	_		•		_
29	+ 2	2	+- 8	+ -1	1	+- 1	0
31	4	2	8	-4	5	1	1
33	6 I	2	8	-10	25	1	2
35	8	2	8	-3	4	1	3
36	9	2	8	8	-1	0	4
(5 rows)							

Normally the shortest path from vertex 2 to vertex 8 would have an aggregate cost of 2, but since there is a large penalty for making the turn needed to get this cost, the route goes through vertex 6 to avoid this turn.

If you cross reference the node column in the dijkstra results with the vertex id mapping table, this will show you that the path goes from v2 -> v5 -> v6 -> v5 -> v8 in the original graph.

See Also \P

- $\bullet \ \ http://en.wikipedia.org/wiki/Line_graph$
- http://en.wikipedia.org/wiki/Complete graph

Indices and tables

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Introduction¶ This family of functions is used for transforming a given input graph $\backslash (G(V,E)\backslash)$ into a new graph $\backslash (G'(V',E')\backslash)$.

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Change $Log\P$

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- pgRouting 2.4.2 Release Notes
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- pgRouting 2.3.2 Release Notes
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Release Notes¶

To see the full list of changes check the list of Git commits on Github.

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- pgRouting 2.4.2 Release Notes

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pgRouting 2.6.2 Release Notes \P To see the issues closed by this release see the Git closed milestone for 2.6.2 on Github.

Bug fixes

- #1152 Fixes driving distance when vertex is not part of the graph
- #1098 Fixes windows test
- #1165 Fixes build for python3 and perl5

pgRouting 2.6.1 Release Notes¶ To see the issues closed by this release see the Git closed milestone for 2.6.1 on Github.

- Fixes server crash on several functions.
 - $\ \mathrm{pgr_floydWarshall}$
 - pgr_johnson
 - pgr_astar
 - pgr_bdAstar
 - pgr bdDijstra
 - pgr_alphashape
 - pgr_dijkstraCostMatrix
 - pgr_dijkstra
 - pgr_dijkstraCost
 - pgr_drivingDistance
 - pgr KSP
 - pgr_dijkstraVia (proposed)
 - pgr_boykovKolmogorov (proposed)
 - pgr_edgeDisjointPaths (proposed)

- pgr_edmondsKarp (proposed)
- pgr_maxCardinalityMatch (proposed)
- pgr_maxFlow (proposed)
- pgr withPoints (proposed)
- pgr_withPointsCost (proposed)
- pgr_withPointsKSP (proposed)
- pgr_withPointsDD (proposed)
- pgr_withPointsCostMatrix (proposed)
- pgr_contractGraph (experimental)
- pgr_pushRelabel (experimental)
- pgr_vrpOneDepot (experimental)
- pgr_gsoc_vrppdtw (experimental)
- Fixes for deprecated functions where also applied but not tested
- Removed compilation warning for g++8
- Fixed a fallthrugh on Astar and bdAstar.

pgRouting 2.6.0 Release Notes¶ To see the issues closed by this release see the Git closed milestone for 2.6.0 on Github.

New fexperimental functions

• pgr_lineGraphFull

Bug fixes

- Fix pgr_trsp(text,integer,double precision,integer,double precision,boolean[,text])
 - without restrictions
 - * calls pgr_dijkstra when both end points have a fraction IN (0,1)
 - * calls pgr_withPoints when at least one fraction NOT IN (0,1)
 - with restrictions
 - * calls original trsp code

Internal code

- Cleaned the internal code of trsp(text,integer,integer,boolean,boolean [, text])
 - Removed the use of pointers
 - Internal code can accept BIGINT
- Cleaned the internal code of withPoints

pgRouting 2.5.5 Release Notes¶ To see the issues closed by this release see the Git closed milestone for 2.5.5 on Github.

Bug fixes

- Fixes driving distance when vertex is not part of the graph
- Fixes windows test
- Fixes build for python3 and perl5

pgRouting 2.5.4 Release Notes¶ To see the issues closed by this release see the Git closed milestone for 2.5.4 on Github.

- Fixes server crash on several functions.
 - pgr_floydWarshall
 - pgr_johnson
 - pgr_astar
 - $pgr_bdAstar$
 - pgr_bdDijstra
 - pgr alphashape
 - pgr_dijkstraCostMatrix
 - pgr_dijkstra
 - pgr_dijkstraCost
 - pgr_drivingDistance
 - pgr_KSP
 - pgr_dijkstraVia (proposed)
 - pgr boykovKolmogorov (proposed)
 - pgr_edgeDisjointPaths (proposed)
 - pgr_edmondsKarp (proposed)
 - pgr_maxCardinalityMatch (proposed)
 - pgr_maxFlow (proposed)
 - pgr_withPoints (proposed)
 - pgr_withPointsCost (proposed)
 - pgr_withPointsKSP (proposed)
 - pgr_withPointsDD (proposed)
 - $-\ pgr_withPointsCostMatrix\ (proposed)$
 - pgr_contractGraph (experimental)
 - pgr_pushRelabel (experimental)
 - pgr_vrpOneDepot (experimental)
 - pgr_gsoc_vrppdtw (experimental)
 - Fixes for deprecated functions where also applied but not tested
- Removed compilation warning for g++8
- Fixed a fallthrugh on Astar and bdAstar.

pgRouting 2.5.3 Release Notes¶ To see the issues closed by this release see the Git closed milestone for 2.5.3 on Github.

Bug fixes

 Fix for postgresql 11: Removed a compilation error when compiling with postgre SQL

pgRouting 2.5.2 Release Notes¶ To see the issues closed by this release see the Git closed milestone for 2.5.2 on Github.

Bug fixes

• Fix for postgresql 10.1: Removed a compiler condition

pgRouting 2.5.1 Release Notes¶ To see the issues closed by this release see the Git closed milestone for 2.5.1 on Github.

Bug fixes

• Fixed prerequisite minimum version of: cmake

pgRouting 2.5.0 Release Notes¶ To see the issues closed by this release see the Git closed issues for 2.5.0 on Github.

enhancement:

 $\bullet\,$ pgr_version is now on SQL language

Breaking change on:

- $\bullet \hspace{0.1cm} pgr_edgeDisjointPaths:$
 - Added path_id, cost and agg_cost columns on the result
 - Parameter names changed
 - The many version results are the union of the one to one version

New Signatures:

• pgr_bdAstar(one to one)

New Proposed functions

• pgr_bdAstar(one to many)

- pgr_bdAstar(many to one)
- pgr_bdAstar(many to many)
- pgr_bdAstarCost(one to one)
- pgr_bdAstarCost(one to many)
- pgr_bdAstarCost(many to one)
- pgr_bdAstarCost(many to many)
- pgr_bdAstarCostMatrix
- pgr_bdDijkstra(one to many)
- pgr_bdDijkstra(many to one)
- pgr bdDijkstra(many to many)
- pgr bdDijkstraCost(one to one)
- pgr_bdDijkstraCost(one to many)
- pgr_bdDijkstraCost(many to one)
- pgr bdDijkstraCost(many to many)
- pgr bdDijkstraCostMatrix
- pgr_lineGraph
- pgr_lineGraphFull
- \bullet pgr_connectedComponents
- pgr_strongComponents
- pgr_biconnectedComponents
- pgr articulationPoints
- pgr_bridges

Deprecated Signatures

• pgr bdastar - use pgr bdAstar instead

Renamed Functions

- $pgr_maxFlowPushRelabel$ use $pgr_pushRelabel$ instead
- pgr_maxFlowEdmondsKarp -use pgr_edmondsKarp instead
- pgr_maxFlowBoykovKolmogorov use pgr_boykovKolmogorov instead
- pgr_maximumCardinalityMatching use pgr_maxCardinalityMatch instead

Deprecated function

• pgr pointToEdgeNode

pgRouting 2.4.2 Release Notes¶ To see the issues closed by this release see the Git closed milestone for 2.4.2 on Github.

Improvement

• Works for postgreSQL 10

Bug fixes

- Fixed: Unexpected error column "cname"
- Replace __linux__ with __GLIBC__ for glibc-specific headers and functions

pgRouting 2.4.1 Release Notes¶ To see the issues closed by this release see the Git closed milestone for 2.4.1 on Github.

Bug fixes

- Fixed compiling error on macOS
- Condition error on pgr_withPoints

pgRouting 2.4.0 Release Notes¶ To see the issues closed by this release see the Git closed issues for 2.4.0 on Github.

New Signatures

• pgr_bdDijkstra

New Proposed Signatures

- pgr_maxFlow
- pgr_astar(one to many)
- pgr_astar(many to one)
- pgr_astar(many to many)
- pgr_astarCost(one to one)
- pgr_astarCost(one to many)
- pgr_astarCost(many to one)pgr_astarCost(many to many)
- pgr_astarCostMatrix

Deprecated Signatures

- pgr_bddijkstra - use pgr_bdDijkstra instead

Deprecated Functions

• pgr_pointsToVids

Bug fixes

- Bug fixes on proposed functions
 - pgr_withPointsKSP: fixed ordering
- TRSP original code is used with no changes on the compilation warnings

pgRouting 2.3.2 Release Notes¶ To see the issues closed by this release see the Git closed issues for 2.3.2 on Github.

Bug Fixes

- Fixed pgr_gsoc_vrppdtw crash when all orders fit on one truck.
- Fixed pgr_trsp:
 - Alternate code is not executed when the point is in reality a vertex
 - Fixed ambiguity on seq

pgRouting 2.3.1 Release Notes \P To see the issues closed by this release see the Git closed issues for 2.3.1 on Github.

Bug Fixes

- Leaks on proposed max_flow functions
- $\bullet~$ Regression error on pgr_trsp
- Types discrepancy on pgr_createVerticesTable

pgRouting 2.3.0 Release Notes¶ To see the issues closed by this release see the Git closed issues for 2.3.0 on Github.

New Signatures

- pgr_TSP
- pgr_aStar

New Functions

• pgr_eucledianTSP

New Proposed functions

- pgr_dijkstraCostMatrix
- $\bullet \hspace{0.1cm} pgr_withPointsCostMatrix$

- pgr maxFlowPushRelabel(one to one)
- pgr_maxFlowPushRelabel(one to many)
- pgr_maxFlowPushRelabel(many to one)
- pgr maxFlowPushRelabel(many to many)
- pgr_maxFlowEdmondsKarp(one to one)
- pgr_maxFlowEdmondsKarp(one to many)
- pgr_maxFlowEdmondsKarp(many to one)
- pgr_maxFlowEdmondsKarp(many to many)
- pgr_maxFlowBoykovKolmogorov (one to one)
- pgr maxFlowBoykovKolmogorov (one to many)
- pgr maxFlowBoykovKolmogorov (many to one)
- pgr_maxFlowBoykovKolmogorov (many to many)
- pgr_maximumCardinalityMatching
- pgr_edgeDisjointPaths(one to one)
- pgr edgeDisjointPaths(one to many)
- pgr_edgeDisjointPaths(many to one)
- pgr_edgeDisjointPaths(many to many)
- pgr_contractGraph

Deprecated Signatures

- pgr_astar use pgr_aStar instead

Deprecated Functions

- pgr_flip_edges
- pgr_vidsToDmatrix
- pgr_pointsToDMatrix
- pgr_textToPoints

pgRouting 2.2.4 Release Notes¶ To see the issues closed by this release see the Git closed issues for 2.2.4 on Github.

Bug Fixes

- Bogus uses of extern "C"
- Build error on Fedora 24 + GCC 6.0
- Regression error pgr_nodeNetwork

pgRouting 2.2.3 Release Notes¶ To see the issues closed by this release see the Git closed issues for 2.2.3 on Github.

Bug Fixes

• Fixed compatibility issues with PostgreSQL 9.6.

pgRouting 2.2.2 Release Notes¶ To see the issues closed by this release see the Git closed issues for 2.2.2 on Github.

Bug Fixes

• Fixed regression error on pgr_drivingDistance

pgRouting 2.2.1 Release Notes \P To see the issues closed by this release see the Git closed issues for 2.2.1 on Github.

Bug Fixes

- Server crash fix on pgr alphaShape
- Bug fix on With Points family of functions

pgRouting 2.2.0 Release Notes¶ To see the issues closed by this release see the Git closed issues for 2.2.0 on Github.

Improvements

- pgr_nodeNetwork
 - Adding a row_where and outall optional parameters
- Signature fix
 - pgr_dijkstra to match what is documented

New Functions

- $\bullet \quad pgr_floydWarshall$
- pgr_Johnson
- pgr_dijkstraCost(one to one)
- pgr_dijkstraCost(one to many)
- pgr_dijkstraCost(many to one)
- pgr dijkstraCost(many to many)

Proposed functionality

- pgr_withPoints(one to one)
- pgr_withPoints(one to many)
- pgr_withPoints(many to one)
- pgr withPoints(many to many)
- pgr_withPointsCost(one to one)
- pgr_withPointsCost(one to many)
- pgr_withPointsCost(many to one)
- pgr_withPointsCost(many to many)
- pgr_withPointsDD(single vertex)
- pgr withPointsDD(multiple vertices)
- pgr withPointsKSP
- pgr_dijkstraVia

Deprecated functions:

- $pgr_apspWarshall$ use $pgr_floydWarshall$ instead
- pgr_apspJohnson use pgr_Johnson instead
- pgr_kDijkstraCost use pgr_dijkstraCost instead
- pgr_kDijkstraPath use pgr_dijkstra instead

Renamed and deprecated function

• pgr_makeDistanceMatrix renamed to _pgr_makeDistanceMatrix

pgRouting 2.1.0 Release Notes¶ To see the issues closed by this release see the Git closed issues for 2.1.0 on Github.

New Signatures

- pgr_dijkstra(one to many)
- pgr_dijkstra(many to one)
- pgr_dijkstra(many to many)
- pgr drivingDistance(multiple vertices)

Refactored

- pgr dijkstra(one to one)
- \bullet pgr_ksp
- pgr_drivingDistance(single vertex)

Improvements

• pgr_alphaShape function now can generate better (multi)polygon with holes and alpha parameter.

Proposed functionality

- Proposed functions from Steve Woodbridge, (Classified as Convenience by the author.)
 - pgr_pointToEdgeNode convert a point geometry to a vertex_id based on closest edge.
 - pgr_flipEdges flip the edges in an array of geometries so the connect end to end.
 - pgr_textToPoints convert a string of x,y;x,y;... locations into point geometries.
 - pgr_pointsToVids convert an array of point geometries into vertex ids
 - pgr_pointsToDMatrix Create a distance matrix from an array of points.
 - pgr_vidsToDMatrix Create a distance matrix from an array of vertix id.
 - pgr_vidsToDMatrix Create a distance matrix from an array of vertix_id.
- Added proposed functions from GSoc Projects:
 - pgr_vrppdtw
 - pgr_vrponedepot

Deprecated functions

- pgr_getColumnName
- $\bullet \quad pgr_getTableName$
- pgr isColumnCndexed
- pgr_isColumnInTable
- $\bullet \hspace{0.1in} pgr_quote_ident$
- \bullet pgr_versionless
- \bullet pgr_startPoint
- \bullet pgr_endPoint
- pgr_pointToId

No longer supported

• Removed the 1.x legacy functions

Bug Fixes

• Some bug fixes in other functions

Refactoring Internal Code

- A C and C++ library for developer was created
 - -encapsulates postgre SQL related functions
 - encapsulates Boost.Graph graphs
 - * Directed Boost.Graph
 - * Undirected Boost.graph.
 - allow any-integer in the id's
 - allow any-numerical on the cost/reverse cost columns
- Instead of generating many libraries: All functions are encapsulated in one library The library has the prefix 2-1-0

pgRouting 2.0.1 Release Notes¶ Minor bug fixes.

Bug Fixes

• No track of the bug fixes were kept.

pgRouting 2.0.0 Release Notes¶ To see the issues closed by this release see the Git closed issues for 2.0.0 on Github.

With the release of pgRouting 2.0.0 the library has a bandoned backwards compatibility to $pgRouting\ 1.x$ releases. The main Goals for this release are:

- Major restructuring of pgRouting.
- Standardization of the function naming
- Preparation of the project for future development.

As a result of this effort:

- pgRouting has a simplified structure
- Significant new functionality has being added
- Documentation has being integrated
- Testing has being integrated
- And made it easier for multiple developers to make contributions.

Important Changes

• Graph Analytics - tools for detecting and fixing connection some problems in a graph

- A collection of useful utility functions
- Two new All Pairs Short Path algorithms (pgr_apspJohnson, pgr_apspWarshall)
- Bi-directional Dijkstra and A-star search algorithms (pgr_bdAstar, pgr_bdDijkstra)
- One to many nodes search (pgr_kDijkstra)
- K alternate paths shortest path (pgr ksp)
- New TSP solver that simplifies the code and the build process (pgr_tsp), dropped "Gaul Library" dependency
- Turn Restricted shortest path (pgr_trsp) that replaces Shooting Star
- Dropped support for Shooting Star
- Built a test infrastructure that is run before major code changes are checked in
- Tested and fixed most all of the outstanding bugs reported against 1.x that existing in the 2.0-dev code base.
- Improved build process for Windows
- Automated testing on Linux and Windows platforms trigger by every commit
- Modular library design
- Compatibility with PostgreSQL 9.1 or newer
- Compatibility with PostGIS 2.0 or newer
- Installs as PostgreSQL EXTENSION
- Return types re factored and unified
- Support for table SCHEMA in function parameters
- Support for st_ PostGIS function prefix
- Added pgr_ prefix to functions and types
- Better documentation: http://docs.pgrouting.org
- shooting_star is discontinued

pgRouting 1.x Release Notes¶ To see the issues closed by this release see the Git closed issues for 1.x on Github. The following release notes have been copied from the previous RELEASE_NOTES file and are kept as a reference.

Changes for release 1.05¶

• Bug fixes

Changes for release 1.03¶

- Much faster topology creation
- Bug fixes

Changes for release 1.02¶

- Shooting* bug fixes
- Compilation problems solved

Changes for release $1.01\P$

• Shooting* bug fixes

Changes for release $1.0\P$

- Core and extra functions are separated
- Cmake build process
- Bug fixes

Changes for release 1.0.0b¶

- Additional SQL file with more simple names for wrapper functions
- Bug fixes

Changes for release 1.0.0a¶

- Shooting* shortest path algorithm for real road networks
- Several SQL bugs were fixed

Changes for release 0.9.9¶

- PostgreSQL 8.2 support
- Shortest path functions return empty result if they could not find any path

Changes for release $0.9.8\P$

- Renumbering scheme was added to shortest path functions
- Directed shortest path functions were added
- $\bullet\,$ routing_postgis.sql was modified to use dijkstra in TSP search

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